Introduction

The liver region segmentation from abdominal computed tomography (CT) scans is relatively old problem. A new challenging problem is automation of segmentation process. Main difficulty preventing accurate liver region segmentation comes along with CT imaging modality. Abdominal CT scans can be characterized as images having very poor contrast. This decides the fact that the absorption level of x-ray beams for some abdominal organs coincides. These organs in image are depicted by the pixels that have very similar gray-level. This fact makes the liver region segmentation very difficult because the liver region boundary becomes hardly identifiable. The organs are stomach and spleen. Kidneys and lungs are two more organs beside listed that surround the liver. Fat and muscles can be included also. This article attempts to apply multifractal analysis (MA) to solve this problem in a semi-automatic way.

Dealing with medical data arises two main problems: time consumed to produce results, and accuracy of the results. The first problem is solved by applying innovative solutions in nowadays. Ant system implemented using software processor Microblaze can be such solution [1], for example. Solution of the second problem begins with research and investigation of new mathematical methods applicable to a problem under consideration. A research presented in this article employs MA which is a new way of work [2]. This reference explains mathematical basics of MA and applies it to segmentation of the images. The first problem is solved by applying adapted MA to a set of liver images. The model of liver region, i.e. the image with manually segmented liver region, was prepared considering recommendations of work [7].

Multifractal analysis basics

There are two main steps of MA [2]. The first step is calculation of singularities for each image pixel. Then the multifractal spectrum is calculated and the segmentation of image under investigation is produced. Calculation of singularities mostly coincides with calculation of Hölder exponent. The Hölder exponent is calculated as follows:

\[ a_i(m,n) = \frac{\ln(\mu_i(m,n))}{\ln(i)}, i = 1, 2, 3, \ldots \]  

where, \( \mu_i(m,n) \) is the quantity of measure in \( i \times i = \{g,g\}; g = 2p + 1, p = 0, 1, 2, \ldots \) sized neighbourhoods centered around each pixel of the image. The estimate of \( a_i(m,n) \) is calculated then from bi-logarithmic graph \( \ln(\mu_i(m,n)) \) vs. \( \ln(i) \), using linear regression. After this procedure applied to whole image, \( a \)-image is obtained. To calculate the multifractal spectrum of the \( a \)-image, distribution of \( a \) value is divided as follows:

\[ a_r = a_{\min} + (r - 1)a_{\max}, r = 1, 2, 3, \ldots, R, \]  

where, \( R \) is the number of sub-ranges in \([a_{\min}, a_{\max}]\). Divi-
sion $\Delta a$, is used uniform as follows:

$$\Delta a_r = a_r = (a_{\text{max}} - a_{\text{min}})/ R.$$  \hspace{1cm} (3)

Afterwards the $a$-image is thresholded continuously in each subrange $r$, leaving only those pixels which lie in sub-range $[a_r, a_r + \Delta a]$. Then the dimension $f(a_r)$ of thresholded $a$-image is calculated for each subrange according to the formula

$$f_j(a_r) = -\frac{\ln(N_j(a_r))}{\ln(j)}, \ j = 1, 2, 3, \ldots,$$  \hspace{1cm} (4)

where $N_j(a_r)$ is number of boxes containing at least one $a$-value belonging to the sub-range $[a_r, a_r + \Delta a]$, when $a$-image is covered with regular grid of boxes having size $j$. $N_j(a_r)$ is calculated repeatedly for different size of boxes $j = \{1, 2, 3, \ldots\}$, and $f(a_r)$ is estimated from bi-logarithmic graph $\ln(N_j(a_r))$ vs. $\ln(j)$ using linear regression. After examination of whole $a$-image, the $f(a)$-image is obtained by replacing the pixels in each sub-range $[a_r, a_r + \Delta a]$ with related value of $f(a)$.

**Multifractal spectrum based analysis**

Liver region segmentation problem is based on the computation of multifractal spectrum and relies on two main assumptions. The liver is textured and mathematically can be characterized as multifractal structure. The last assumption allows us to employ MA to the liver region segmentation problem. This article proposes to use image contrast enhancement as a prerequisite in the liver region segmentation problem. The prerequisite gives twofold advantage. The shape of multifractal spectrum obtained is more accurate which gives more accurate reproduction of the liver region contour. This effect is explained in Fig. 1 and Fig. 2. Fig. 1 shows the effect of contrast enhancement on distribution of value $a$ which highly impacts the calculation results of value $f(a)$. Crosses mark value $a$ specific to the liver region. Circles mark value $a$ specific to the liver surrounding, i.e. stomach, spleen, fat, muscles. Value $a$ tends to cluster and clusters around value 0.5 according to the Fig. 1(b) in comparison with Fig. 1(a). These values tend to split between also which effects the spread of value $f(a)$ (Fig. 2). Theoretically, the multifractal spectrum has parabola shape. Exact reproduction of this shape is not possible in practice. Thus, the aim is to ensure more even spread of pairs $(a, f(a))$ that describe the parabola. This affects the shape of the parabola. Contrast enhancement ensures this as it is seen in Fig. 2. The contrast enhancement value needed to ensure requirements listed above is calculated as follows:

$$C = f_1 - f_a,$$  \hspace{1cm} (5)

where $C$ – contrast enhancement value; $f_1$ – the mean of gray-levels of the liver area; $f_a$ – the mean of gray-levels to adjoining area to the liver.

The investigation shows that the effect is achieved when contrast enhancement value ranges from 0.2 to 0.4 for the first data set under investigation. Contrast enhancement value ranges from 0.2 to 0.8 for the second data set under investigation. More thorough description of these data sets is presented in experimental results section. Contrast enhancement is ensured with a contrast-stretching transformation.

This article takes into account four measures mostly used in MA [8]:

Maximum: $\mu_1(m, n) = \max_{(k,l) \in \Omega} g(k,l)$,  \hspace{1cm} (6)

Minimum: $\mu_1(m, n) = \min_{(k,l) \in \Omega} g(k,l)$,  \hspace{1cm} (7)

Sum: $\mu_1(m, n) = \sum_{(k,l) \in \Omega} g(k,l)$,  \hspace{1cm} (8)

Iso: $\mu_1(m, n) = \# \{(k,l) | g(m,n) = g(k,l), (k,l) \in \Omega\}$,  \hspace{1cm} (9)

where $\mu_i(m, n)$ – indicates capacity measures; $g(k,l)$ – indicates gray-scale intensity values of pixels $(k,l)$ belonging to the set of neighborhoods $\Omega$, having size $i$; $g(k,l)$ does not take zero value in case of minimum measure.

The sum measure is used in this article. This allows to manipulate parameter $i$ more freely. Thus, simpler and faster reconstruction of segmented image can be proposed. The other measure faces some problems. Maximum and minimum measures are restricted to use $i = \{1, 3, 5\}$ which means that the reconstruction of segmented image should be more sophisticated. The iso measure is not suitable for the task because segmentation results, produced using it, are not always meaningful.

According to the MA theory the liver region can be segmented by looking for $f(a) \approx 2$ because the liver area is...
textured. Value $\alpha$ should be set equal to 0.5 because this value corresponds to the liver region in distribution $\alpha(m, n)$. The multifractal spectrum was calculated using parameters: $i = \{1, 3, 5, \ldots, 13\}, j = \{1, 2, 3, \ldots, 20\}$. This ensures that the multifractal spectrum will have smooth shape with one clear peak. Then, the peak corresponds to the liver region. Values $i$ were chosen to ensure local enhancement of singularities in image. Values $j$ were chosen to favor small singularities. These values supplement prerequisite and ensures that correct liver contour will be produced. Value $R = 16$ is chosen experimentally to produce smooth multifractal spectrum shape.

Analysis of multifractal spectrum leads to segmented liver image which has some drawbacks. These are: boundary of liver region is mostly discontinuous; regions of adjoining organs reside in image; image has small, spare objects. To avoid these drawbacks morphological reconstruction of liver region from segmented image is proposed.

**Morphological reconstruction**

Algorithm to reconstruct liver region from segmented image is based on mathematical morphology. It consists of four steps:
1. Initial extraction of liver region;
2. Removal of small, spare objects from image;
3. Separation of contacting regions that correspond to areas of adjoining organs;
4. Final extraction of liver region.

These steps are implemented as binary morphological operations. The first, second and the last steps are implemented using morphological reconstruction. The third step is implemented using erosion followed by dilation. The round structuring element with radius equal to four was used in these morphological operations.

**Experimental results**

The results produced by adapted multifractal analysis technique were assessed using three criteria: dice similarity coefficient (DSC), false positive ratio (FPR), false negative ratio (FNR). These criteria are described by the formulas:

\[
E_{DSC} = 2 \frac{|S_L \cap \hat{S}_L|}{|S_L| + |\hat{S}_L|},
\]

\[
E_{FPR} = 2 \frac{|S_R \cap \hat{S}_L|}{|S_R| + |\hat{S}_L|},
\]

\[
E_{FNR} = 2 \frac{|S_L \cap \hat{S}_B|}{|S_L| + |\hat{S}_B|},
\]

where $E_{DSC}$ – dice similarity index; $E_{FPR}$ – false positive ration; $E_{FNR}$ – false negative ratio; $S_L$ – image with manually segmented liver region; $\hat{S}_L$ – image produced with multifractal analysis; $S_B$ – background pixels of manually segmented image; $\hat{S}_B$ – background pixels of image produced with multifractal analysis.

Fig. 3. Evaluation of the liver region segmentation from CT scans using three criteria.

Two well known limiting values of criteria Eq. 10 – Eq. 12 were used to discuss the liver region segmentation results. Higher percentage than 70 % for the DSC shows an acceptable overlap between segmented liver region and the model of liver region. FPR and FNR under 5 % are considered as an acceptable error ratio. CT images under investigation are sized 512×512 pixels with depth of 8 bits per pixel. These images were resized to 256×256 pixels to speed up calculations. Using this procedure only small image details are removed from the image. These are not important in liver segmentation task. Then only part of the whole image, containing the liver region, was examined.

The investigation of adapted multifractal analysis is summarized on two distinct data sets. This approach can be thought positive because these data sets cover all important difficulties arising due to segmentation of the liver region. The difficulties arising are:

- Poor image contrast.
- Weak liver region boundaries.
- Various sizes of liver contact areas that adjoin the neighborhood organs.

Estimation of segmentation results using the three criteria from Eq. 10 – Eq. 12 are shown in Fig. 3. The figure shows high average DSC percentage for both data sets. DSC equals to 82.7 % (±6.9 %) for data set 1 and equals to 95.8 % (±1.7 %) for data set 2. This means good overlap between segmented liver region and the model of liver region. It is seen that it never falls under 70 %. This is good result in comparison to the other works published in this area. The DSC usually falls into range from 75.8 %
to 97.5%. Average FPR percentage equals to 5.9% (±8.9%) for data set 1 and equals to 4.6% (±2%) for data set 2. This shows acceptable error ratio in FPR case. Average FNR percentage equals to 28.9% (±19%) for data set 1, which means unacceptable error ratio. Average FNR equals to 3.8% (±3.3%) for data set 2. These values can be explained partially by some errors introduced with the model of liver region. Some error sources can be seen in Fig. 4. The figure depicts some segmented liver regions using bright contour. Dark contour delineate the contour of the liver region model. It is seen, that the liver vessel areas violate homogeneous structure of the liver area segmented. While average DSC shows promising results, it should be found a way how to have low both FPR and FNR.

Conclusions

This article discusses problems related to the application of multifractal analysis to segment liver region from CT scans. It shows, that the liver region can be described by the peak of multifractal spectrum. To be this true, an accurate multifractal spectrum should be produced. This is ensured by enhancing contrast of CT images that allows setting the proper parameters needed for the calculation of multifractal spectrum.

According to the dice similarity coefficient the liver region segmentation results evaluated are limited in range with the start value 75.8 % and the end value 97.5 %. These results coincide with achievements in this research area published by the other scientists.

References

1. Laptik R., Arminas V., Navakauskas D. Ant System Implementation using Microblaze: Some Preliminary Results on


Multifractal analysis is suggested to solve the liver region segmentation problem from CT scans. It is claimed that image contrast enhancement should be taken as a preprocessing step. It is shown that this ensures more exact reproduction of multifractal spectrum. According to the dice similarity coefficient the liver region segmentation results evaluated are limited in range with the start value 75.8 % and the end value 97.5 %. These results coincide with achievements in this research area published by the other scientists. This suggests that multifractal analysis can be valuable tool to segment the liver region from CT scans.


Kepeñų srities segmentavimą kompiuterinės tomografijos atvaizduose siūloma atlikti taikant multifraktalinės parametrizacijos metodiką. Teigiama, kad taikant šią metodiką, tiksliam kepeñų srities segmentavimui užtikrinti reikalingas kompiuterinės tomografijos atvaizdų kontrasto pagerinimas. Tai leidžia gauti tiksliaiškį singularumų skirstinį, kuris leima galutinį segmentavimo rezultatą. Atliktas tyrimas rodo, kad taikant multifraktalinės parametrizacijos metodiką gautami segmentavimo rezultatai pagal panašumo indeksą yra panasiškos eilės kaip ir kituose šios srities darbuose, t. y. intervale nuo 75,8 % iki 97,5 %. Ii. 4, bibl. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).