Statistics of Macro SC Diversity System with Two Micro EGC Diversity Systems and Fast Fading

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Introduction

Diversity methods have long been used in wireless systems to mitigate the detrimental effects of multipath fading and co-channel interference. Diversity schemes can be classified according to the type of combining employed at the receiver, namely, maximal-ratio combining (MRC), equal gain combining (EGC), selection diversity combining (SDC) and generalized selection combining (GSC). In an EGC combiner, the outputs of different diversity branches are first co-phased and weighted equally before being summed to give the resultant output. EGC combiner does not require the estimation of the channel gains, and hence it results in reduced receiver complexity relative to the MRC scheme. However, the performance of EGC is inferior to that of MRC since the branch weights are all set to unity. The Rayleigh distribution is the most widely used distribution to describe the received envelope value. The Rayleigh flat fading channel model assumes that all make up the resultant received signal are reflected or scattered and there is no direct path from the transmitter to the receiver. The Rayleigh distributed envelope of a received signal is given by:

\[ P_x(x) = \begin{cases} \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), & 0 \leq x < \infty \\ 0, & x < 0 \end{cases} \]

where \( \sigma \) – the received root-mean-square (rms) envelope level; \( \sigma^2 \) – the average power of the received signal.

As mentioned above, diversity has long been recognized as a powerful communication receiver technique for mitigating the detrimental effects of channel fading and co-channel interference. The underlying premise is that if several uncorrelated replicas of a signal are received over multiple diversity paths with comparable signal strengths, then it is improbable that these signals will experience simultaneous deep fades. Diversity methods can be employed either at the base station (macroscopic diversity) or at the mobile station (microscopic diversity), although the antenna separation required differs for each case. In practice, microscopic diversity reception techniques are employed to combat the fast fading variations in the received signal strength caused by fast fading, whereas macroscopic diversity is used to mitigate the slower fading variations caused by shadowing.

Model of the receiver

Macro diversity system with two micro diversity systems is being discussed in this paper. The model of the discussed system is shown in Fig. 1. Signals at the first micro diversity combiner input are \( x_1 \) and \( x_2 \), and signals at the second micro diversity combiner input are \( y_1 \) and \( y_2 \). The signal at the first micro diversity system output is \( x \), and the signal at the second micro diversity system output is \( y \). Micro diversity combiners are EGC and the macrocombiner is SC. At the macro diversity combiners input there is a substantive Rayleigh fading. Micro diversity combiners are used to reduce the influence of fast Rayleigh fading to the system performances.

Fig. 1. The model of the system

\[ x \]

\[ y \]

\[ z \]
Probability density of random variable \( z \) after macrolevel combining is:

\[
p_z(z) = \int_0^\infty \int_0^{\Omega_1} \int_0^{\Omega_2} p_x(z/\Omega_i) p_{\Omega_i\Omega_j}(\Omega_1\Omega_2) \, d\Omega_1 \, d\Omega_2 \, dz,
\]

where \( p_{\Omega_i\Omega_j}(\Omega_1\Omega_2) = \frac{\rho^{\frac{1}{2}(c-1)}}{\Gamma(\Omega_1\Omega_2) \gamma_1(1-\rho)} e^{\frac{\Omega_1\Omega_2}{2(1-\rho)}} \times\]

\[
\times I_{c-1}\left(\frac{\sqrt{\rho\Omega_2}}{\gamma_0(1-\rho)}\right)
\]

and \( c \geq \frac{1}{2} \) denotes the fading severity index, \( I_c(\cdot) \) is the modified Bessel function of the first kind of order \( c \), \( \Gamma(\cdot) \) is the Gamma function, notation \( E[.\] denotes the statistical average of its argument, \( \rho \) (\( 0 < \rho < 1 \)) corresponds to the power correlation coefficient between the signals received on the two closely spaced antennas.

The probability density function of the EGC microdiversity output in the presence of Rayleigh fading is given with [8]

\[
p_x(r) = \frac{r^{M-1} e^{-\frac{r^2}{\Omega^2}}}{\Omega^{M-1} (M-1)!},
\]

where \( M \) – denotes the number of microcombiner branches; \( \Omega \) – denotes equivalent signal power at the microcombiner’s output which is also defined with [8]:

\[
p_x(x/\Omega_i) = \frac{xe^{-\frac{x^2}{\Omega_i^2}}}{\Omega_i^2},
\]

and the pdfs at the microcombener inputs are:

\[
p_x(y/\Omega_j) = \frac{ye^{-\frac{y^2}{\Omega_j^2}}}{\Omega_j^2},
\]

after substituting previous expressions into (2) with the respect to (3) we obtain following expression:

\[
p_x(z) = \int_0^\infty \int_0^{\Omega_1} \int_0^{\Omega_2} z e^{-\frac{z^2}{\Omega_i^2}} \rho^{\frac{1}{2}(c-1)} \gamma_1(1-\rho) \gamma_0^2 e^{\frac{\Omega_1\Omega_2}{2(1-\rho)}} \times
\]

\[
\times I_{c-1}\left(\frac{\sqrt{\rho\Omega_2}}{\gamma_0(1-\rho)}\right) + \int_0^\infty \int_0^{\Omega_2} \int_0^{\Omega_1} z e^{-\frac{z^2}{\Omega_i^2}} \rho^{\frac{1}{2}(c-1)} \gamma_1(1-\rho) \gamma_0^2 e^{\frac{\Omega_1\Omega_2}{2(1-\rho)}} \times
\]

\[
\times I_{c-1}\left(\frac{\sqrt{\rho\Omega_1}}{\gamma_0(1-\rho)}\right) = J_1 + J_2.
\]

The integrals from the previous expressions can be solved by developing modified Bessel function of the first kind of order \( c \) by using well-known transformation:

\[
I_c(x) = \sum_{k=0}^{\infty} \frac{x^{2k+c}}{2^{2k} k! \Gamma(k+c+1)}.
\]

Now expression for \( J_1 \) can be written

\[
J_1 = \int_0^\infty \int_0^{\Omega_1} \int_0^{\Omega_2} z e^{-\frac{z^2}{\Omega_i^2}} \rho^{\frac{1}{2}(c-1)} \gamma_1(1-\rho) \gamma_0^2 e^{\frac{\Omega_1\Omega_2}{2(1-\rho)}} \times
\]

\[
\times I_{c-1}\left(\frac{\sqrt{\rho\Omega_2}}{\gamma_0(1-\rho)}\right) \frac{y_0(1-\rho)}{\Gamma(\gamma_1(1-\rho))(1-\rho)} \gamma_0^k e^{\frac{\Omega_1\Omega_2}{2(1-\rho)}} \times
\]

\[
\times I_{c-1}\left(\frac{\sqrt{\rho\Omega_1}}{\gamma_0(1-\rho)}\right) = J_1 + J_2.
\]

The pdf of macrocombiner output is presented at Fig. 2 for same values of fading parameters

\[
\rho = 0.6, \quad c = 2, \quad y_0 = 20
\]

![Fig. 2. Probability density function](image)
As above, we calculated Cumulative Probability Density function for random variable \( z \) and it is shown in Fig. 3:

\[
F_z(x) = \int_0^\infty \int_0^{\frac{x}{\Omega_1}} \int_0^{\frac{y}{\Omega_2}} p_{\Omega_1,\Omega_2} (\Omega_1,\Omega_2) d\Omega_1 d\Omega_2 \] 

\[
+ \int_0^\infty \int_0^{\frac{x}{\Omega_1}} \int_0^{\frac{y}{\Omega_2}} p_{\Omega_1,\Omega_2} (\Omega_1,\Omega_2) d\Omega_1 d\Omega_2 \int_0^x p_{\Omega_1,\Omega_2} (\Omega_1,\Omega_2) d\Omega_2. 
\]

(16)

The amount of fading can be calculated by:

\[
A_F = \frac{m_2}{m_1} - 1. 
\]

(19)

The bit error probability for correlated system is shown in Fig. 8 and can be calculated

\[
Pe_c = \int_0^\infty \text{erfc}(z^2) p_z(z) dz. 
\]

(21)

The channel capacity is given by formula (20) and shown in Fig. 7:

\[
C = \int_0^\infty \ln(1 + z^2) p_z(z) dz. 
\]

(20)
The bit error probability for QPSK system can be calculated by (22) and it is shown in Fig. 9:

\[ Pe_2 = \int_0^\infty (\text{erfc}(z^2) + \text{erfc}^2(z^2)) p_z(z) dz. \] \hspace{1cm} (22)

Fig. 9. Bit error probability for QPSK

**Conclusion**

The Probability density function is calculated by closed form of expression, and so: Cumulative probability density function, First order Moment of random variable, Second order Moment of random variable, Amount of fading, Channel capacity, Bit error rate, are calculated in this paper. The output combiner signal is simply the sum of combiner input signals. It was needed to use the binomial formula for calculating n-order moment. Besides this, the general Rayleigh moments of accidental variables are calculated too. The average signal from output of EGC combiner with two or more branches, the square average output signal and the signal variance can be calculated by obtained formulas. Also, the signal probability density can be calculated by, on that way, obtained output signal moments and some assembly of orthogonal functions.

**References**


In this paper the statistics of macro SC diversity systems with two micro EGC diversity systems and Rayleigh fading are given. We calculated and simulated: The Probability Density Function, The Cumulative Probability Density Function, The first order Moment of Random Variable, The second order Moment of Random Variable, The Amount of Fading, The Channel Capacity, The Bit Error Rate. Ill. 9, bibl. 8 (in English; abstracts in English, Russian and Lithuanian).


Исследованы: система выборочного сочетания, система двух микро EGC, а также фединг Релея. Расчитано и сформировано: функция плотности вероятности; интегральная функция распределения; момент переменных первого порядка; момент переменных второго порядка; сумма фединга; емкость канала и коэффициент двоичных погрешностей. Ил. 9, бибl. 8 (на английском языке; рефераты на английском, русском и литовском языках).


Ištirta atrankinio derinimo sistema, dviejų EGC mikro sistema bei Reiljėjaus fedingas. Apskaičiuota ir sumodeliuota tikimybės tankio funkcija, suminė tikimybės tankio funkcija, pirmos eilės kintamųjų momentas, antros eilės kintamųjų momentas, fedingo suma, kanalo talpa, dviejų bei dvejų kaitų koeficientas. Iš. 9, bibl. 8 (anglų kalba; santaukos anglų, rusų ir lietuvių k.).