

## Optimal Interval of Pulse and Breathing Signal Formation

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### Introduction

There is the kind of workers those who work alone and whose actions influence on safety of the other workers or technological processes and can result huge economical waste. There is a need to equip these workers with a physiological parameter monitoring system. It is particularly relevant in fabrication with continuous technological cycles when attendant people quantity is minimal. Requirements for the measurement reliability and speed for such monitoring systems are high. Unreliable measurement results or insufficient speed of receiving of the information about the important parameters could preclude from establishing fast health impairment.

Kaunas University of Technology (KTU) has developed human physiological parameter monitoring system which is implemented as wearable device and continuously monitors two of the main parameters for human physiological state evaluation: heart rate and respiration rate [1]. Physical nature of the parameters is such, that measurement time for these parameters is quite long – from part of the second to few tenths seconds. A lot of noise passes to the input of measurement blocks. Sensors of these parameters are wearable and human motions induce high noise levels.

Therefore, there is a need of the theoretical research of the problem how to process optimally these signals in such way that parameters would be measured enough precisely as they vary a little but sudden and significant change or overlapping limits of the parameters would be captured as fast as possible.

### Structural scheme of the signal processing

*Measurement signal processing path.* Heart rate and respiration rate will be input values in the optimization problem formulation and will be noted  $x(t) = f(t)$ . Technically, input value will be considered as a continuous value. Heart rate or respiration rate could change in any

time because of the changes of biophysical or biochemical processes in the human organism. However, frequency signal always is measured in a discrete way. Impulse is formed in measurement transducers in the specific phase of heart systole or inhalation/exhalation phase in the handling case. Consequently both of these input values  $x(t)$  are converted to the time interval  $T_k$ , between the two consecutive time moments  $t_{k-1}$  and  $t_k$  by a measurement transducer.  $t_{k-1}$  and  $t_k$  are the time moments when two consecutive analogical heart systole or inhalation/exhalation phases repeats:

$$T_k = t_k - t_{k-1}, f_k = f(t_k) = 1/T_k. \quad (1)$$

Disturbances are present in measurement path for real conditions.

*Disturbances that take affect on a signal.* Measurement transducers mounted in clothing are affected by different noise. The main sources of the noise are: 1) noise which affect the processes, from which the measured values are distinguished (electrocardiogram or thorax motion in the area where measurement belt is placed); 2) noise that appear in the sensor inputs during human motion; 3) disturbances caused by the near acting devices; 4) period duration evaluation error; 5) sampling error. Reduced to a signal noise value will be noted  $\xi(t)$ . We will assume the noise are additive to the input signal  $x(t)$ .

Signal processing could be represented by the structural scheme shown in the figure 1. Signals  $x(t)$  and  $\xi(t)$  are summed in a summation unit SE. The sum passes to the input of the impulse modulator. Discrete signal  $N_x(t_k)$  proportional to the time interval  $T_k$  is formed in the time moments  $t_k$ ,  $k = 1, 2, \dots, n-1, n$ , in the output of the modulator. Impulse modulator IM controls measurement transducer MT. Digital signal  $N_x(t_k)$  proportional to the time interval  $T_k$  is obtained in the measurement transducer:

$$N_x(t_k) = E\{MT_k\}; \quad (2)$$

here  $M = const$ ,  $E\{\dots\}$  – is an integer part of the a real number present in a brackets. The signal passes to the signal processing block SAB, where the digital signal  $N_y$  proportional to the input signal  $x(t)$  is formed.

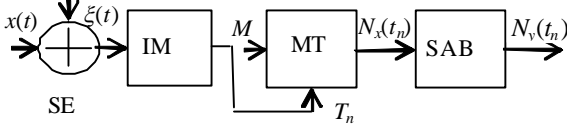


Fig. 1. Block scheme of the input value measurement

### Problem formulation

Further we will research how to process the signal  $x(t)$  which is influenced by the noise  $\xi(t)$  optimally in such way that if the signal is almost constant measurement precision continually would become better and if change of the signal appears, it will be noticed on time.

The impulse modulator IM shown in figure 1 is the frequency generator. Such modulator is nonlinear because of the reasons: because of the salutatory work of the impulse modulator and because of the nonlinear relationship between the input value and the period. It makes the synthesis of the optimal filter for a general case inconvenient. However, IM could be considered as a linear for the particular cases.

Suppose average value of a measured parameter is  $x(t) = x_0$  in the normal conditions. Time interval  $T_n = T_{nid}$  is influenced just by the input value  $x(t)$  for an ideal case. Then we will get the value in the measurement transducer:

$$N_{x0} = MT_0 = \frac{M}{x_0}. \quad (3)$$

Optimization problem will be solved for the case when an input signal changes in a negligible range:

$$x(t) = x_0 + \Delta x + \xi, \quad \Delta x/x_0 \ll 1, \quad \xi/x_0 \ll 1. \quad (4)$$

This regime will be called static. Output signal of the impulse modulator in the static regime by applying the equations (2) and (4) could be expressed:

$$\Delta\varphi_n = \frac{M}{x_0 + \Delta x + \xi} - \frac{M}{x_0} \approx -\frac{M}{x_0^2} \cdot (\Delta x + \xi). \quad (5)$$

We could assume that the period of the modulator is constant  $T_k \approx T_0$  in this regime. Then  $t_k = kT_0$ .

The signal processing block SAB must be designed in such way that impact of the  $\xi(t)$  would be maximally damped. Therefore, it is also very important that the fast changes of the inputs would be transferred to the output of the system as fast as possible and as precisely as possible.

Noise is stochastic process, so the system is also stochastic. Therefore, further we will investigate a synthesis problem of the design of the correction device which would be optimal in a sense of a noise damping and in a sense of quick acting in a stochastic system. First we will investigate a synthesis problem of the design of the correction device which would be optimal in a sense of a

noise damping. This problem is also known as a problem of optimal filtering [2].

### Transfer function of the optimal filter in a static regime

The system in a static regime will be researched as a linear and impulse system.

Method of design of the optimal filter transfer function for the linear continuous systems is well known [2]. We will adapt this method for the impulse system.

We will use z transformation to describe signals and a transfer functions. Suppose it must be implemented the impulse transfer function – given operator –  $H(z)$  applied to the input impacts in the system. The response of the system to the input value  $r_s(nT_0)$  must be value  $c_d(nT_0)$ . System transfer function  $W_0(z)$  must be determined in such way that if disturbance  $r_\xi(nT)$  is present the sum of the system reaction  $c_s(nT_0)$  and  $c_\xi(nT_0)$  to accordingly the input and disruptive impacts standard deviation  $e_0^2(nT)$  from  $c_d(nT)$  would be as small as possible. Scheme for a deviation  $e_0(nT)$  formation is presented in fig. 2.

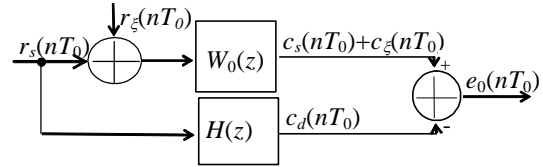


Fig. 2. Structural scheme of a deviation formation

Deviation signal could be expressed:

$$e_0(nT) = c_s(nT) + c_\xi(nT) - c_d(nT). \quad (6)$$

Expression of the standard deviation:

$$\begin{aligned} e_0^2(nT_0) = & \overline{c_s^2(nT_0) + c_\xi^2(nT_0) + c_d^2(nT_0) +} \\ & \overline{c_s(nT_0)c_\xi(nT_0) + c_\xi(nT_0)c_s(nT_0) -} \\ & \overline{c_s(nT_0)c_d(nT_0) - c_d(nT_0)c_s(nT_0) -} \\ & \overline{c_\xi(nT_0)c_d(nT_0) - c_d(nT_0)c_\xi(nT_0)}. \end{aligned} \quad (7)$$

Averages  $\overline{c_s^2(nT_0)}$ ,  $\overline{c_\xi^2(nT_0)}$ ,  $\overline{c_d^2(nT_0)}$ ,  $\overline{c_s(nT_0)c_\xi(nT_0)}$ ,  $\overline{c_\xi(nT_0)c_s(nT_0)}$ ,  $\overline{c_s(nT_0)c_d(nT_0)}$ ,  $\overline{c_d(nT_0)c_s(nT_0)}$ ,  $\overline{c_\xi(nT_0)c_d(nT_0)}$  and  $\overline{c_d(nT_0)c_\xi(nT_0)}$  could be expressed by the corresponding correlation functions, and these functions could be expressed by the discrete spectral densities  $\Phi_{ss}$  and  $\Phi_{\xi\xi}$  of the correspondingly to the input and disturbance signals, and by reciprocal discrete spectral densities  $\Phi_{s\xi}$  and  $\Phi_{\xi s}$  and by the reciprocal transfer functions. Positive and negative  $n$  values are summed for the standard deviation  $e^2(nT)$  calculation. Therefore, two-way z transformation must be applied for the evaluation of the  $\overline{e^2(nT)}$  and for its constituent part spectral densities. Sum of the input,

disturbance and reciprocal spectral densities will be marked as  $\Phi(z)$ :

$$\Phi(z) = \Phi_{ss}(z) + \Phi_{\xi\xi}(z) + \Phi_{s\xi}(z) + \Phi_{\xi s}(z). \quad (8)$$

For the system under consideration we could assume that signal and noise are not correlated, therefore  $\Phi_{s\xi}(z) = \Phi_{\xi s}(z) = 0$ . Function  $\Phi(z)$  is obtained by two-way  $z$  transform and could be expressed:

$$\Phi(z) = F(z) \cdot F(z^{-1}), \quad (9)$$

where  $F(z)$  is a function which has poles and zeroes inside the unit circle,  $F(z^{-1})$  is a function which has poles and zeroes outside the unit circle.

Frequency characteristic of the optimal filter for a continuous system is obtained [2]. In a case a useful signal  $r_s(t)$  and disturbance signal  $r_\xi(t)$  are not correlated frequency characteristic could be expressed:

$$W_0(j\omega) = \frac{1}{F(j\omega)} \left[ \frac{H(j\omega)\Phi_{ss}(\omega)}{F(-j\omega)} \right]_+ . \quad (10)$$

Transfer function of the optimal filter  $W_0(z)$  for the discrete system could be expressed:

$$W_0(z) = \frac{1}{F(z)} \left\{ \frac{H(z)[\Phi_{ss}(z)]}{F(z^{-1})} \right\}_+ . \quad (11)$$

The symbol  $\{*\}_+$  means that from all the function inside the brackets just a part of the function which has a poles inside of the unit circle of the complex plane is taken.

Because  $F(z) = \{F(z)\}_+$  and if we apply equation (9), the latter equation could be expressed:

$$W_0(z) = \left\{ \frac{H(z)[\Phi_{ss}(z)]}{F(z)F(z^{-1})} \right\}_+ = \left\{ \frac{H(z)[\Phi_{ss}(z)]}{\Phi(z)} \right\}_+ . \quad (12)$$

Therefore,  $W_0(z)$  is the function of the one side  $z$  transformation.

### Transfer function of optimal filter when disturbance is a white noise

We will assume that disruptive impact  $\xi(t)$  has characteristics: arithmetic mean  $M\xi(t) = 0$  and spectral density is constant value  $K$ . Consequently,  $\xi(t)$  is a white noise. This hypothesis is close to reality for the respiration rate measurement case.

Lets express the transfer function  $W_0(z)$  when input is step:  $x(t) = C \cdot 1(t)$ , where  $C = const$ . In this case,  $\Phi_{ss}(z) = \frac{Cz}{z-1}$ , and  $\Phi_{\xi\xi}(z) = K$ . If we put these expressions to the equations (9) and (10) we will get a transfer function  $W_0(z)$ :

$$W_0(z) = \frac{H(z) \cdot \frac{Cz}{z-1}}{\frac{Cz}{z-1} + K} = H(z) \cdot \frac{C}{K+C} \cdot \frac{z}{z - \frac{K}{K+C}} . \quad (13)$$

Therefore, if we want that system under investigation would be optimal in a sense of disturbance damping, the device which implements chosen control  $H(z)$  law must be complemented by the device which transfer function is:

$$W_K(z) = \frac{C}{K+C} \cdot \frac{z}{z - K/(K+C)} . \quad (14)$$

This transfer function could be considered as a transfer function of a serial connection of the inertial and proportional units. Continuous transfer functions of the inertial and proportional units are  $W_T(s)$  and  $W_P(s)$ :

$$W_T(s) = \frac{1}{s + (1/T) \ln \frac{C+K}{K}} , \quad (15)$$

$$W_P(s) = \frac{C}{C+K} , \quad (16)$$

Properties of the functions depend on the ratio  $\frac{C}{K}$ . If a

disturbance impact is much greater than input impact  $\frac{C}{K} \ll 1$ , the transfer function of the inertial unit is close to

the transfer function of the integrating unit. The transfer function of the proportional unit  $W_P(s) \approx \frac{C}{K} \ll 1$  will

decrease if  $C$  decreases. This coincides with increase of comparative integration constant. This matches a well known fact that damping of the disturbances is effectively performed by integrating a signal especially in the cases when a signal is weak in comparison to noise. The integration must become longer if a signal becomes weaker in comparison to disturbances to transfer an input impact as match precisely as possible. When input is absent ( $C = 0$ ), no signal must be transferred to output  $W_p(s) = 0$  despite there is some level of noise.

If the ratio  $\frac{C}{K} > 1$  increases, rate of the proportional unit becomes close to 1. Logarithmic characteristic of the inertial unit is shown in fig. 3. We see that  $W_p(\omega) = 1$  while  $\omega < 1/T_e$ . Sample period must be chosen in a way that a spectrum of the useful signal would be in a frequency band  $[0, 2/T]$  according to the criteria of Nyquist. Inertial unit becomes noninertial  $W_p(\omega) = 1$  in the frequency range  $[0, 2/T]$ . Therefore, no correction is necessary in this case.

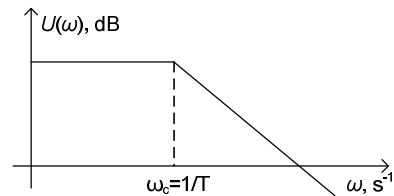


Fig. 3. Logarithmic characteristic of the inertial unit

### Implementation of the optimal filter

Optimal filter is implemented in a microprocessor by processing the information about the input signal obtained at the end of each time period  $T_k$ .

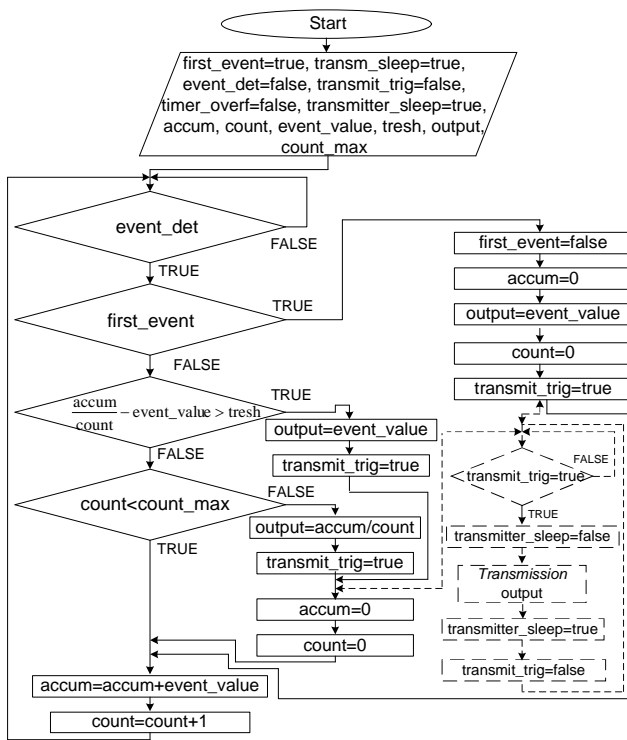


Fig. 4. Algorithm of the optimal correction block implementation

While regime is static, the input value must be integrated and average of the integrated value must be passed to the output. Theoretically, in a pure static regime, integration must proceed while time goes to infinity. Practically, number of the integrating periods is various and is chosen according to the level of the noise and desired measurement precision. In a system described in [3], 5 period of the integration is enough. In a dynamic regime when the change of the input exceeds, the

predefined level information about the measured value must be transferred at the end of each period. Algorithm implemented in a processor of the microcontroller is shown in fig. 4. It is used in real monitoring system.

## Conclusions

1. When the pulse and breathing frequency of manual workers is monitored it is typically that the measurement of these parameters could be suitably accurate, vice versa the sudden variation or surpassing of admissible limits could be fixed without lag.

2. The optimal by noise suppression and quick-acting transfer function of input signal filter can be obtained applying the optimal filtering theory for discrete systems.

3. The signal processing in this case will be optimal, when the algorithm of signal processing will vary dependently on ratio of input signal variation and noise level – in static regime input signal is integrating, in dynamical regime input signal is transferred to output with minimal delay.

## References

1. Lukočius R., Virbalis J. A., Daunoras J., Vegys A. Interactive Textile's Signals Processing System // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 5(61). – P. 13–17 (in lithuanian).
2. Dambrauskas A. Simplex Search Statistical Theory. – Vilnius: Technika. – 2007. – 259 p. (in lithuanian).
3. Lukočius R., Virbalis J. A., Vegys A. Sensors of the Physiological Parameter Monitoring System Applied in a Human Daily Activity // Proceedings of the 3RD International Conference on Electrical and Control Technologies ECT. – Kaunas. – 2008. – P. 317–320.

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## R. Lukočius, J. A. Virbalis, A. Vegys. Optimal Interval of Pulse and Breathing Signal Formation // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 2(90). – P. 103–106.

The human pulse or breathing frequency monitoring is topical for workers who work alone. The suitably accurate measurements must be performed when the level of different parasite signals is significant. There again the sudden variation of parameters or surpassing of admissible limits must be fixed without lag. It is obtained the optimal by noise suppression and quick-acting transfer function of input signal filter based on the optimal filtering theory for discrete systems. This transfer function is realized in processor of microcontroller: the algorithm of signal processing is varied dependently on ratio of input signal variation and noise level - in static regime input signal is integrating, in dynamical regime input signal is transferred to output with minimal delay. Ill. 4, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

## Р. Лукочюс, Ю. А. Вирбалис, А. Вегис. Оптимальный интервал формирования сигнала с учетом пульса и частоты дыхания человека // Электроника и электротехника. - Каунас: Технологія, 2009. – № 2(90). – С. 103–106.

Если контролируется пульс и частота дыхания человека, выполняющего определенную физическую работу, актуальной задачей является достаточно точное измерение этих параметров при высоком уровне разнообразных шумов. В случае резкого изменения или превышения допустимых пределов этих параметров это должно быть зафиксировано максимально быстро. На основе теории оптимальной фильтрации в дискретных системах получена передаточная функция звена, оптимального в смысле подавления помех и быстрей действия. Эта передаточная функция реализована в процессоре микроконтроллера, меняя алгоритм обработки входного сигнала в зависимости от его динамики: интегрируя входной сигнал в статическом режиме и передавая его на выход с минимальной задержкой в динамическом режиме. Ил. 4, библи. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

## R. Lukočius, J. A. Virbalis, A. Vegys. Optimalus žmogaus pulso ir kvėpavimo dažnio signalo formavimo intervalas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 2(90). – P. 103–106.

Kontroliuojant žmonių, atliekančių tam tikrą fizinį darbą, pulsą ir kvėpavimo dažnį, svarbu šiuos parametrus išmatuoti pakankamai tiksliai ir esant aukštam triukšmų lygiui. Be to, šių parametrų staigus pokytis ir leistinų ribų viršijimas turi būti užfiksuotas kaip galima greičiau. Remiantis optimalaus filtravimo teorija diskretinėms sistemoms, gauta optimalaus trikdančiojo poveikio slopinimo ir greitaveikos prasme įrenginio perdavimo funkcija. Ji realizuota mikrovaldiklio procesoriuje, keičiant signalo apdorojimo algoritmą priklausomai nuo įėjimo ir trikdančiojo poveikio santykio: statiniu režimu integruojant įėjimo signalą, o dinaminio režimu – įėjimo signalą į išėjimą perduodant su minimaliu vėlinimu. Il. 4, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).