ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2010. No. 1(97)
ELEKTRONIKA IR ELEKTROTECHNIKA

T 121 SIGNAL TECHNOLOGY
SIGNALŲ TECHNOLOGIJA

Arrhythmic ECG Signals Extraction by Blind Source Separation

K. Pukėnas
Department of Information Science and Languages, Lithuanian Academy of Physical Education, Sporto str. 6, LT-44221, Kaunas, Lithuania, phone +370 37 302668; e-mail: k.pukenas@lkka.lt

Introduction

Blind source separation (BSS) and blind source extraction (BSE)) has received much research attention because of their potential utility in a wide range of applications including many in biomedical signals (ECG, EEG, fMRI, etc.) processing. The objective of BSS is to simultaneously recover all mutually independent unknown source signals only from observations obtained through an unknown linear mixture system, while the objective of BSE – to extract specific sources of interest. This requires the proper use of prior information about the sources or the mixing operation in forcing the algorithm to extract the sources of interest rather than any arbitrary sources. Given observation matrix $\mathbf{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N)] \in \mathbb{C}^{M \times N}$ the general linear instantaneous mixing signal model is [1], [2]

$$\mathbf{X} = \mathbf{AS} + \mathbf{V}, \qquad (1)$$

where N is the number of available samples, M denoting the number of observations (output dimension), K denoting the number of sources (input dimension), $\mathbf{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(N)] \in \mathbb{C}^{K \times N}$ contains the corresponding latent (hidden) components which represent unknown source signals, $\mathbf{A} \in \mathbb{C}^{M \times K}$ represents the unknown mixing matrix describing the input-output relation and $\mathbf{V} \in \mathbb{C}^{M \times N}$ is a matrix of additive noises which are mutually uncorrelated and are also uncorrelated with the sources. The goal is therefore to estimate both unknowns ($\mathbf{A}$ and $\mathbf{S}$) from the measurements $\mathbf{X}$ and in principle all there is to do is to invert the mixing process

$$\hat{\mathbf{S}} = \mathbf{WX}, \qquad (2)$$

where $\mathbf{W} \approx \mathbf{A}^{-1}$ is called the separating matrix. The general BSS problem requires $\mathbf{A}$ to be an $M \times K$ matrix of full rank, with $M \geq K$ (i.e. there are at least as many mixtures as independent sources). In most algorithmic derivations, an equal number of sources and sensors is assumed. Several approaches have been developed for the solution of both BSS and BSE problems, which are based on either second-order or higher order statistics of the data (see [1] for instance). Second-order methods operate in a semiblind context, since their derivation usually requires that certain additional assumptions are made on the nature of the original signals, such as presence of temporal structure in stationary signals, statistical nonstationarity of the sources or cyclostationarity [2]. Such information is usually available in certain biomedical applications, for instance, in physiological signals such as the ECG, and should be exploited. The approximate joint diagonalization (AJD) of a set of real *m*-square symmetrical covariance matrices estimated on different time windows (second-order statistics) is an essential tool in blind source separation algorithms [2]–[4].

Given a group of M sensor signals with N samples $\mathbf{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N)] \in \mathbb{C}^{M \times N}$, the time-delayed covariance matrixes $\mathbf{R}_k \in C^{M \times M}$ of the observation data has the form

$$\mathbf{R}_k = E[\mathbf{x}(t) \cdot \mathbf{x}^T(t + \tau_k)] = \frac{1}{N-1} \mathbf{X}\mathbf{X}_k^T, \qquad (3)$$

$k = 1, 2, 3, \cdots, K$, where K is the index of the maximum time lag, i.e., τ_K and $E[\cdot]$ denotes the statistical expectation operator. Given a set of time-delayed covariance matrixes $\mathcal{R} = \{\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_K\}$, where $\mathbf{R}_k \in C^{M \times M}$, $1 \leq k \leq K$, the approximate joint diagonalization problem seeks a nonsingular diagonalizing matrix $\mathbf{W} \in C^{M \times M}$ and K associated diagonal matrix $\mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \cdots, \mathbf{\Lambda}_K \in C^{M \times M}$ such that the following common structures are best fitted [5]:

$$\mathbf{R}_k = \mathbf{W} \mathbf{\Lambda}_k \mathbf{W}^{\mathbf{T}} \text{ or } \mathbf{W}^{\mathbf{T}} \mathbf{R}_k \mathbf{W} = \mathbf{\Lambda}_k. \qquad (4)$$

The "goodness of fit" is evaluated by some criterion (cost or objective function). It is proved, that matrix $\mathbf{W}$ is closely related to $\mathbf{A}^{-1}$ – inverse of mixture matrix $\mathbf{A}$. The existing algorithms for approximate joint diagonalization are generally divided into two categories: orthogonal and nonorthogonal diagonalizations. In BSS, using orthogonal diagonalization, observations are prewhitened so that they are uncorrelated and have unity variance [1]. However, due to the limitations of orthogonal joint diagonalization, many authors developed nonorthogonal approximate joint diagonalization (NOAJD) methods to avoid prewhitening [5]–[7]. Recently, a sequential blind source extraction algorithm was developed for a class of periodic signals [8] and another – for quasi-periodic signals with time-varying period, based on the partial approximate joint diagonalization of covariance matrices and the time-varying lag calculation procedure [9]. One of the advantages of the sequential algorithms lies in they computational simplicity. But for small number of sources (for example, two sources, where one of them is additive noise), the nonorthogonal joint diagonalization of the covariance matrices with lag dictated by the fundamental period of one of the sources can be easily and succesfully applied to extract the source of interest, i. e. actually the BSE task can be resolved by BSS without large redundant calculations. The main objective of this paper is to: (i) demonstrate, that BSS algorithm using second-order statistics based on the approximate nonorthogonal joint diagonalization of time-delayed covariance matrices (for example FFDIAG [6]) at time varying lag, which is recalculated on a cycle-by cycle basis, is able to extract arrhythmic ECG signals with time-varying period from noise by performing non-orthogonal approximate joint diagonalization only on minimum number of covariance matrices and (ii) present a modified and adapted for ECG signals time-varying lag calculation procedure [9] based on the two stage phase allocation method.

Description of the algorithm

For the source of interest extraction some additional priory information is necessary. If the fundamental period, or its approximation, of the source of interest is fixed and known, then its covariance matrix will have the same value at time lags corresponding to integer multiples of the fundamental period. However, if the source of interest has a period that varies from period to period (for instance, arrhythmic ECG), then joint diagonalization of the autocorrelation matrices at time lags corresponding to integer multiples of the fundamental period leads to erroneous results. The method, proposed in [9], entails detecting the peaks of the quasi-periodic signal that are assumed to define the period of the source of interest, as is the case in ECG signals, and allowing a linear phase signature $\Theta(t)$, to span the range, for example, from 0 to 2π between the peaks. The phase signature is then allocated to each sample of the signal, with the positions of the R-peaks being fixed at $\Theta(t) = 0$, as shown in Fig. 1. It follows that the samples corresponding to a certain specific phase angle are compared along the signal. Therefore, the covariance matrix take into account the variable period τ_t, which is calculated from $\Theta(t)$ from cycle-to-cycle of the signal

$$\mathbf{R}_\tau = E\left[\mathbf{x}(t) \cdot \mathbf{x}^T(t + \tau_t)\right], \qquad (5)$$

where τ_t is defined [8]

$$\tau_t = \min\{\tau \mid \Theta(t+\tau) = \Theta(t),\ \tau > 0\}. \qquad (6)$$

However, for certain arrhythmias (premature atrial contractions, for example) the variation of the ECG signal across R-R interval is not uniform – the portion of ECG signal (for the duration of QT interval approximately) remain relatively fixed length when R-R interval vary noticeably. Therefore, the time varying lag, applicable for varying R-R interval, is not acceptable for this portion of R-R interval.

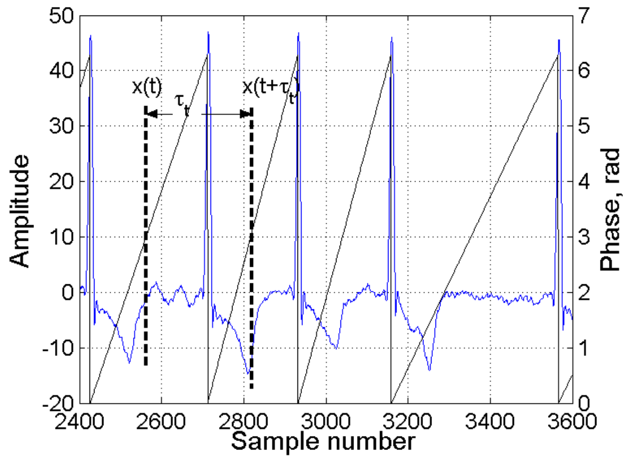

Fig. 1. Phase allocation procedure for computing τ_t. The sawtooth signal depicts the phase signature $\Theta(t)$ ranging from 0 to 2π. A sample at time instant t is compared with the sample at $t + \tau_t$, where τ_t is recalculated on a cycle-by-cycle basis

Considering this, two stage linear phase signature $\Theta(t)$ is proposed – one stage with constant slope in order to span the range, for example, from 0 to π for QT interval (approximately) or more simply – for RT interval (part τ_1 in Fig.2) and another stage in order to span the range from π to 2π for remaining (varying) part of R-R interval (part τ_2 in Fig. 2). This prevents unadequate defining of the relativly stable part of ECG by covariance matrix calculation. Again the covariance matrix is calculated according (5), but τ_t is defined on the basis of two stage linear phase signature $\Theta(t)$. In parallel to calculation of the covariance matrix at time-varying lag, in this work the high performance numerical algorithm FFDIAG (Fast Frobenius Diagonalization) [6], [10] for approximate nonorthogonal joint diagonalization of a set of matrices (5) is used. FFDIAG method is a iterative scheme to approximate the solution of the following optimization problem (more details see [5])

$$\min_{\mathrm{W} \in \mathrm{C}^{\mathrm{M \times M}}} \sum_{k=1}^{K} \sum_{i \neq j} \left(\left(\mathbf{W}\mathbf{R}_k \mathbf{W}^T\right)_{ij}\right)^2. \qquad (7)$$

The matrix of source signals is estimated as

$$\hat{\mathbf{S}} = \mathbf{W}\mathbf{X}, \qquad (8)$$

in which each row represent a separate signal.

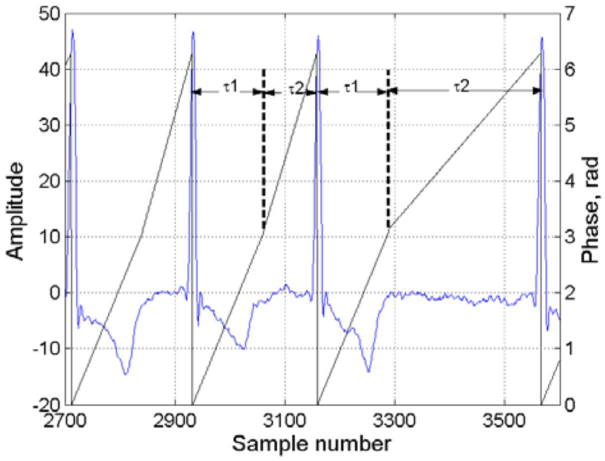

Fig. 2. Modified two stage phase allocation procedure for computing τ_t. The sawtooth signal depicts the phase signature $\Theta(t)$ ranging from 0 to π for part τ_1 of R-R interval (RT interval aproximately) and ranging from π to 2π for remain part (τ_2) of R-R interval

Simulation results

Computer simulations were carried out to illustrate the performance of the proposed algorithm. In the simulation, blind extraction of the ECG signals obtained from MIT-BIH Arrhythmia database (available at: http://www.physionet.org/cgi-bin/ATM) was considered. In the first experiment the ECG signals were mixed with white Gaussian noise (WGN) and in the second experiment – with *x* component of the well-know chaotic Hénon map. The coupling (mixing) matrix $\mathbf{A}$ in both cases is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0{,}15 \\ 1{,}2 & 0{,}2 \end{bmatrix}, \quad (9)$$

i. e. two ECG signals (the number of sensors *M* is equal to the number of signals *K*) with noise ratio (SNR) $SNR \approx 15\text{dB}$ were used. Thus, the performance criteria were evaluated. for one covariance matrix for $lag = 0$ and another – for $lag = \tau_t$ (by performing sequential blind extraction algorithm [8] the number of covariance matrices varies from 5 to 20). For the sawtooth signal of the phase signature $\Theta(t)$ τ_1 = 350 ms was defined. In a simulation environment (the true matrix $\mathbf{A}$ is known) the performance of blind separation can be characterized by one single performance index defined by [11]

$$J(\mathbf{P}) = \frac{\left\| \mathbf{P} - \text{diag}(\mathbf{P}) \right\|^2}{\left\| \mathbf{P} \right\|^2}, \quad (10)$$

where the permutation matrix $\mathbf{P} = \mathbf{WA}$, $\mathbf{P} \in C^{M \times M}$ and $\|\cdot\|$ denotes the Frobenius norm of a matrix. Note that $J(\mathbf{P})$ is non negative and $J(\mathbf{P}) = 0$ holds if $\mathbf{W} = \mathbf{A}^{-1}$.

Fig. 3(a) shows the mixture of arrhythmic ECG signal with time-varying period and WGN, Fig. 3(b) – extracted ECG signal. At chaotic Henon map noise, the picture is similar.

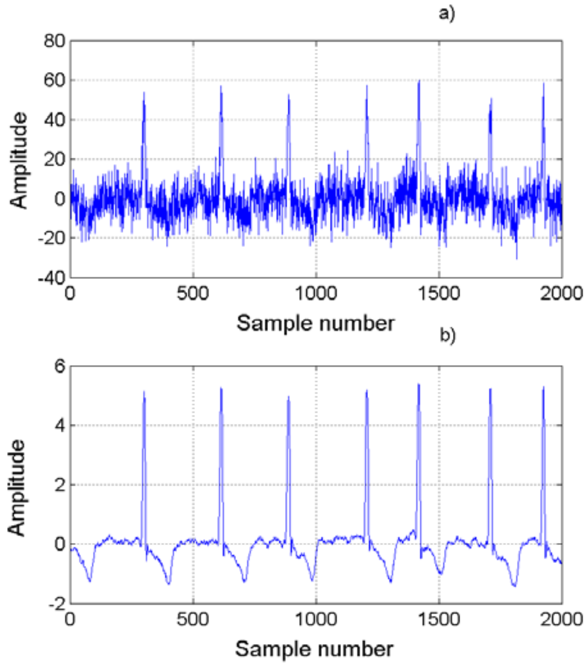

Fig. 3. Mixture of arrhythmic ECG signals with time-varying period and white Gaussian noise (a) and extracted ECG signal using proposed algorithm (b)

The performance index $J(\mathbf{P})$ for extraction three arrhythmic ECG signals with time-varying period from white Gaussian noise and from chaotic Henon map noise is shown in Table 1. In order to compare also $J(\mathbf{P})$ for one stage phase allocation method [9] is calculated.

Table 1. The performance index for extraction arrhythmic ECG signals from white Gaussian and chaotic Henon map noise

No. of ECG	ECG length (samples)/ sample rate, Hz	Additive noise	Performance index	
			One stage phase allocation	Two stage phase allocation
1	3000/128	WGN	0,00312	0,00137
		Henon	0,02653	0,02638
2	10000/360	WGN	0,00621	1,4 E -4
		Henon	0,00024	2,3 E -5
3	10000/360	WGN	0,00969	1,7 E -5
		Henon	0,00034	1,9 E -6

We can see that the combination of FFDIAG and both (based on one– and two stage phase allocation) calculation methods of covariance matrix performed well at moderate amounts of noise. Two stage phase allocation method allows to increase performance of separation.

Conclusions

Blind source separation algorithm using second-order statistics based on the nonorthogonal approximate joint diagonalization of autocovariance matrices at time-varying lag, which is recalculated on a cycle-by cycle basis, and the time-varying lag calculation procedure based on the two stage phase allocation method have been studied in this paper. Herewith, a novel blind source separation algorithm for the extraction of quasi-periodic signals with time-varying period (arrhythmic ECG) is introduced. Simulation results show that algorithm is able to extract arrhythmic ECG signals with time-varying period mixed with white Gaussian noise and with chaotic Hénon map noise (at $SNR \approx 15\mathrm{dB}$) by performing nonorthogonal approximate joint diagonalization using only two covariance matrices – one covariance matrix at zero lag and another constructed at a lag corresponding to the the variable period, which is calculated from phase signature from cycle-to-cycle of the signal. Apllying the proposed two stage phase allocation method for ECG - signals allows to increase the performance of algorithm comparing to one stage phase allocation method.

References

1. **Choi S., Cichocki A., Park H. M., Lee S–Y.** Blind Source Separation and Independent Component Analysis: A Review // Neural Information Processing – Letters and Reviews. – 2005. – Vol. 6, No. 1. – P. 1–57.
2. **Belouchrani A., Meraim K. A., Cardoso J. F., Moulines E.** A Blind Source Separation Technique Using Second–Order Statistics // IEEE Trans. Signal Process. – 1997. – Vol. 45, Iss. 2. – P. 434–444.
3. **Degerine S., Kane E.** A Comparative Study of Approximate Joint Diagonalization Algorithms for Blind Source Separation in Presence of Additive Noise // IEEE Trans. Signal Process. – 2007. – Vol. 55, Iss. 6. – P. 3022–3031.
4. **Pukėnas K.** Blind Source Separation of a Mixture of Pseudoperiodic Chaotic Signals. // Electronics and Electrical Engineering. – 2008. – No. 8 (88). – P. 77–80.
5. **Li X.–L., Zhang X.–D.** Nonorthogonal Joint Diagonalization Free of Degenerate Solution // IEEE Trans. Signal Process. – 2007. – Vol. 55, Iss. 5. – P. 1803–1814.
6. **Ziehe A., Laskov P., Nolte G., Muller K. R.** A Fast Algorithm for Joint Diagonalization with Non–orthogonal Transformations and its Application to Blind Source Separation // Journal of Machine Learning Research – 2004. – Vol. 5 – P. 777–800.
7. **Souloumiac A.** Nonorthogonal Joint Diagonalization by Combining Givens and Hyperbolic Rotations // IEEE Trans. Signal Process. – 2009. – Vol. 57, Iss. 6. – P. 2222–2231.
8. **Jafari M. G., Wang W., Chambers J. A., Hoya T., Cichocki A.** Sequential Blind Source Separation Based Exclusively on Second–Order Statistics Developed for a Class of Periodic Signals // IEEE Trans. Signal Process. – 2006. – Vol. 54, No. 3. – P. 1028–1040.
9. **Tsalaile T., Sameni R., Sanei S., Jutten Ch., Chambers J.** Sequential Blind Source Extraction For Quasi–Periodic Signals With Time–Varying Period // IEEE Trans. On Biomedical Engineering. – 2009. – Vol. 56, No. 3. – P. 646–655..
10. The Matlab code for FFDIAG algorithm is publicly available at http://ida.first.fraunhofer.de/~ziehe/research/ffdiag.html.
11. **Tanaka A., Imai H., Miyakoshi M.** Second–Order–Statistics–Based Blind Source Separation for Non–Stationary Sources with Stationary Noise // Signal Processing and Information Technology, 2006 IEEE International Symposium on. – 2006. – P. 647–650.

Received 2009 09 05

K. Pukėnas. Arrhythmic ECG Signals Extraction by Blind Source Separation // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 1(97). – P. 19–22.

In this paper the blind source separation (BSS) algorithm based on the known nonorthogonal approximate joint diagonalization of several time-delayed covariance matrices and the time-varying lag calculation procedure based on the proposed two stage phase allocation method is investigated by applying it to arrhythmic ECG signals with time-varying period mixed with white Gaussian noise or with chaotic Hénon map noise (at SNR=15 dB). Simulation results show that algorithm is able to extract arrhythmic ECG signals with time-varying period from noise by performing nonorthogonal approximate joint diagonalization of only two covariance matrices – one covariance matrix at zero lag and another – at a lag corresponding to the the variable period, which is calculated from phase signature from cycle-to-cycle of the ECG signal. Ill. 3, bibl. 11, tabl. 1 (in English; abstracts in English, Russian and Lithuanian).

К. Пукенас. Выделение арритмических электрокардиосигналов методом “слепого разделения” // Электроника и электротехника. – Каунас: Технология, 2010. – № 1(97). – С. 19–22.

Исследуется алгоритм “слепого разделения источников” (*Blind Source Separation* – BSS), основанный на сочетании известного метода совместной приблизительной неортогональной диагонализации нескольких ковариационных матриц и метода переменного сдвига по времени при расчете ковариационных матриц, основанного на предлагаемом двухступенчатом распределении фазы переменного периода. Путем анализа смеси реальных сигналов аритмических ЭКГ и белого шума или хаотического сигнала отображения Эно показывается, что алгоритм обеспечивает хорошее выделение сигнала ЭКГ при отношении сигнал-шум порядка 15 дБ и использовании только двух ковариационных матриц – при нулевом сдвиге и переменном сдвиге на один период (интервал R-R), определяемом на основании фазы интервала R-R. Ил. 3, библ. 11, табл. 1 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Aritminių elektrokardiosignalų signalų išskyrimas akluoju metodu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 1(97). – P. 19–22.

Tiriamas aklo šaltinių atskyrimo algoritmas aritminiams elektrokardiosignalams išskirti, gautas derinant žinomą bendros apytikslės kovariacinių matricų neortogonalios diagonalizacijos metodą ir kintamo postūmio laiko atžvilgiu metodą kovariacinėms matricoms apskaičiuoti, pagrįstą siūlomu dviejų pakopų signalo periodo fazės nustatymo būdu. Atlikti tyrimai su realiais aritminiais EKG signalais, sumaišytais su baltuoju Gauso triukšmu arba chaotiniu Henono atvaizdo signalu, rodo, kad algoritmas įgalina gerai išskirti EKG signalą, kai signalo ir triukšmo santykis 15 dB, panaudojant tik dvi kovariacines matricas – esant nuliniam postūmiui ir kintamam postūmiui per vieną periodą (R-R intervalą), nustatomam pagal R-R intervalo fazę. Il. 3, bibl. 11, lent. 1 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).