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The Reflection of Plane Electromagnetic Wave from Dielectric Slab with Metallic Lattice Inside

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Introduction

For wave propagation simulation in room the mathematical models of walls are necessary [1]. Practically, walls are not simple dielectric layers with simply measurable complex permittivity. Generally, walls contain certain number of layers with different permittivity and conductivity, for example, gas–concrete wall with brick boarding and rockwool heat–insulation. For microwave frequencies, plastering also has influence on the reflection and transmission of ray. The simulation will be more complicated if layered structure contains different obstacles, for example, metallic structures in concrete. If metallic structures are threading the wall, the model will be more complex [2], [3], [4]. Usually, metallic structures in composite materials or in walls have regular placement. For example, in reinforced–concrete, metallic wires are shaped as a regular grating (Fig. 1).

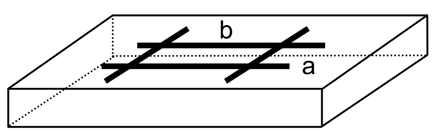

Fig. 1. The slab of reinforced concrete

Slab parameters depend on *a* and *b* grating dimensions. If dimension is comparable or greater as wavelength in concrete, than permittivity will be periodical function of space co–ordinate. If dimension is smaller as wavelength in concrete, than permittivity will be approximate constant for equivalent slab. The orientation of field polarisation with respect to wire orientation is important too, because regular grating is a combination from two perpendicular diffraction gratings. This paper contains our findings of concrete structure with copper grating, which dimensions are small, comparing with wavelength.

Metallic structure in concrete

Model for research contains slab of concrete with thickness *d* and small metallic structure inside (Fig. 2). The thickness of metallic structure d_2.

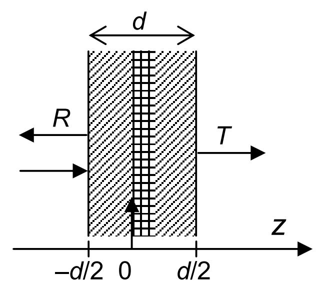

Fig. 2. Cross–section of slab

Let electromagnetic wave incidence be normal to slab. Every layer is characterised by thickness and complex dielectric permittivity at frequency *f*

$$\varepsilon_{ci} = \varepsilon_i - i \cdot \frac{\sigma_i}{2 \cdot \pi \cdot f \cdot \varepsilon_0}, \tag{1}$$

wave impedance

$$Z_i = Z_0 / \sqrt{\varepsilon_{ci}} \tag{2}$$

and wave number

$$k_i = k_0 \cdot \sqrt{\varepsilon_{ci}}, \tag{3}$$

where $Z_0 = \sqrt{\mu_0 / \varepsilon_0}$;
$k_0 = 2\pi \cdot f \sqrt{\varepsilon_0 \cdot \mu_0}$;
ε_i – relative dielectric constant of slab *i*;
σ_i – conductivity [$S \cdot m^{-1}$] of slab *i*;
$\mu_0 = 4\pi \cdot 10^{-7}$ [$H \cdot m^{-1}$];
$\varepsilon_0 = 10^{-9} / (36\pi)$ [$F \cdot m^{-1}$].

R and *T* are reflection and transmission coefficients of slab. For simple materials *R* and *T* can bee easy calculated by solving system of equations. The mathematical model of wall is a system of equations, resulting from boundary conditions for electrical and magnetic field intensity continuities on layer boundaries.

Electrical field intensity in layer i can be represented as a pair of travelling waves:

$$A_i \cdot \exp(-i \cdot k_i \cdot z) + B_i \cdot \exp(i \cdot k_i \cdot z)\,. \qquad (4)$$

For example, A_0 stands for incident wave intensity on the wall and B_0/A_0 represents reflection coefficient. After solving system of equations the field distribution in every layer is definite.

In our case middle layer has non–homogeneous structure. There are tempting attempts in literature to create equivalent permittivity for metallic grid. To get a simple estimation, the ratio between the area occupied by the metallic structure in x–y plane and the total area of the slab ρ_w can bee applied. The complex permittivity of slab with grid with dimensions that are smaller than the wavelength is estimated as per [2]:

$$\varepsilon_{ekv} = (1-\rho_w)\cdot\varepsilon_0\cdot\varepsilon_{concrete} - i\cdot\rho_w\cdot\sigma_{metall}/\omega\,, \quad (5)$$

where σ_{metall} – conductivity of wires; ω – angular frequency.

Let structure have dimensions d_1=0.2 [m], $d_2=10^{-5}$ [m], d_3=0.2[m]. Concrete has permittivity $\varepsilon_{concrete} = 6.25$ and conductivity $\sigma_{concrete} = 0.037$ [S·m^{-1}]. Grid is created from Cu with conductivity $\sigma_{metall} = 5.8\cdot10^7$ [S·m^{-1}]. Calculations are carried out at frequency f=2.4 [GHz].

Table 1. Results

ρ_w	$\lvert R\rvert$	$\lvert T\rvert$
0	0.390557	0.272871
10^{-6}	0.405266	0.258157
10^{-4}	0.622684	0.040728
10^{-2}	0.662922	$4.7481\cdot10^{-4}$
0.1	0.663304	$3.0771\cdot10^{-5}$
1	0.663378	$6.3895\cdot10^{-8}$

The results of calculating show that for small area, occupied by metal, the grating works like a continuous metallic plane. These results cast some doubt upon this method. Therefore we suggest method for well–founded study of this problem.

Metallic structure is selected as flat structure because for cylindrical wire grid the ratio of occupied area depends on co–ordinate. For simplicity, the metallic grid is presented as a thin plane with rectangular windows and thickness d_2. The width of frame is denoted d_e. The system of Cartesian co–ordinates x_1 and y_1 are coupled with grid and can be rotated in regard to system, coupled with incident electromagnetic field (Fig. 3). The incident field polarisation coupled with Cartesian system co–ordinates x and y. The angle between axis x and x_1 is φ.

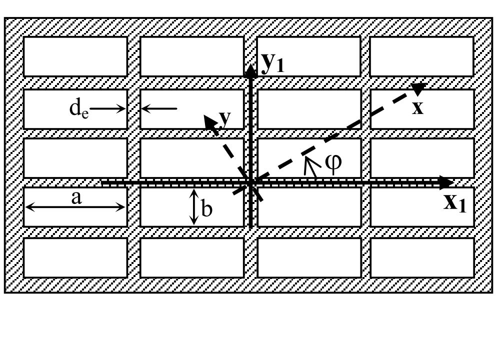

Fig.3. The shape of grid

The idea of the studied method is that the incident field creates current density in metallic frames of the structure. These currents create reflected and transmitted waves. In addition, the field in the windows creates the field behind the grid, according to Huygens principle. The currents in frames and field intensity in windows are fixed approximately. The current density in frames is obtained by solving 3 layers problem for ρ_w=1. Electrical field intensity in metallic layer gives $\vec{\mathbf{j}} = \sigma_{metall}\cdot\vec{\mathbf{E}}_2$. The electromagnetic field intensities in windows are obtained by solving simple boundary problem for free space–concrete, and calculating electric and magnetic fields in concrete at dept $d/2$.

If incident field polarized along x axis E_x the

$$E_{x1} = E_x\cdot\cos(\varphi) \text{ and } E_{y1} = E_x\cdot\sin(\varphi)\,. \qquad (6)$$

The current density $\vec{\mathbf{j}}$ has components j_{x1} and j_{y1}, and creates electromagnetic field that can be expressed by vector potential $\vec{\mathbf{A}}$. Vector potential satisfies equation:

$$\nabla^2\mathbf{A}_{x_1,y_1} + k_1^2\cdot\mathbf{A}_{x_1,y_1} = -\mu_0\cdot\mathbf{j}_{x_1,y_1}\,. \qquad (7)$$

The solution of Helmholz equation (2) for boundless room can be expressed by Green function [5]. This is vector potential for radiated field. The solving for A_{x1}:

$$A_{x1}(x_1,y_1,z) = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} j_{x_1}(\alpha,\beta,\gamma)\cdot G(x_1,y_1,z,\alpha,\beta,\gamma)\cdot d\alpha\cdot d\beta\cdot d\gamma\,, \quad (8)$$

where $G(x_1,y_1,z,\alpha,\beta,\gamma) = \dfrac{\exp(-i\cdot k_1\cdot R)}{R}$,

$$R = \sqrt{(x_1-\alpha)^2+(y_1-\beta)^2+(z-\gamma)^2}\,.$$

Analogous computed component A_{y1} allows to write a vector potential that determine electrical field intensity of radiated field:

$$\vec{\mathbf{E}}_{rad}(x_1,y_1,z) = -\frac{i\cdot\omega}{k_1^2}\nabla\left(\nabla\bullet\vec{\mathbf{A}}\right) - i\cdot\omega\cdot\vec{\mathbf{A}}\,. \qquad (9)$$

The radiated field in concrete coincides with the field, reflected from the metallic structure. For such interpretation on surface (z=–d/2), the electromagnetic field, created by current, must have a flat wavefront. This idea of average boundary conditions was developed in [6]. The flat wavefront can be obtained for small cells, or for enough large distance between metallic structure and surface. In our case for d/2=0.2 [m], the size of cell was obtained by compare computing for different a and b. For all case d_e=0.002 [m]. For a=b=0.03 [m] relative deviation over cell area do not exceed 0.7 %. For greater a and b deviation increases. Not significant deviation along the cell allows to present radiated field like flat wave sum of the incident and reflected fields on boundary. To summarise, we must return to co–ordinate couple with incident wave:

$$\begin{cases} E_x = E_{x1} \cdot \cos(\varphi) - E_{y1} \cdot \sin(\varphi), \\ E_y = E_{x1} \cdot \sin(\varphi) + E_{y1} \cdot \cos(\varphi). \end{cases} \tag{10}$$

For $\varphi = \pi/4$, a=b=0.03 on surface E_x=–0.0191489–j·0.0003534, E_y=0 incidents to surface concrete – free room with transfer coefficient T=1.42882–j·0.00905353. We must add transferred field to the field, reflected from boundary free room–concrete R=–0.428815+j·0.00905353. Now we have a corrected reflection coefficient R_{cor}=–0.456178+j·0.0087219. This outcome doesn't depend from the angle between polarisation of incident wave and metallic structure geometry.

For non–square cells, the reflected field contains also other polarisation, that, in general, depends on angle φ. The calculating for a=0.02 [m], b=0.03 [m], $\varphi = \pi/4$ gives R_{cor}=–0.461806+j·0.00838485 and reflected field with other polarisation $E_{y\,refl}$=–0.00562761+j·0.000337073.

The transfer coefficient of slab depends on penetration of a field via windows. We assume that the field intensity in windows is the same as in the middle of slab of concrete (without metallic structure). According to Huygens principle, this field creates wave that at enough great distance can be considered as a flat wave and transferred through boundary by simple conditions.

For calculating field, radiated from surface elements, it is handy to apply spherical system co–ordinates (Fig. 4), that, afterwards, can be transformed to Cartesian system.

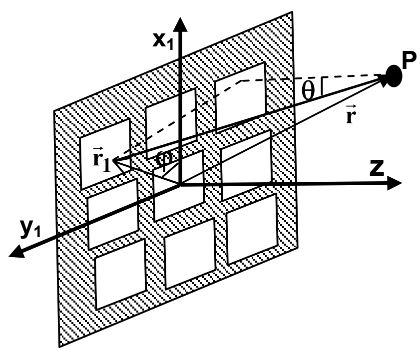

Fig. 4. Spherical system co–ordinates

In observation point P, the radiated field $\vec{\mathbf{E}}_P$ is determined by integrating results over all windows of structure. For field intensity component on surface E^S_{x1} and E^S_{y1}:

$$\begin{cases} \vec{\mathbf{E}}_{Px1} = H \cdot \int_S [1+\cos(\theta)] \cdot [\vec{\mathbf{e}}_\theta \cdot \cos(\varphi) - \vec{\mathbf{e}}_\varphi \cdot \sin(\varphi)] \cdot G(\vec{\mathbf{r}}, \vec{\mathbf{r}}_1) \cdot dS_1 , \\ \vec{\mathbf{E}}_{Py1} = J \cdot \int_S [1+\cos(\theta)] \cdot [\vec{\mathbf{e}}_\theta \cdot \sin(\varphi) + \vec{\mathbf{e}}_\varphi \cdot \cos(\varphi)] \cdot G(\vec{\mathbf{r}}, \vec{\mathbf{r}}_1) \cdot dS_1 , \end{cases} \tag{11}$$

where $G(\vec{\mathbf{r}}, \vec{\mathbf{r}}_1) = \dfrac{\exp(-i \cdot k_1 \cdot |\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|)}{4\pi \cdot |\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|}$,

$H = i \cdot k_1 \cdot E^S_{x1}$, $J = i \cdot k_1 \cdot E^S_{y1}$.

The field intensities E^S_{x1} and E^S_{y1} are over region constant only for normal incident case.

We assume that the value of the flat wavefront, radiated from cell waves at co–ordinate z=d/2, was obtained accurately, because the field is created from a lot of radiated regions, located at enough great distance from surface.

The field intensity in windows E_x=0.33358+j·0.0027725. For calculating, this field was transformed to co–ordinate system, coupled with grid, and then, transformed to system, coupled with waves. For $\varphi = \pi/4$, a=b=0.03 on surface z=d/2 E_x=0.162289+j·0.00693169, E_y=0. This field incidents to surface concrete – free room and partially will be added to field, transmitted through slab, and partially reflected towards the lattice. The reflected part can cause multiple reflection between grid and surface, but, in our case, reflected field at metallic structure is E_{second}= =0.0398793+j·0.000613665. This is approximately ten times smaller in magnitude than initial field in windows, and, being the first approximation we can take no notice of it. For transmission coefficient of slab T, the contribution is significant. Now T_{cor}=0.231944+j·0.00843481. This outcome doesn't depend on the angle between polarisation of incident wave and metallic structure geometry.

As distinct from radiation currents in frames, the non–square form of windows creates non cross polarisation, but changes only radiated field.

Conclusion

This paper contains findings of authors, which concern reflection and transmission electromagnetic waves for dielectric slab with metallic structures inside. The metallic structure is periodic, with the relatively small period in comparison with wavelength. The necessity to limit a period is caused mainly by exigency to obtain an even field distribution at slab boundary. The idea of studied method is that the incident field creates current density in metallic frames of structure. These currents create reflected and transmitted waves. In addition the field in windows creates field behind grid in accordance to Huygens principle. The currents in frames and field intensity in windows are set approximately.

Current radiated fields give little than 10% deviation for slab reflection coefficient for our selected geometry. It doesn't depend on angle between polarisation of incident wave and frames. For non–square cells only, the reflected field contains small different polarisation field.

The field, radiated by field distribution in windows, significantly changes transmission coefficient but, non–square form of windows doesn't create cross polarisation.

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J. Ziemelis, T. Solovjova. Plokščiosios elektromagnetinės bangos atsispindėjimas nuo dielektriko sluoksnio su įtaisyta metaline gardele // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 6(62). – P. 9–12.

Pateikti atspindėjimo ir dielektrinio sluoksnio su vidine metalo struktūra elektromagnetinių bangų tyrimai. Parinkta periodinė metalo struktūra, kurio periodas mažas palyginti su bangos ilgiu. Tyrimams naudojamas elektrinis laukas, sukuriamas struktūros metalinėse gardelėse, kuriame susidaro atsispindėjusioji ir perėjusioji banga. Be to, lauko susidarymas struktūros languose pagal Giuigeno principą sukuria lauką už gardelės. Srovės metalinėse gardelėse tankis ir lauko išsisklaidymas ant langų yra nustatyti apytiksliai. Srovės sukurtas atsispindėjęs laukas struktūros geometrijoje pakeičia atsispindėjimo koeficientą mažiau nei 10 %, tačiau lauko sklaida languose gerokai padidino perėjimo koeficientą. Il. 4, bibl. 6 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

J. Ziemelis, T. Solovjova. **The Reflection of Plane Electromagnetic Wave from Dielectric Slab with Metallic Lattice Inside // Electronics and Electrical Engineering.– Kaunas: Technologija, 2005. – No. 6(62). – P. 9–12.**

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Ю. Зиемелис, Т. Соловьёва. Отражение плоской электромагнитной волны от слоя диэлектрика со встроенной металлической решёткой // Электроника и электротехника. – Каунас: Технолгия, 2005. – № 6(62). – С. 9–12.

Работа содержит исследования отражения и прохождения электромагнитных волн для диэлектрического слоя с внутренней металлической структурой. Металлическая структура выбрана периодической, с периодом малым по сравнению с длиной волны. Для исследований используется поле, созданное током в металлических рамках структуры, который создаёт отражённую и прошедшую волны. Кроме того, распределение поля в окнах структуры согласно принципу Гюйгенса создаёт поле за решёткой. Плотность тока в металлических рамках и распределение поля в окнах определены приближенно. Созданное токами отражённое поле для выбранной в работе геометрии структуры изменяет коэффициент отражения менее чем на 10 %, но распределение поля в окнах существенно увеличило коэффициент прохождения. Ил. 4, библ. 6 (на английском языке; рефераты на литовском, английском и русском яз.).