

Nonlinear Chirp Pulse Excitation for the Fast Impedance Spectroscopy

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Introduction

Excitation with a short duration pulse is often used to perform the fast spectroscopy – to cover the wide frequency range within short timeframe [1-5].

Variety of well known short excitation waveforms can produce wideband spectral distribution of power e.g. rectangular pulses, pseudo-random maximum length sequence (MLS) of rectangular pulses, Gaussian pulses and its derivatives, sinc and others [1-6]. Though a number of different waveforms are used, the chirp wave is expected to be the most suitable one in many cases [2-5]. The power spectral density (PSD) of the chirp excitation pulse can cover tailored frequency range within given duration in time domain [4-5]. Recently shorter chirp pulses, having only some of cycles or even a fraction of a single cycle, have been analyzed and proposed to be used for the broadband spectroscopy of dynamic impedance. These short waveforms are named “titlets“ by the authors [5].

Linear chirp is a well-known wideband excitation waveform, where the instantaneous frequency f_i , changes linearly during the excitation interval T_{exc}

$$f_i(t) = ktf_0, \quad (1)$$

where f_0 is the starting frequency (at time $t = 0$), and k is the rate of the frequency increase or chirp rate. The instantaneous frequency could also be defined in terms of the derivative of the phase θ

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}. \quad (2)$$

Chirps with a nonlinear rate of the frequency change are also known [7,8]. Unlike the linear chirp, which has a constant chirp rate, a nonlinear chirp has varying chirp rate during the excitation interval. Spectral properties of the chirp waveform depend on

quickenning (or slowing) of the frequency change i.e. acceleration or deceleration of the frequency change.

Rayleigh energy criterion and Parseval's theorem infer that for a linear chirp with a constant bandwidth, PSD must be proportional to pulse width. Consequently, under conditions of constant bandwidth, the PSD must be inversely proportional to chirp rate. Furthermore, the principle of stationary phase infers that the major contribution to the spectrum at any frequency f is made by that part of the signal which has instantaneous frequency f_i . This means that for the nonlinearly modulated chirp, the PSD in the particular frequency range is inversely proportional to the chirp rate in that particular frequency [9].

In the field of impedance spectroscopy an equal spectral density at all frequencies of interest is desirable though smooth PSD curve above level of 0.5 is often also satisfactory. It is also considered necessary that most of the excitation energy will fall into bandwidth of interest [2-4].

Since the spectrum at any frequency f is made by that part of the signal which has corresponding instantaneous frequency, f_i it is possible to shape the spectral density curve by controlling the speed of the frequency change of the chirp. This principle is usable also for titlets - short chirp waveforms.

It is possible to control the chirp rate by combining several exponential signals with different slopes when driving the voltage controlled sinusoidal oscillator (VCO) as described below.

Nonlinearly Modulated Chirp Excitation

Nonlinearly modulated chirp excitation is defined in general as a waveform where the instantaneous frequency f_i changes nonlinearly during the excitation interval T_{exc} . If the growth (or reduction) of the frequency change is exponential then the frequency of the signal varies exponentially as a function of time t :

$$f_i(t) = k^t f_0. \quad (3)$$

The advantage of using an exponential relationship is in ease of realization, Fig. 1. depicts a simplified structure of the exponentially modulated chirp waveform source.

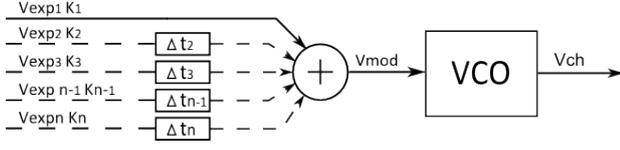


Fig. 1. Exponentially modulated chirp waveform source

Output signal V_{ch} of the VCO can be expressed as follows

$$V_{ch} = V_p \sin(2\pi(f_0 t + \int_0^t K_f V_{mod} dt)), \quad (4)$$

where V_p – a peak amplitude of the signal; K_f – the frequency sensitivity of the VCO in Hz/V. In current study V_p is kept constant at the 1V level. Peak value of the modulating signal V_{mod} is also kept at the 1V level. Input voltages V_{exp} can be expressed as follows

$$V_{exp} = V_o e^{kt} = V_o e^{t/\tau}, \quad (5)$$

where V_o express an initial voltage; k – growth constant; τ – the e-folding time. $\tau = 1/k$ and is called „time-constant“ later in this paper.

In the simplest case only one exponentially changing voltage V_{exp} drives a VCO. However, it was found, that by combining two or more exponential signals with different growth factors it is possible to generate chirp waveforms with substantially steeper slope of spectral curve above the cut-off frequency, compared to linearly modulated chirp or nonlinearly modulated chirp with one exponentially changing voltage V_{exp} .

Linearly modulated chirps have typically -40 dB/decade fall of the amplitude spectrum curve above the cut-off frequency [2–5]. Simple, exponentially modulated chirp marked as Exp.1 has similar characteristics (Fig. 2). Spectrum of the chirp marked as Exp.2 using combination of two exponential signals for modulating the VCO has -60 dB/decade fall of the amplitude above the cut-off frequency. As it can be seen, spectral curves of the exponentially modulated chirps are also more flat in a usable frequency bandwidth.

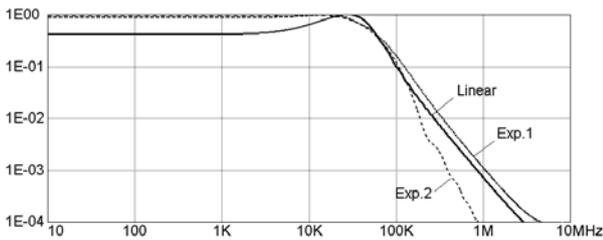


Fig. 2. Normalized RMS spectral density of the exponentially modulated single cycle titlets

All chirps described in this paper are short, one cycle chirps – one cycle titlets.

Further investigation shows, that combination of three exponential signals allows to control the shape of the spectral density curve more flexibly. As it was noted before the peak amplitude of the modulating signal V_{mod} is kept constant, variables are the time delays Δt_n , time-constant τ , direction and sign of the exponential signals and the frequency sensitivity of the VCO, K_f . Resulting waveforms of the modulating signals used in following examples are shown in Fig. 3. Time-constant τ of the signal Exp.1 is 11 μ s. Exp.2 is a sum of upward-sloping exponential signal V_1 with $\tau_1=12 \mu$ s and downward-sloping exponential signal V_2 with $\tau_2=12 \mu$ s, delay $\Delta t_2=5 \mu$ s. Exp.3 is a sum of upward-sloping exponential signal V_1 with $\tau_1=3 \mu$ s and downward-sloping exponential signals V_2 and V_3 with $\tau_2=12 \mu$ s, $\tau_3=5 \mu$ s, delays $\Delta t_2= \Delta t_3= 4 \mu$ s.

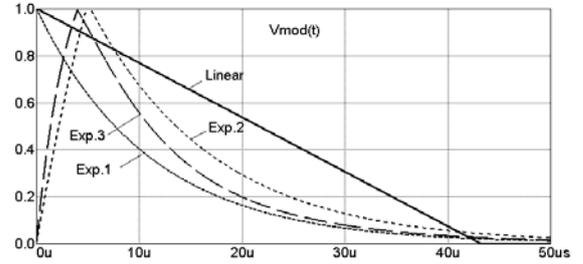


Fig. 3. Modulating waveforms

Corresponding waveforms of titlets at the output of the VCO are shown in Fig. 4.

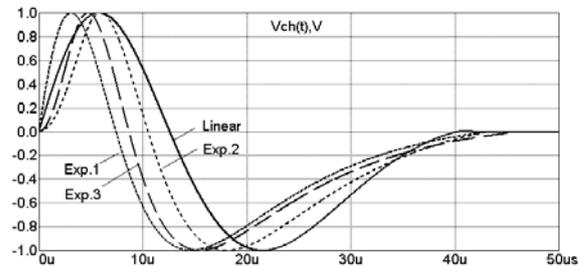


Fig. 4. Titlet waveforms

Frequency sensitivity K_f of the VCO for the linear titlet is 46.5 kHz/V, 92.7 kHz/V for the Exp.1 titlet, 69 kHz/V for the Exp.2 titlet and 80.5 kHz/V for the Exp.3 titlet.

PSD and Energy Distribution of Linearly and Exponentially modulated titlets

Computer simulation give following PSD curves presented in Fig. 5. and Fig. 6. Peak amplitudes of all waveforms are same, +/- 1V. Duration of signals is in the region of 40-50 μ s and was varied intentionally to get the same cut-off frequency where PSD falls to the level of 0.5 (Fig. 6).

The shape of the signal was varied by changing the parameters of the modulating signals to achieve two goals: maximize the energy content and get a flat

and smooth PSD curve in the frequency range of interest.

As it can be seen in Figs 2 and 5, a multislope exponential modulating signal allows substantially steeper slope above the cut-off frequency. However, looking at the situation in the linear scale, see Fig. 6 effect is not so impressive any more. Exponentially modulated titlets perform better than linearly modulated waveforms, but the difference between the simplest Exp.1 and Exp.3 is not significant. However, it must be noted that by changing the parameters of the modulating signal in case of Exp.3, it is possible to raise the first part of the PSD curve above the level of 0.9, but this will drop accordingly the last part of the PSD curve, and this means that overall bandwidth will be narrower.

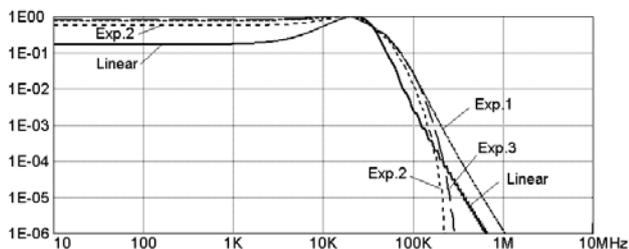


Fig. 5. Normalized PSD of linearly and exponentially modulated titlets, logarithmic scale

PSD curves, where the spectra are normalized separately for each signal are not suitable for the comparison of the energy content of different signals since the maximum values are not equal. To compare the spectral content of different signals, they must be normalized against one of them. PSD curves normalized against the maximum of the linear titlet are presented in Fig. 7.

According to Parseval's theorem, the total energy in the frequency domain must be the same as in time domain. Due to the higher frequency components, a certain part of the signal energy falls outside of the useful bandwidth. Summing up of the PSD values using a small predetermined frequency interval Δf over the full frequency range and dividing the sum of the useful frequency range with total sum, we can find the ratio of useful energy content to the total energy content of the excitation signal. Computer simulation using $\Delta f = 10\text{Hz}$ and full bandwidth of 320 kHz gives following results for the one cycle titlets:

1. Linear titlet – 73 % *
2. Exp.1 titlet – 73 %
3. Exp.2 titlet – 76%
4. Exp.3 titlet – 72 %.

* Note: energy from the starting frequency which is below 0.5 level is excluded.

The slope of distribution curve above the cut-off frequency characterizes the ratio of the energies laying in the desired frequency range and above of it. However, it is also important to pay attention also to the PSD in the desired frequency range.

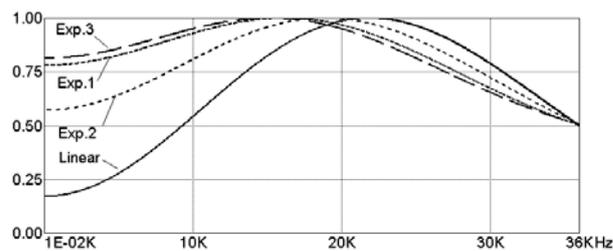


Fig. 6. Normalized PSD of linearly and exponentially modulated titlets, linear scale

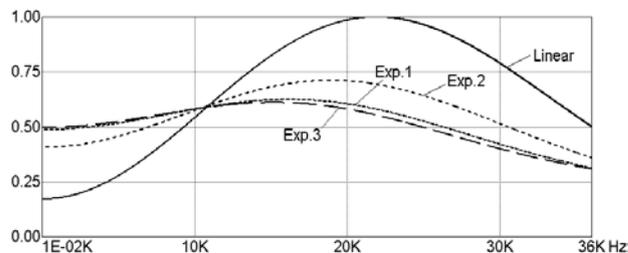


Fig. 7. PSD of linearly and exponentially modulated titlets, linear scale, normalized against the maximum value of the linear titlet

Most likely it is more adequate to compare distribution of the energy of different waveforms in the frequency range of interest since in the field of impedance spectroscopy better signal to noise ratio is usually more important than small power loss outside of the useful frequency range. We can compare the energy content of different waveforms in the frequency range of interest. Computer simulation using frequency interval $\Delta f = 10\text{Hz}$ and cut-off frequency 36kHz, gives following results:

1. Linear titlet – 60 % *
2. Exp.1 titlet – 83 %
3. Exp.2 titlet – 78%
4. Exp.3 titlet – 83 %.

* Note: energy from the starting frequency which is below 0.5 level is excluded.

Conclusions

Nonlinear modulation of the VCO allows to shape the PSD curve of the chirp excitation waveforms. Usage of the exponential modulation signals improves one cycle chirp's energy content in the usable frequency range by more than 20 %, compared to its linear counterpart.

Acknowledgements

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Wideband excitation is used to perform the fast spectroscopy – to cover the wide frequency range within short timeframe. Power spectrum of the chirp excitation pulse can cover tailored frequency range within given duration in the time domain. Linear chirp is a well-known wideband excitation waveform, where the instantaneous frequency varies linearly with time. Chirps with nonlinear relationship between time and frequency are also well known. This group of waveforms shape the power spectrum differently from its linear counterpart. The paper describes the results of the study of spectral properties of the nonlinearly modulated chirp signals, where the frequency of the signal varies according to multi-slope nonlinear relationship over the time. The advantage of this type of chirp signals is that the power spectrum of these signals has substantially steeper slope above the cut-off frequency, compared to its common linear and nonlinear counterparts. Ill. 7, bibl. 9 (in English; abstracts in English, Russian and Lithuanian).

Я. Ояранд, П. Аннус, Р. Ланд, Т. Парве, М. Мин. Нелинейный „chirp” сигнал для быстродействующей импеданс спектроскопии // Электроника и электротехника. – Каунас: Технология, 2010. – № 4(100). – С. 73–76.

Широкополосный генератор используется для получения результата в быстрой импеданс спектроскопии, целью которой является покрытие широкой полосы частот за короткий промежуток времени. Спектр мощности возбуждающего „chirp” сигнала может покрывать необходимый частотный диапазон в течении отведенного времени во временной области. Широко известен линейный chirp-сигнал (сигнал с линейной частотной модуляцией). „Chirp” сигнал с нелинейной зависимостью частоты от времени также хорошо известен и имеет форму спектра мощности отличную от спектра мощности линейного „chirp” сигнала. Данная работа описывает результаты исследований спектральных особенностей нелинейного „chirp” сигнала. Изменение частоты такого сигнала задано несколькими отличающимися друг от друга временными зависимостями. Достоинство рассматриваемых сигналов состоит в лучшем распределении энергетического спектра. Ил. 7, библи. 9 (на английском языке; рефераты на английском, русском и литовском яз.).

J. Ojarand, P. Annus, R. Land, T. Parve, M. Min. Netiesinių „chirp” signalų taikymas greitojo impedanso spektroskopijoje // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2010. – Nr. 4(100). – P. 73–76.

Plačiajuostis generatorius naudojamas rezultatams gauti greitojo impedanso spektroskopijoje, kuria siekiama aprėpti platų dažnių diapazoną per trumpą laiko momentą. Galios spektras gali per nustatytą laiką aprėpti reikiamą dažnių diapazoną, esant „chirp” žadinimo signalui. Taikomas plačiai žinomas tiesinis „chirp” signalas (su tiesine dažnine moduliacija). Netiesinis „chirp” signalas taip pat taikomas ir turi galios spektro formą, kuri skiriasi nuo tiesinio „chirp” signalo formos. Aprašyti netiesinio „chirp” signalo spektriniai tyrimai. Nustatyta, kad, pasikeitus tokio signalo dažniui, patys signalai skiriasi tik laiko parametrais. Il. 7, bibl. 9 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).