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Outage Probability of System with Selection Combining over Correlated Weibull Fading Channels in the Presence of Rayleigh Cochannel Interference

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Introduction

Diversity combining [1] is a well–known method to overcome the destructive effects of fading in wireless communication systems. It is usually implemented by using an antenna array consisted of two or more receiving antennas. The most popular space-diversity techniques are selection combining (SC), equal-gain combining (EGC), and maximal-ratio combining (MRC). SC is the least complicated among these types of diversity combining since it processes only one of the diversity branches. In general, SC combiner chooses the branch with the highest signal-to-noise ratio (SNR). However, it can be said that SC combiner selects the branch with the highest signal-to-interference ratio (SIR) because cochannel interference (CCI) is more significant than the thermal noise in wireless communication systems.

The most frequently used models for describing fading in wireless environments are Rayleigh, Rice, Nakagami-m, and Weibull. Dual SC receiver over correlated Rician fading channels in the presence of correlated Rayleigh distributed CCI was studied in paper [2]. Dual SIR based SC over correlated Nakagami-m fading channels is presented in [3]. Regardless Weibul fading model confirms experimentally attained fading channel measurements, for both indoor [4], as well as for outdoor environments [5], there are not so many papers which studied this model. Correlated Weibull fading channels in the presence of Weibull CCI was studied in [6].

In this paper, dual SIR-based SC receiver over correlated Weibull fading channels in the presence of correlated Rayleigh distributed CCI is considered. System performance has been studied using outage probability derived in novel closed-form.

Outage probability

Dual-diversity system with selection combining is considered. Desired signal envelope follows correlative Weibull fading whose probability density function (PDF) is given by [7]

$$p_{R_1R_2}(R_1,R_2)=\frac{\beta_1\beta_2 R_1^{\beta_1-1}R_2^{\beta_2-1}}{\Omega_{d1}\Omega_{d2}(1-\rho)}\times$$
$$\times\exp\left\{-\frac{1}{1-\rho}\left[\frac{R_1^{\beta_1}}{\Omega_{d1}}+\frac{R_2^{\beta_2}}{\Omega_{d2}}\right]\right\}\times$$
$$\times I_0\left(\frac{2\sqrt{\rho}R_1^{\frac{\beta_1}{2}}R_2^{\frac{\beta_2}{2}}}{(1-\rho)\sqrt{\Omega_{d1}\Omega_{d2}}}\right), \tag{1}$$

where β_i is Weibull fading parameter (β_i > 0), ρ is correlation coefficient, $\Omega_{di}=\overline{R_i^2}$ is average power of desired signal at *i*th branch and $I_0(\cdot)$ is modified Bessel function of the first kind and zero order [8, eq. 9.6.10].

$$I_0(x)=\sum_{n=0}^{+\infty}\frac{x^{2n}}{2^{2n}n!\Gamma(n+1)}. \tag{2}$$

We assume that the envelope of CCI on diversity branches is Rayleigh distributed with PDF [1, eq. 6.2]:

$$p_{A_1A_2}(A_1,A_2)=\frac{4A_1A_2}{\Omega_{c1}\Omega_{c2}(1-r_A)}\times$$
$$\times\exp\left\{-\frac{1}{1-r_A}\left(\frac{A_1^2}{\Omega_{c1}}+\frac{A_2^2}{\Omega_{c2}}\right)\right\}\times$$

$$\times I_0\left(\frac{2\sqrt{r_A}A_1A_2}{\left(1-r_A\right)\sqrt{\Omega_{c1}\Omega_{c2}}}\right), \qquad (3)$$

where r_A is correlation coefficient between A_1 and A_2 and $\Omega_{ci}=\overline{A_i^2}$ is average power of interference.

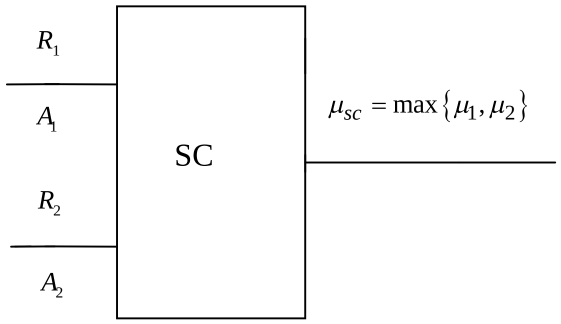

Fig. 1. Dual SC diversity receiver

Let $\mu_1=R_1/A_1$ and $\mu_2=R_2/A_2$ be instantaneous SIRs at the input diversity branches. The joint PDF of these random variables can be obtained as

$$p_{\mu_1\mu_2}(\mu_1,\mu_2)=\int_0^\infty\int_0^\infty |J|\; p_{R_1R_2}(\mu_1A_1,\mu_2A_2)\times$$
$$\times p_{A_1A_2}(A_1,A_2)\; dA_1dA_2, \qquad (4)$$

where $|J|$ Jacobian transformation

$$|J|=\begin{vmatrix}\dfrac{dR_1}{d\mu_1} & \dfrac{dR_1}{d\mu_2}\\ \dfrac{dR_2}{d\mu_1} & \dfrac{dR_2}{d\mu_2}\end{vmatrix}=A_1A_2. \qquad (5)$$

Substituting (1), (3) and (5) in (4) and using the infinite series representation of the modified Bessel function (2) and [7, eq. (8)] (see Appendix I), the joint PDF becomes:

$$p_{\mu_1\mu_2}\left(\mu_1,\mu_2\right)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{\beta_1\beta_2}{n!k!\Gamma\left(n+1\right)\Gamma\left(k+1\right)}\frac{\rho^n}{\left(1-\rho\right)^{2n+1}}\times$$
$$\times\frac{\mu_1^{\beta_1(n+1)-1}\mu_2^{\beta_2(n+1)-1}}{\Omega_{d1}^{n+1}\Omega_{d2}^{n+1}}r_A^k\left(1-r_A\right)^{\frac{(\beta_1+\beta_2)(1+n)}{2}+1}\times$$
$$\times\Omega_{c1}^{\frac{\beta_1(1+n)}{2}}\Omega_{c2}^{\frac{\beta_2(1+n)}{2}}\sqrt{\frac{hl}{\lambda\gamma}}\frac{\lambda^{\frac{\beta_1(1+n)+2(k+1)}{2}}\gamma^{\frac{\beta_2(1+n)+2(k+1)}{2}}}{\left(\sqrt{2\pi}\right)^{h+l+\lambda+\gamma-4}}\times$$
$$\times G_{\lambda,h}^{h,\lambda}\left[\frac{W_1^h\lambda^\lambda\mu_1^{\beta_1h}}{h^h}\left|\begin{matrix}\frac{1-S}{\lambda},\frac{2-S}{\lambda},\ldots,\frac{\lambda-S}{\lambda}\\ 0,\frac{1}{h},\quad\ldots\quad,\frac{h-1}{h}\end{matrix}\right.\right]\times$$
$$\times G_{\gamma,l}^{l,\gamma}\left[\frac{W_2{}^l\gamma^\gamma\mu_2^{\beta_2l}}{l^l}\left|\begin{matrix}\frac{1-T}{\gamma},\frac{2-T}{\gamma},\ldots,\frac{\gamma-T}{\gamma}\\ 0,\frac{1}{l},\quad\ldots\quad,\frac{l-1}{l}\end{matrix}\right.\right], \qquad (6)$$

where $G[\cdot]$ is Meijer's G-function [9], λ, h, γ and l are minimum positive integer values so that $\lambda/h=\beta_1/2$ and $\gamma/l=\beta_2/2$ and

$S=\left[\beta_1\left(1+n\right)+2\left(k+1\right)\right]/2$,

$T=\left[\beta_2\left(1+n\right)+2\left(k+1\right)\right]/2$,

$$W_1=\frac{\left(1-r_A\right)^{\frac{\beta_1}{2}}\Omega_{c1}^{\frac{\beta_1}{2}}}{\left(1-\rho\right)\Omega_{d1}},$$

$$W_2=\frac{\left(1-r_A\right)^{\frac{\beta_2}{2}}\Omega_{c2}^{\frac{\beta_2}{2}}}{\left(1-\rho\right)\Omega_{d2}}.$$

The bivariate cumulative distribution function (CDF) is by definition

$$F_{\mu_1\mu_2}(\mu_1,\mu_2)=\int_0^{\mu_1}\int_0^{\mu_2}p_{\mu_1\mu_2}(x_1,x_2)\; dx_1dx_2. \qquad (7)$$

Substituting (6) into (7), integrals can be solved with the use of [10 eq. (26)] resulting (8).

$$F_{\mu_1\mu_2}\left(\mu_1,\mu_2\right)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{n!k!\Gamma\left(n+1\right)\Gamma\left(k+1\right)}\frac{\rho^n}{\left(1-\rho\right)^{2n+1}}\times$$
$$\times\frac{\mu_1^{\beta_1(n+1)}\mu_2^{\beta_2(n+1)}}{\Omega_{d1}^{n+1}\Omega_{d2}^{n+1}}r_A^k\left(1-r_A\right)^{\frac{(\beta_1+\beta_2)(1+n)}{2}+1}\times$$
$$\times\Omega_{c1}^{\frac{\beta_1(1+n)}{2}}\Omega_{c2}^{\frac{\beta_2(1+n)}{2}}\frac{\lambda^{\frac{\beta_1(1+n)+2(k+1)}{2}}\gamma^{\frac{\beta_2(1+n)+2(k+1)}{2}}}{\left(\sqrt{2\pi}\right)^{h+l+\lambda+\gamma-4}\sqrt{\lambda\gamma lh}}\times$$
$$\times G_{\lambda+1,h+1}^{h,\lambda+1}\left[\frac{W_1^h\lambda^\lambda\mu_1^{\beta_1h}}{h^h}\left|\begin{matrix}\frac{1-S}{\lambda},\frac{2-S}{\lambda},\ldots,\frac{\lambda-S}{\lambda},1-\frac{n+1}{h}\\ 0,\frac{1}{h},\quad\ldots\quad,\frac{h-1}{h},-\frac{n+1}{h}\end{matrix}\right.\right]\times$$
$$\times G_{\gamma+1,l+1}^{l,\gamma+1}\left[\frac{W_2{}^l\gamma^\gamma\mu_2^{\beta_2l}}{l^l}\left|\begin{matrix}\frac{1-T}{\gamma},\frac{2-T}{\gamma},\ldots,\frac{\gamma-T}{\gamma},1-\frac{n+1}{l}\\ 0,\frac{1}{l},\quad\ldots\quad,\frac{l-1}{l},-\frac{n+1}{l}\end{matrix}\right.\right]. \qquad (8)$$

The selection combiner chooses and outputs the branch with the largest SIR

$$\mu_{sc}=\max\left\{\mu_1,\mu_2\right\} \qquad (9)$$

The CDF of the dual SIR based SC output can be expressed as

$$F_{\mu_{sc}}\left(\mu\right)=F_{\mu_1\mu_2}\left(\mu,\mu\right)=$$
$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{n!k!\Gamma\left(n+1\right)\Gamma\left(k+1\right)}\frac{\rho^n}{\left(1-\rho\right)^{2n+1}}\times$$

$$\times \frac{\mu^{(\beta_1+\beta_2)(n+1)}}{\Omega_{d1}^{n+1}\Omega_{d2}^{n+1}} r_A^k \left(1-r_A\right)^{\frac{(\beta_1+\beta_2)(1+n)}{2}+1} \times$$

$$\times \Omega_{c1}^{\frac{\beta_1(1+n)}{2}} \Omega_{c2}^{\frac{\beta_2(1+n)}{2}} \frac{\lambda^{\frac{\beta_1(1+n)+2(k+1)}{2}} \gamma^{\frac{\beta_2(1+n)+2(k+1)}{2}}}{\left(\sqrt{2\pi}\right)^{h+l+\lambda+\gamma-4} \sqrt{\lambda\gamma l h}} \times$$

$$\times G_{\lambda+1,h+1}^{h,\lambda+1}\left[\frac{W_1{}^h \lambda^\lambda \mu^{\beta_1 h}}{h^h} \left| \begin{matrix} \frac{1-S}{\lambda}, \frac{2-S}{\lambda}, \ldots, \frac{\lambda-S}{\lambda}, 1-\frac{n+1}{h} \\ 0, \frac{1}{h}, \quad \ldots \quad, \frac{h-1}{h}, -\frac{n+1}{h} \end{matrix} \right. \right] \times$$

$$\times G_{\gamma+1,l+1}^{l,\gamma+1}\left[\frac{W_2{}^l \gamma^\gamma \mu^{\beta_2 l}}{l^l} \left| \begin{matrix} \frac{1-T}{\gamma}, \frac{2-T}{\gamma}, \ldots, \frac{\gamma-T}{\gamma}, 1-\frac{n+1}{l} \\ 0, \frac{1}{l}, \quad \ldots \quad, \frac{l-1}{l}, -\frac{n+1}{l} \end{matrix} \right. \right]. \quad (10)$$

Outage probability (P_{out}) is an important measure of system's performance defined as the probability that the output SIR of the SC falls below a given outage threshold μ_{th}. P_{out} can be expressed as

$$P_{out} = P_R(\mu_{sc} < \mu_{th}) = \int_0^{\mu_{th}} p_{\mu_{sc}}(\mu)\; d\mu = F_{\mu_{sc}}(\mu_{th}) =$$

$$\times \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{n!k!\Gamma(n+1)\Gamma(k+1)} \frac{\rho^n}{(1-\rho)^{2n+1}} \times$$

$$\times \frac{\mu_{th}^{(\beta_1+\beta_2)(n+1)}}{\Omega_{d1}^{n+1}\Omega_{d2}^{n+1}} r_A^k \left(1-r_A\right)^{\frac{(\beta_1+\beta_2)(1+n)}{2}+1} \times$$

$$\times \Omega_{c1}^{\frac{\beta_1(1+n)}{2}} \Omega_{c2}^{\frac{\beta_2(1+n)}{2}} \frac{\lambda^{\frac{\beta_1(1+n)+2(k+1)}{2}} \gamma^{\frac{\beta_2(1+n)+2(k+1)}{2}}}{\left(\sqrt{2\pi}\right)^{h+l+\lambda+\gamma-4} \sqrt{\lambda\gamma l h}} \times$$

$$\times G_{\lambda+1,h+1}^{h,\lambda+1}\left[\frac{W_1{}^h \lambda^\lambda \mu_{th}{}^{\beta_1 h}}{h^h} \left| \begin{matrix} \frac{1-S}{\lambda}, \frac{2-S}{\lambda}, \ldots, \frac{\lambda-S}{\lambda}, 1-\frac{n+1}{h} \\ 0, \frac{1}{h}, \quad \ldots \quad, \frac{h-1}{h}, -\frac{n+1}{h} \end{matrix} \right. \right] \times$$

$$\times G_{\gamma+1,l+1}^{l,\gamma+1}\left[\frac{W_2{}^l \gamma^\gamma \mu_{th}{}^{\beta_2 l}}{l^l} \left| \begin{matrix} \frac{1-T}{\gamma}, \frac{2-T}{\gamma}, \ldots, \frac{\gamma-T}{\gamma}, 1-\frac{n+1}{l} \\ 0, \frac{1}{l}, \quad \ldots \quad, \frac{l-1}{l}, -\frac{n+1}{l} \end{matrix} \right. \right]. \quad (11)$$

Numerical results

Using Eq. (11) outage probability is plotted in Fig.1 as a function of the outage threshold μ_{th} for different system parameters. We can note that outage probability decreases as Weibull fading parametres increase and increasing of correlation coefficients leads to deterioration of system performance.

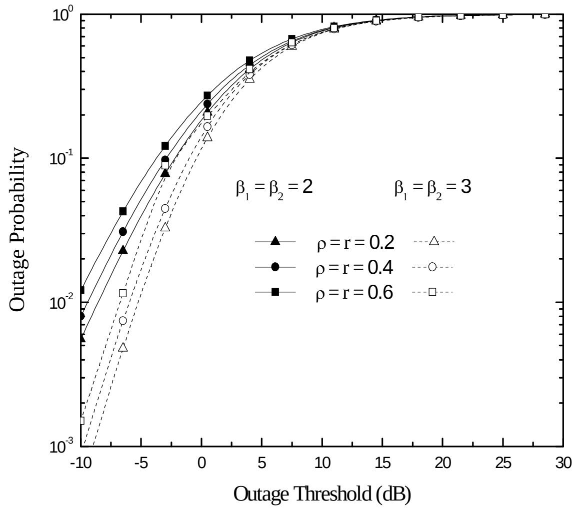

Fig. 2. Outage probability versus outage threshold

Conclusion

In this paper, the performance of dual SIR-based SC diversity system operating over correlated Weibull fading channels in the presence of Rayleigh distibuted CCI, was studied. Useful analytical formulae for outage probability was obtained. Various performance evaluation results for different fading channel conditions were also presented.

Appendix I

Solving integrals using Meijer's G function

Integrals which form is

$$I = \int_0^\infty x^{\alpha-1} \exp\left(-x - \beta x^\gamma\right) dx, \quad (12)$$

where α and β are positive values have been solved in [7] using Meijer's G function as

$$I = \frac{\lambda^\alpha \sqrt{\frac{k}{\lambda}}}{\left(\sqrt{2\pi}\right)^{k+\lambda-2}} \times$$

$$\times G_{\lambda,k}^{k,\lambda}\left[\beta^k \frac{\lambda^\lambda}{k^k} \left| \begin{matrix} \frac{1-\alpha}{\lambda}, \frac{2-\alpha}{\lambda}, \ldots, \frac{\lambda-\alpha}{\lambda}, \\ 0, \frac{1}{k}, \quad \ldots \quad, \frac{k-1}{k} \end{matrix} \right. \right], \quad (13)$$

where γ rational number, and λ and k are positive integer numbers so that

$$\gamma = \frac{\lambda}{k} \quad (14)$$

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Dual signal-to-interference-ratio (SIR)-based selection diversity is often used in wireless communication systems due to its simplicity and ability to mitigate fading and cochannel interference. In this paper, desired signal follows the correlated Weibull distribution and interfering signal follows the correlated Rayleigh distribution. Outage probability presents one of the most used wireless system performance criterion. We derived cumulative distribution function (CDF) and outage probability in closed form using special Meijer's G functions. Numerical results are also presented to show the effects of fading severity and level of correlation to the system's performance. Ill. 2, bibl. 10 (In English; summaries in English, Russian and Lithuanian).

Д. Алексич, Н. Секулович, М. Стефанович. Исследование вероятности выключения системы при Вейбулском распределении сигнала и воздействии Релейских шумов соседних каналов // Электроника и электротехника. – Каунас: Технология, 2009. – № 2(90). – С. 7–10.

Анализируется воздействие шумов типа Рэлея на процесс выключения системы. Показано, что в беспроводных коммутационных системах вероятность соотношения сигнала и шума целесообразно описать законом распределения Рэлея с учетом, что сигнал подчиняется закону Вейбула. Найдено, что функцию распределения и вероятность выключения системы можно описать функциями Маяра. Полученный результат позволяет расчитать эффективность системы. Ил. 2, библ. 10 (на английском языке; рефераты на английском, русском и литовском яз.).

D. Aleksić, N. Sekulović, M. Stefanović. Sistemos išjungimo tikimybė derinant Veibulo kanalų nykimą veikiant Relėjaus gretimų kanalų trikdžiams // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 2(90). – P. 7–10.

Dvigubo signalo ir triukšmo santykis, pagrįstas pasirinkimo įvairove, yra dažnai naudojamas belaidėse komunikacijos sistemose dėl jo paprastumo, gebėjimo sušvelninti išnykimą ir interferencijos su gretimu kanalu. Pageidaujamas signalas atitinka koreliuotą Veibulo skirstinį, interferuojantysis signalas atitinka koreliuotą Relėjaus skirstinį. Išjungimo tikimybė atspindi vieną iš labiausiai naudojamų belaidės sistemos atlikimo kriterijų. Gauta jungtinė skirstinio funkcija ir išjungimo tikimybė uždaroje sistemoje naudojant specialias Meijerio funkcijas. Skaitinia rezultatai taip pat leidžia apskaičiuoti koreliuotų sistemų efektyvumą. Il. 2, bibl. 10 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).