

Probability Distribution Transformation in Continuous Production Control

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Introduction

Continuous inter-operational quality control is commonly applied in manufacture process of mechatronics (especially electronic) products. Selective control peculiarities of such products are analyzed in publications [1–5] on the grounds of color picture tube manufacture specifics. In this paper we will describe the performance of multistage continuous inter-operational control with the help of stochastic models, when production classification errors of the first and second kind are present. Main attention is paid to the transformation of production defectivity level probability distributions, which in turn allows to estimate the efficiency of inter-operational control in the way of modeling, and to select the required number of control stages and their characteristics.

Control scheme and models with fixed values

Inter-operational control fragment is presented in Fig. 1, which involves two stages of continuous control K_1 and K_2 (in both stages products are classified according to analogical decision rules).

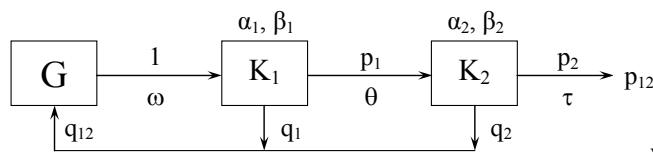


Fig. 1. The scheme of two-stage inter-operational continuous control

There is a probability ω that a defective product after manufacture operations G will enter the control stage K_1 which is characterized by classification error probabilities α_1 and β_1 ; probability ω is transformed to parameter θ after control operations. Product acceptance probability is p_1 and rejection probability is q_1 .

Analogously in the second stage K_2 with errors α_2 and β_2 parameter θ is transformed to τ , when probabilities p_2 , q_2 are fixed. Product in such scheme is accepted with probability q_{12} . Rejected products are returned for reparation to manufacture process G (specifically such scheme is used in manufacture of color picture tubes and their components).

According to models created in the work [6], we receive such direct and reverse dependencies, which are needed for the further analysis:

$$\left\{ \begin{array}{l} p_1 = p_1(\omega) = 1 - \alpha_1 - (1 - \alpha_1 - \beta_1)\omega = \\ = (1 - \alpha_1)[1 - (1 - \tilde{\beta}_1)\omega] \\ p_2 = p_2(\theta) = (1 - \alpha_2)[1 - (1 - \tilde{\beta}_2)\theta] \\ p_{12} = p_1 p_2; \end{array} \right. \quad (1)$$

$$\text{where } \tilde{\beta}_i = \frac{\beta_i}{1 - \alpha_i}, \quad i = 1, 2;$$

$$q_1 = 1 - p_1, \quad q_2 = 1 - p_2, \quad q_{12} = q_1 + p_1 q_2 = 1 - p_{12}; \quad (2)$$

$$\begin{cases} \theta = \theta(\omega) = \frac{\beta_1 \omega}{p_1(\omega)} = \frac{\tilde{\beta}_1 \omega}{1 - (1 - \tilde{\beta}_1)\omega}, \\ \tau = \tau(\theta) = \frac{\beta_2 \theta}{p_2(\theta)} = \frac{\tilde{\beta}_2 \theta}{1 - (1 - \tilde{\beta}_2)\theta}, \\ \tau = \tau(\omega) = \frac{\tilde{\beta}_1 \tilde{\beta}_2 \omega}{1 - (1 - \tilde{\beta}_1 \tilde{\beta}_2)\omega} = \frac{\tilde{\beta}_{12} \omega}{1 - (1 - \tilde{\beta}_{12})\omega}; \end{cases} \quad (3)$$

where $\tilde{\beta}_{12} = \tilde{\beta}_1 \tilde{\beta}_2$ – generalized two-stage control transformation constant;

$$\begin{cases} \theta = \theta(\tau) = \frac{\tau}{\tilde{\beta}_2 + (1 - \tilde{\beta}_2)\tau}, \\ \omega = \omega(\theta) = \frac{\theta}{\tilde{\beta}_1 + (1 - \tilde{\beta}_1)\theta}, \\ \omega = \omega(\tau) = \frac{\tau}{\tilde{\beta}_{12} + (1 - \tilde{\beta}_{12})\tau}. \end{cases} \quad (4)$$

It is easy to ascertain, that when the third control stage with errors α_3 , β_3 is introduced, we have $\tilde{\beta}_{13} = \tilde{\beta}_1 \tilde{\beta}_2 \tilde{\beta}_3$, and in general case the generalized transformation constant $\tilde{\beta}_{1S}$ for control scheme consisting of S stages is

$$\tilde{\beta}_{1S} = \prod_{i=1}^S \tilde{\beta}_i, \quad i = 1 - s. \quad (5)$$

In separate occurrence, when $\tilde{\beta}_1 = \tilde{\beta}_2 = \dots = \tilde{\beta}_S = \tilde{\beta}$, we have $\tilde{\beta}_{1S} = \tilde{\beta}^S$.

Models with random values

The defective product probability in the entirety of products, as a random value [3], before control K_1 is characterized by density $f(\omega)$ (see Fig. 2), before K_2 – by density $g(\theta)$ and after control K_2 – by density $h(\tau)$.

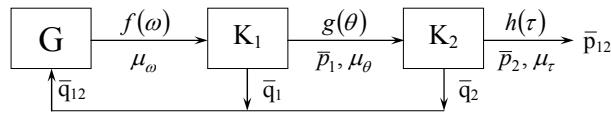


Fig. 2. Stochastic characteristics of two-stage control

The means of defectivity level respectively are μ_ω , μ_θ , μ_τ :

$$\mu_\omega = \int_0^1 \omega f(\omega) d\omega, \quad \mu_\theta = \int_0^1 \theta g(\theta) d\theta, \quad \mu_\tau = \int_0^1 \tau h(\tau) d\tau. \quad (6)$$

According to (1) and (6) we receive average sizes of accepted flows $\bar{p}_1, \bar{p}_2, \bar{p}_{12}$, and according to (2) – average flows of rejected products $\bar{q}_1, \bar{q}_2, \bar{q}_{12}$:

$$\begin{cases} \bar{p}_1 = (1 - \alpha_1) [1 - (1 - \tilde{\beta}_1) \mu_\omega], \\ \bar{p}_2 = (1 - \alpha_2) [1 - (1 - \tilde{\beta}_2) \mu_\theta], \\ \bar{p}_{12} = \bar{p}_1 \bar{p}_2; \end{cases} \quad (7)$$

$$\bar{q}_1 = 1 - \bar{p}_1, \quad \bar{q}_2 = 1 - \bar{p}_2, \quad \bar{q}_{12} = 1 - \bar{p}_{12}. \quad (8)$$

Assume, that $g(\theta)$ is the density of beta distribution [3] with parameters $a, b (a \geq 1, b \geq 1)$.

$$g(\theta) = B^{-1}(a, b) \theta^{a-1} (1-\theta)^{b-1}, \quad 0 \leq \theta \leq 1, \quad (9)$$

where $B(a, b) = \int_0^1 \theta^{a-1} (1-\theta)^{b-1} d\theta = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ – beta function, $\Gamma(z)$ – gamma function. When z is whole positive number, we have $\Gamma(z) = (z-1)!$.

The mean μ_θ and dispersion σ_θ^2 of defectivity level equal [7]

$$\mu_\theta = \frac{a}{a+b}, \quad \sigma_\theta^2 = \frac{\mu_\theta(1-\mu_\theta)}{a+b+1}. \quad (10)$$

In this case density $h(\tau)$ is directly transformed density $g(\theta)$ of (T) with transformation coefficient (constant) $\tilde{\beta}_2$, and $f(\omega)$ - conversely transformed mean $g(\theta)$ of (A) with transformation constant $\tilde{\beta}_1$:

$$\mathbf{T:} h(\tau) = |\theta'(\tau)| g[\theta(\tau)] =$$

$$= \frac{\tilde{\beta}_2^b \tau^{a-1} (1-\tau)^{b-1}}{B(a,b) [\tilde{\beta}_2 + (1 - \tilde{\beta}_2)\tau]^{a+b}}, \quad 0 \leq \tau \leq 1; \quad (11)$$

where $|\theta'(\tau)| = \left| \frac{\partial \theta(\tau)}{\partial \tau} \right| = \frac{\tilde{\beta}_2}{[\tilde{\beta}_2 + (1 - \tilde{\beta}_2)\tau]^2}$, $\theta(\tau)$ - keeping

to (4), $g[\theta(\tau)] = \frac{1}{B(a,b)} [\theta(\tau)]^{a-1} [1 - \theta(\tau)]^{b-1}$;

$$\mathbf{A:} f(\omega) = |\theta'(\omega)| g[\theta(\omega)] = \frac{\tilde{\beta}_1^a \omega^{a-1} (1-\omega)^{b-1}}{B(a,b) [1 - (1 - \tilde{\beta}_1)\omega]^{a+b}}; \quad 0 \leq \omega \leq 1; \quad (12)$$

where $|\theta'(\omega)| = \frac{\tilde{\beta}_1}{[1 - (1 - \tilde{\beta}_1)\omega]^2}$, $\theta(\omega)$ – keeping to (3).

Densities $h(\tau)$ and $f(\omega)$ are called the second rate beta densities [8]. Maximum points (modes) of densities θ_M , ω_M , τ_M are equal:

$$\left\{ \begin{array}{l} \theta_M = \frac{a-1}{a+b-1}, \\ \omega_M = \frac{1}{4\tilde{\gamma}} \left\{ \sqrt{[a+b-2-(b+1)\tilde{\gamma}]^2 + 8\tilde{\gamma}(a-1)} - \right. \\ \left. - [a+b-2-(b+1)\tilde{\gamma}] \right\}, \\ \tau_M = \frac{1}{4c} \left\{ \frac{[a+b-2+(b+1)c] -}{\sqrt{-[a+b-2+(b+1)c]^2 - 8c(a-1)}} \right\}; \end{array} \right. \quad (13)$$

where $\tilde{\gamma} = 1 - \tilde{\beta}$; $c = \frac{1}{\tilde{\beta}} - 1$.

The means μ_ω, μ_τ , differently from (6), are convenient to express as a functions of random values using density $g(\theta)$:

$$\left\{ \begin{array}{l} \mu_\omega = \int_0^1 \omega(\theta)g(\theta)d\theta = \frac{1}{B(a,b)} \int_0^1 \frac{\theta^a(1-\theta)^{b-1}}{\tilde{\beta}_1 + (1 - \tilde{\beta}_1)\theta} d\theta, \\ \mu_\tau = \int_0^1 \tau(\theta)g(\theta)d\theta = \frac{\tilde{\beta}_2}{B(a,b)} \int_0^1 \frac{\theta^a(1-\theta)^{b-1}}{1 - (1 - \tilde{\beta}_2)\theta} d\theta. \end{array} \right. \quad (14)$$

Dispersions also $\sigma_\omega^2, \sigma_\tau^2$ can be described analogously:

$$\left\{ \begin{array}{l} \sigma_\omega^2 = \int_0^1 [\omega(\theta)]^2 g(\theta)d\theta - \mu_\omega^2, \\ \sigma_\tau^2 = \int_0^1 [\tau(\theta)]^2 g(\theta)d\theta - \mu_\tau^2. \end{array} \right. \quad (15)$$

It is obvious, that the reverse transformation A or direct transformation T can be multi-stage in general case. Let's say, that we are interested in direct transformation of S stages $T = T(T_1, T_2, \dots, T_S)$, i.e. the stage K_1 (Fig. 2) does not function, and K_2 is formed of S control stages K'_1, K'_2, \dots, K'_S connected in series (Fig. 3).

It is sufficient to use only whole number values of parameters a and b ($a \geq 1, b \geq 1$) during the modeling [3, 4].

After integration of (14) we have

$$\left\{ \begin{array}{l} \mu_\omega = \frac{(a+b-1)!}{(a-1)!(b-1)!} \cdot \frac{\tilde{\beta}_1}{(1 - \tilde{\beta}_1)^{a+1}} \sum_{i=0}^{b-1} C_{b-1}^i (-1)^i \left(\frac{\tilde{\beta}_1}{1 - \tilde{\beta}_1} \right)^i \\ \cdot \left[\sum_{j=1}^{a+i} C_{a+i}^j (-1)^{a+i-j} \frac{1 - \tilde{\beta}_1^j}{j \tilde{\beta}_1^j} + (-1)^{a+i} \ln \frac{1}{\tilde{\beta}_1} \right], \\ \mu_\tau = \frac{(a+b-1)!}{(a-1)!(b-1)!} \cdot \frac{\tilde{\beta}_2}{(1 - \tilde{\beta}_2)^{a+1}} \sum_{i=0}^{b-1} C_{b-1}^i (-1)^i \left(\frac{-1}{1 - \tilde{\beta}_2} \right)^i \\ \cdot \left[\sum_{j=1}^{a+i} C_{a+i}^j (-1)^j \frac{1 - \tilde{\beta}_2^j}{j} + \ln \frac{1}{\tilde{\beta}_2} \right]; \end{array} \right. \quad (16)$$

where $B(a,b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$, $C_n^m = \frac{n!}{m!(n-m)!}$ – combinations from n taken m at a time.

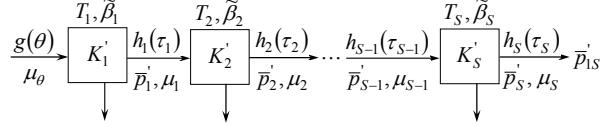


Fig. 3. Direct transformation T : S-stage scheme

Then after the S-stage we receive:

$$h_S(\tau_S) = |\theta'(\tau_S)| g[\theta(\tau_S)] = \frac{\tilde{\beta}_{1S}^b \tau_S^{a-1} (1 - \tau_S)^{b-1}}{B(a,b) [\tilde{\beta}_{1S} + (1 - \tilde{\beta}_{1S}) \tau_S]^{(a+b)}},$$

$$0 \leq \tau_S \leq 1; \quad (17)$$

where

$$\theta(\tau_S) = \frac{\tau_S}{\tilde{\beta}_{1S} + (1 - \tilde{\beta}_{1S}) \tau_S}, \quad \theta'(\tau_S) = \frac{\tilde{\beta}_{1S}}{[\tilde{\beta}_{1S} + (1 - \tilde{\beta}_{1S}) \tau_S]^2},$$

$\tilde{\beta}_{1S}$ - keeping to (5), $S=1, 2, 3\dots$;

$$\mu_S = \int_0^1 \tau_S(\theta)g(\theta)d\theta = \frac{\tilde{\beta}_{1S}}{B(a,b)} \int_0^1 \frac{\theta^a(1-\theta)^{b-1}}{1 - (1 - \tilde{\beta}_{1S})\theta} d\theta; \quad (18)$$

$$\text{where } \tau_S(\theta) = \frac{\tilde{\beta}_{1S}\theta}{1 - (1 - \tilde{\beta}_{1S})\theta};$$

$$\left\{ \begin{array}{l} \bar{p}'_S = (1 - \alpha_S) [1 - (1 - \tilde{\beta}_{1S}) \mu_{S-1}] \mu_o = \mu_\theta, \text{ if } S = 1; \\ \bar{p}'_{1S} = \prod_{i=1}^S \bar{p}'_i = \bar{p}'_1 \bar{p}'_2 \dots \bar{p}'_S; \end{array} \right. \quad (19)$$

Practical implementations

Example 1: Control scheme according to Fig. 2 with error probabilities $\alpha_1 = 0.12$, $\beta_1 = 0.22$ (A) and $\alpha_2 = 0.08$, $\beta_2 = 0.23$ (T). Parameters of densities: $a = 2, b = 3$.

Modeling results:

$$\mu_\omega = \frac{1}{1-\tilde{\beta}} \left\{ 1 - \frac{4\tilde{\beta}^4}{(1-\tilde{\beta})^4} \left[\begin{array}{l} \frac{1}{\tilde{\beta}_3} \left(1 - 3\tilde{\beta} \ln \frac{1}{\tilde{\beta}} \right) + \\ + \frac{3}{2} \left(\frac{\tilde{\beta}}{1-\tilde{\beta}} \right)^2 - 1 \end{array} \right] \right\},$$

$$\mu_\tau = \frac{\tilde{\beta}}{1-\tilde{\beta}} \left\{ \frac{4}{(1-\tilde{\beta})^4} \left[\tilde{\beta}^2 \left(\tilde{\beta} + 3 \ln \frac{1}{\tilde{\beta}} \right) + \frac{3}{2} (1-\tilde{\beta})^2 - 1 \right] - 1 \right\},$$

$$g(\theta) = 12\theta(1-\theta)^2, \mu_\theta = 0.4, \theta_M = \frac{1}{3}, \tilde{\beta}_1 = \tilde{\beta}_2 = \tilde{\beta} = \frac{1}{4};$$

$$A: f(\omega) = \frac{768\omega(1-\omega)^2}{(4-3\omega)^5}, \mu_\omega = 0.6777, \omega_M = 0.8167;$$

$$T: h(\tau) = \frac{192\tau(1-\tau)^2}{(1+3\tau)^5}, \mu_\tau = 0.1694, \tau_M = 0.0686;$$

$$\bar{p}_1 = 0.4327, \bar{p}_2 = 0.644, \bar{p}_{12} = 0.2787.$$

Densities $f(\omega)$, $g(\theta)$, $h(\tau)$ are presented in Fig. 4.

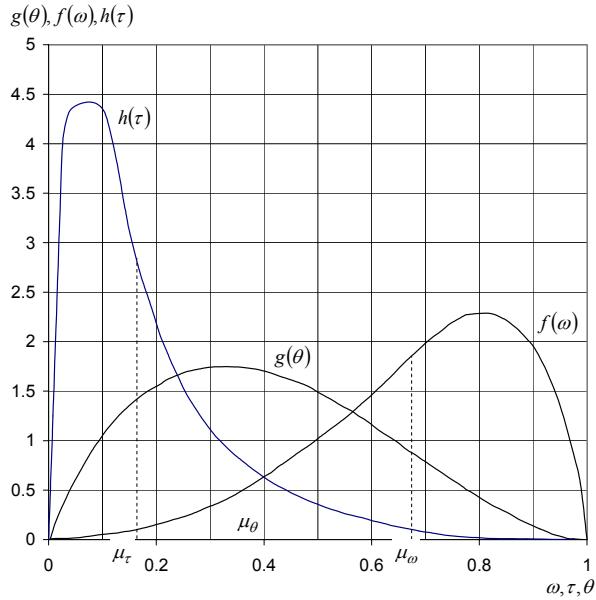


Fig. 4. Densities of defectivity levels, when $a = 2$, $b = 3$,

$$\tilde{\beta} = 0.25$$

Example 2: Control scheme according to Fig. 3 when $S = 1, 2, 3$. In all stages $\alpha = 0.2$, $\beta = 0.4$; $a = b = 2$.

Modeling results:

$$\tilde{\beta} = \frac{1}{2}, \tilde{\beta}_{12} = \frac{1}{4}; \tilde{\beta}_{13} = \tilde{\beta}^3 = \frac{1}{8};$$

$$\mu_S = \frac{\tilde{\beta}^S}{1-\tilde{\beta}^S} \left\{ \frac{3}{(1-\tilde{\beta}^S)^2} \left[1 - \tilde{\beta}^S \left(\frac{2}{1-\tilde{\beta}^S} \ln \frac{1}{\tilde{\beta}^S} - 1 \right) \right] - 1 \right\},$$

$$S = 1, 2, 3;$$

$$g(\theta) = 6\theta(1-\theta); \mu_\theta = \theta_M = \frac{1}{2};$$

$$T_1: h_1(\tau_1) = \frac{24\tau_1(1-\tau_1)}{(1+\tau_1)^4}, \mu_1 = 0.3645, \tau_{1M} = 0.2192;$$

$$T_2: h_2(\tau_2) = \frac{96\tau_2(1-\tau_2)}{(1+3\tau_2)^4}, \mu_2 = 0.2459, \tau_{2M} = 0.0959;$$

$$T_3: h_3(\tau_3) = \frac{384\tau_3(1-\tau_3)}{(1+7\tau_3)^4}, \mu_3 = 0.1543, \tau_{3M} = 0.0447;$$

$$\bar{p}'_1 = 0.6; \bar{p}'_2 = 0.6542, \bar{p}'_3 = 0.7016;$$

$$\bar{p}'_{12} = 0.3925; \bar{p}'_{13} = 0.2754.$$

Densities $g(\theta)$, $h_1(\tau_1)$, $h_2(\tau_2)$, $h_3(\tau_3)$ are presented in Fig. 5.

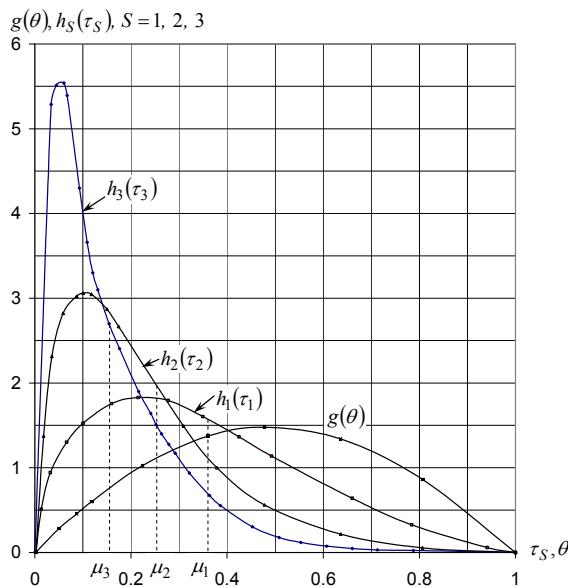


Fig. 5. Densities of defectivity levels, when $a = b = 2$, $\tilde{\beta} = \frac{1}{2}$

Conclusions

1. Received expressions give opportunity to model various desirable situations in the scheme of inter-operational control in visual manner, according to transformations of density of defectivity levels, by defining density parameters at desired point of scheme (density of beta distribution) and selecting real error probability values of separate control stages.

2. Main criterion during selection of number of control stages is the average defectivity level after control, which is critically influenced by the generalized error probability $\tilde{\beta}$ and the defectivity level before control. Average flow size of accepted products show, what part of products return to the technological process for regeneration; this parameter determines the flow of repeatedly “rotating” products in the technological process and losses caused by that.

3. The generalized transformation constant of control scheme is equal to the product of generalized error probabilities of separate stages; that allows to apply the same models in separate stages, after replacing the transformation constant into product of constants or raising to the power if constants are equal.

4. The prospective direction of inter-operational control analysis is the creation of models for the scheme of multi-parametric control scheme with different decision rules at the separate stages.

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R. Kalnius, A. Vaišvila, D. Eidukas. Probability Distribution Transformation in Continuous Production Control // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006.– No.4(68). – P. 29–34.

Mathematical models of main stochastic characteristics of continuous multi-stage inter-operational control for the products of mechatronics were created, when the flows of rejected products are returned to the manufacture process for regeneration, and products classification rules are analogous at the separate control stages, and when undeniable product classification errors of the first and second kind are present (good products are rejected and defective products are accepted as good). The efficiency of control operation is estimated in the way of modeling according to transformations of density functions of defectivity level at separate stages, by applying beta distribution and dynamics of defectivity level mean variation. It was shown that returning flows are determined by error probabilities and the level of defectivity before control, and the models of separate stages differ only in the magnitude of generalized error, which equals the product of generalized error probabilities of separate stages. Manufacturing peculiarities of picture tubes and their components were employed during creation of models. Ill. 5, bibl. 8 (in English; summaries in English, Russian and Lithuanian).

Р. Кальниус, А. Вайшвила, Д. Эйдукас. Трансформация вероятностных распределений на сплошном контроле изделий // Электроника и электротехника. – Каунас: Технология, 2006. – С. 4(68). – С. 29–34.

Представлены математические модели основных вероятностных характеристик многоступенчатого сплошного межоперационного контроля мехатронных изделий, когда потоки забракованных изделий возвращаются в производственный процесс для регенерации и правила классификации изделий одинаковы для всех ступеней контроля при наличии существенных ошибок классификации (годные изделия бракуются, а дефектные – признаются годными). Эффективность функционирования контроля оценивается путем моделирования на основе трансформации функции плотности вероятностей уровня дефектности на отдельных ступенях контроля, используя бета распределение, а также по динамике изменения среднего уровня дефектности. Показано, что обратные потоки обусловлены вероятностями ошибок и уровнем дефектности потока, поступающего на контроль, а модели для отдельных ступеней различаются только величиной обобщенной ошибки, вероятность которой равна перемножению вероятностей обобщенных ошибок отдельных ступеней. Для создания моделей были использованы особенности производства кинескопов и их основных компонентов. Ил. 5, библ. 8 (на английском, русском и литовском яз.).

R. Kalnius, A. Vaišvila, D. Eidukas. Tikimybų skirstinių transformacija gaminų ištisinėje kontrolėje // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 4(68). – P. 29-34.

Sudaryti ištisinės daugiapakopės tarpoperacinės kontrolės pagrindinių tikimybinių charakteristikų matematiniai modeliai mechatroniniams gaminiams, kai išbrokuotų gaminų srautai grąžinami į gamybos procesą regeneracijai, o gaminų klasifikavimo taisyklės atskirose pakopose analogiškos, esant nepaneigtinoms gaminų klasifikavimo pirmos ir antros rūšies klaidoms (išbrokuojami geri gaminiai ir pripažintami gerais defektiniais gaminiais). Kontrolės funkcionalumas efektyvumas vertinamas modeliavimo būdu pagal defektingumo lygio tankio transformacijas atskirose pakopose, taikant beta skirstinį bei defektingumo lygiu prieš kontrolę, o atskirų pakopų modeliai skiriasi tik apibendrintos klaidos dydžiu, kuri lygi atskirų pakopų apibendrintų kladų tikimybų sandaugai. Modelių sudarymui buvo pasinaudota kineskopų ir jų komponenčių gamybos proceso ypatumais. Il. 5, bibl. 8, (lietuvių kalba; santraukos anglų, rusų ir lietuvių k.).