http://dx.doi.org/10.5755/j01.eee.20.5.7103 ELEKTRONIKA IR ELEKTROTECHNIKA, ISSN 1392–1215, VOL. 20, NO. 5, 2014

Exploring Types of Instabilities in Switching Power Converters: the Complete Bifurcation Analysis

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***Abstract*—This paper is dedicated to investigation of different types of instabilities in switching power converters under peak current mode control on the basis of improved discrete-time model. The general aspects of construction of complete bifurcation diagrams are supplemented by examples of possible practical applications in the field of power electronics. It is demonstrated that the data obtained from the properties of numerically calculated unstable periodic regimes can be efficiently utilized to estimate some practically relevant parameters of the converter. Special attention is paid to uncommon nonlinear effects occurring in the dynamics of boost converter as the compensation ramp is applied. The possibilities of application of different chaotic modes to the improvement of electromagnetic compatibility are defined in terms of proved robustness of chaos.**

***Index Terms*—Bifurcation, chaos, stability, switching converters.**

I. INTRODUCTION

Switching power converters (SPC), utilizing semiconductor switching elements operated in either saturated or cut-off regions became predominant power conversion circuits in the majority of modern electronic devices mainly due to high efficiency, small weight and physical size. Despite the widespread use, SPC pose serious challenges to power supply designers because of the nonlinearity originated from the switching properties of the converter, varying its configuration every switching cycle.

It is desirable that this type of energy conversion circuits operate periodically with constant switching frequency equal to that of the external driving clock signal. However, variations of system parameters lead to the loss of stability of main period-1 (P1) regime and the occurrence of great variety of instabilities and nonlinear phenomena [1]. It has been demonstrated in [2]–[3] that widely used averaged models are not capable of predicting the vast majority of smooth and non-smooth instabilities observed in SPC. Thus the growing number of research papers and monographs, devoted to the prediction and control of these undesirable operating modes by means of nonlinear models, has been published during last decade [1]–[8].

Most of the proposed techniques exploit very incomplete and rather simple graphs – the brute-force bifurcation diagrams – representing changes in the dynamics of the system as one or several parameters are varied [2]. More advanced methodologies are based on the construction of complete bifurcation diagrams (CBD), obtained by means of numerical calculations, continuation techniques and Floquet theory [7], [9]. The main advantages and general applicability of the CBD to the investigation of complex nonlinear phenomena in most popular SPC have been presented in previous papers [5]–[8]. It has been shown that the construction of CBD, including numerically and analytically calculated stable and unstable periodic regimes, allows prediction and explanation of various types of previously unexplored or incorrectly interpreted complex nonlinear phenomena observed in SPC under current mode and voltage mode control, operating in continuous and discontinuous current modes.

It has been declared by several researchers [3]–[4] that most of the results concerning the investigation of nonlinear phenomena in SPC by means of some generally adopted techniques still have very limited potentiality in solving practical design problems, providing useful information only to the specialists in the field of nonlinear dynamics, but not power electronics. The current research is dedicated to the demonstration of importance of construction of CBD in applications that are of direct relevance to some practical aspects in the design of switching power converters.

Nevertheless the chaotic modes of operation are generally undesirable in SPC, it has been shown that the robust chaos may be utilized to improve the electromagnetic compatibility (EMC) by means of spread spectrum techniques [10], [11]. Moreover, the ability of generation of chaotic patterns directly by SPC could be used by other interconnected circuitry, eliminating the necessity of special chaos generators within the system (e.g. in data encryption schemes). Thus it could be practically preferable to have the SPC that is capable of operating in both – stable P1 regime within wide range of parameters and robust chaotic mode.

The object of current investigation is the peak current mode controlled boost converter, which is proved to become unstable as the duty cycle exceeds 0.5 [2]. The most usual

Manuscript received January 13, 2014; accepted April 09, 2014.
Support for this work was provided by the Riga Technical University through the Scientific Research Project Competition for Young Researchers No. ZP-2013/05.

practical technique used to eliminate the instability of the current loop is the introduction of compensation ramp, which is subtracted from the reference current, extending the range of stable P1 operation of the converter. However, the impact of this signal on other periodic regimes, as well as changes introduced to the topology of the chaotic region, still remain unexplored. The aim of this paper is to discover the influence of the compensation ramp on the mechanisms of instability and robustness of chaos in boost type SPC, highlighting some possible practical applications of the obtained results.

The paper is organized as follows. Section II presents the modified discrete-time model of the current mode controlled boost converter, considering the implementation of compensation ramp. The results of complete bifurcation analysis are presented and discussed in Section III. The concluding remarks of the present work are given in Section IV.

II. Current Mode Controlled Boost Converter With Compensation Ramp

Typical boost converter under peak current control with compensation ramp is shown in the Fig. 1. The outer voltage feedback loop (always present in practical circuit) is eliminated as it has been proved, that the presence of this circuitry does not affect the high-frequency nonlinear dynamics of inner current loop of boost SPC [1], [2].

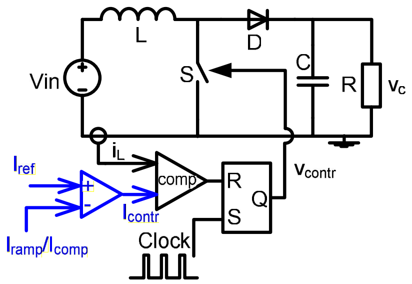

Fig. 1. Simple schematic diagram of the peak current controlled boost converter with compensation ramp.

The switch is turned *ON* periodically in accordance to the clock signal pulses, and turned *OFF* when the value of the inductor current reaches the control signal, composed from the I_{ref} and the compensation ramp I_{ramp} (Fig. 2(a)). Any clock pulses arriving during the *ON* interval are ignored.

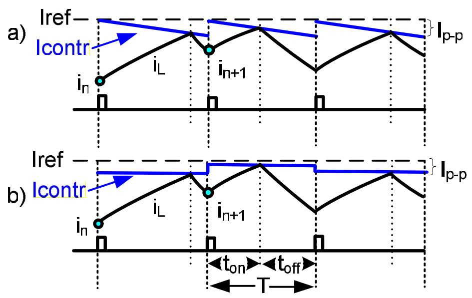

Fig. 2. Interaction of inductor current with compensation ramp: a) analogue; b) digital implementation.

During the analysis of nonlinear dynamics of SPC the use of discrete-time models, obtained in the closed form, proved to be extremely useful, allowing the avoidance of complicated and time-consuming computations associated with systems of differential equations [12]. The discrete-time model of current controlled boost converter consists of the two sets of difference equations according to the two possible sequences of switching events. If the inductor current reaches the control value after the arrival of the next clock pulse, the operation of converter could be described by the following discrete-time model:

$$v_{n+1} = v_n e^{-T/(RC)}, \quad (1)$$

$$i_{n+1} = i_n + V_{in} T L. \quad (2)$$

However, if the clock pulse arrives after the i_L reaches the control value, the model of the system is the following:

$$v_{n+1} = e^{-mt_{off}} \left[K_1 \cos(«t_{off}) + K_2 \sin(«t_{off}) \right] + V_{in}, \quad (3)$$

$$i_{n+1} = e^{-mt_{off}} [C[-m(K_1 \cos(«t_{off}) + K_2 \sin(«t_{off})) + «(-K_1 \sin(«t_{off}) + K_2 \cos(«t_{off}))] + (K_1 \cos(«t_{off}) + K_2 \sin(«t_{off}))/R] + V_{in}/R, \quad (4)$$

where $t_{on} = (I_{ref} - i_n)/(V_{in}/L + S_c)$; $K_1 = v_n e^{-2mt_{on}} - V_{in}$; $t_{off} = T - t_{on}$; $K_2 = \left[I_{ref}/C - \left(v_n e^{-2mt_{on}} + V_{in} \right) \right] / «$; $m = 1/(2RC)$; $p = 1/\sqrt{LC}$; $« = \sqrt{p^2 - m^2}$; $S_c = I_{p-p}/T$.

The borderline I_{border} defines the case, when the clock pulse arrives exactly at the time instance the inductor current reaches the control signal

$$I_{border} = I_{ref} - T\left(V_{in}/L + S_c\right). \quad (5)$$

As it could be seen, the obtained model (1)–(5) includes the term S_c, representing the introduced compensation ramp. After examination of the improved discrete-time model it has been revealed, that the implementation of the compensation ramp within this model is very similar to the practical methodology explored in [13]. The authors propose digital slope compensating technique not using the generated analog ramp signal (see Fig. 2(a)), but rather pre-calculating the comparator's thresholds, depending on the value of valley inductor current i_n (see Fig. 2(b)). Thus the proposed improved model (1)–(5) could be especially useful when the digital implementation of the control circuitry is advisable.

The model presented will be used in order to construct the complete bifurcation diagrams, depicting the qualitative changes in the dynamics of boost SPC as the value of reference current is varied.

The next section represents the results of complete bifurcation analysis with and without the use of compensation ramp for the boost converter with following parameters: R = 40 [Ω]; L = 1.5 [mH]; C = 10 [μF]; T = 100 [μS]; V_{in} = 5 [V]; I_{ref} = [0.4…1.2] [A]; S_c = 0 and S_c = 3000 [A/s].

III. Results of the Complete Bifurcation Analysis

The inherent nonlinearity of switching converters is the consequence of piecewise linear operation in the time domain, giving rise to the dynamics that has not yet been observed in other previously studied nonlinear systems [2]. This special kind of nonlinearity does not allow providing the complete analysis of the phenomena observed in SPC by means of commonly used software packages intended for the investigation of nonlinear dynamics (see e.g. [14]). Due to the incompleteness of existing software tools, the results of this research have been obtained using special MATLAB package, developed by the author within his thesis [5].

The complete bifurcation analysis utilized within this research is based on the numerical calculations of stable and unstable periodic regimes by means of modified Newton-Raphson approach. The stability of regimes is defined using numerical and analytical calculations of monodromy matrixes and evaluation of characteristic multipliers at every single point of the diagram. This approach allows the investigation of the development and interconnections of different subharmonic regimes under the variation of system parameters [6]–[9]. The remaining part of this section is dedicated to the analysis of the constructed complete bifurcation diagrams.

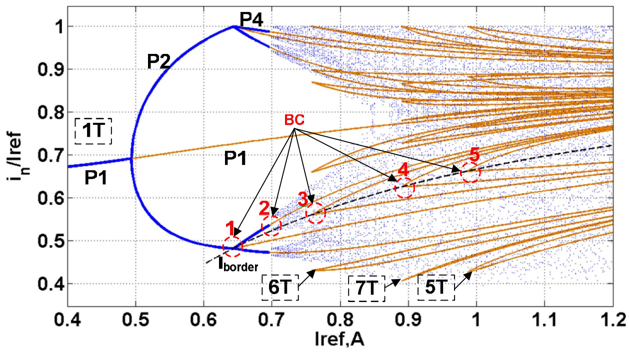

Fig. 3. The complete bifurcation diagram for the boost converter without compensation ramp.

The reference current has been selected as bifurcation parameter being changed in the range [0.4…1.2] [A]. The CBD of the boost converter without compensation ramp is shown in the Fig. 3. Dark lines represent stable periodic regimes, light lines – numerically calculated unstable ones, dashed line depicts the I_{border} (see (5)) and the shaded area represents the chaotic area of operation of SPC.

The constructed diagram shows that as the value of I_{ref} is increased, the system demonstrates smooth transition from stable P1 to stable P2 regime. At the point of this period-doubling bifurcation the P1 does not disappear, but rather becomes unstable, continuing to exist in the whole range of I_{ref} under consideration. Nevertheless the unstable periodic orbits could not be experimentally observed, the study of these regimes is extremely important in explanation of the underlying causes of a great variety of nonlinear effects. As a matter of fact, periodic regimes, even becoming unstable, still define the topological structure of bifurcation diagrams, uncovering the nature of observed phenomena. The mentioned affirmation could be demonstrated by means of the following example.

Figure 4(a), Fig. 4(b) show the unstable manifolds of P1 saddle (unstable) points accordingly for I_{ref} = 0.6 [A] and I_{ref} = 0.75 [A], permitting a thorough geometrical understanding of the structure of existing practically observable attractors in the system [2]. The CBD in the Fig. 3 shows that for I_{ref} = 0.6 [A] the P1 orbit is unstable and the dynamics of the system is governed by stable P2 regime. Nevertheless, Fig. 4(a) depicts the fact, that both P2 points are located on the unstable manifold of P1 saddle. Under variations of system parameters these two points will still be attracted to this manifold just shifting along it. The same observations could be made for the attractors of other regimes (see Fig. 4(b)).

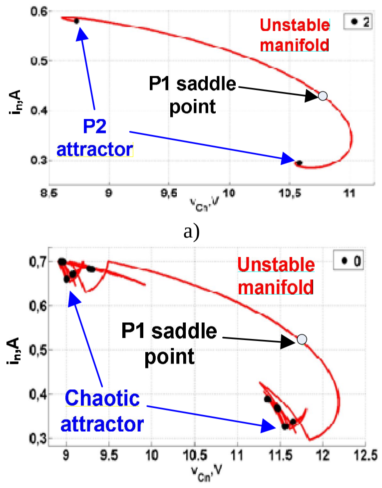

Fig. 4. The unstable manifolds of P1 saddle points and corresponding attractors for (a) I_{ref} = 0.6 [A]; (b) I_{ref} = 0.75 [A].

From the practical point of view the mentioned interconnection means that the characteristics of unstable regime unambiguously define the maximal amplitude of output voltage and inductor current ripples in the SPC, operating in a variety of subharmonic and chaotic regimes, allowing at least the rough estimate of these practically relevant parameters.

The most distinct features of the CBD in the Fig. 3 are the mechanisms of chaotification and the formation of the subsequent structure of chaotic region. The common route to chaos in smooth nonlinear systems is through the infinite number of period-doubling bifurcations, contracting in parameter space until the chaotic attractor is formed [1]. In the case of boost converter under current mode control (Fig. 3) only first period doubling represents smooth transition from stable P1 to stable P2 regime. All the consequent bifurcations are formed by interaction with the borderline defined in (5), representing different kinds of so called border collisions (BC). The first BC causes sudden change of stability of P2 regime and the appearance of stable P4 operation. All the following BC (marked from 2 to 5 in the Fig. 3) lead to the development of uncommon non-smooth bifurcations, destabilizing all previously stable periodic regimes and creating only sets of purely unstable branches leading to the abrupt chaotification.

The chaotic mode of operation in this case is called robust in the sense that there are no small regions of stable periodic regimes (also called periodic windows) within the chaos. The absence of mentioned windows could be explained in terms of interaction of periodic regimes with the borderline, leading to the situation, when only unstable orbits arise (see points 3–5 in Fig. 3). It should be noted that only this robust chaos could be efficiently applied, eliminating the EMC problems within SPC, as well as providing reliable source of chaotic oscillations for other possible applications.

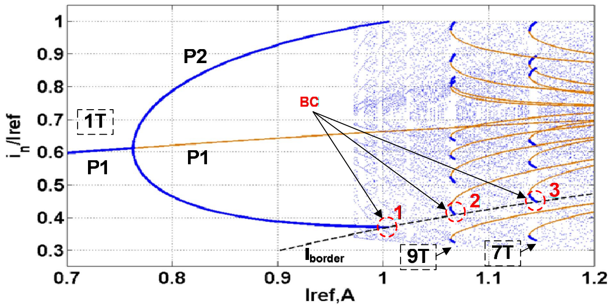

Fig. 5. The complete bifurcation diagram for the boost converter with compensation ramp.

The complete bifurcation diagram shown in the Fig. 5 depicts the evolution of dynamics of boost SPC with implemented compensation ramp. Firstly, it is obvious that the mentioned ramp signal extends the range of stable P1 operating mode (from $I_{ref} \approx 0.5$ [A] in the Fig. 3 to $I_{ref} \approx 0.77$ [A] in the Fig. 5). However, the introduction of this compensation also noticeably changes chaotification scenario and the overall topology of chaotic region.

The first bifurcation, as in the Fig. 3, is also smooth period-doubling, appearing at $I_{ref} \approx 0.77$ [A]. As the I_{ref} is increased the direct transition to chaotic mode of operation and the absence of common period-doubling cascade could be observed. Moreover, the region of coexistence of P2 and chaotic mode is shown, designating the appearance of intermittent chaotic operation under the influence of even small external noise.

The subsequent chaotic regime in this case is not robust as the construction of unstable branches allowed detection of small regions of stable periodic regimes (see 9T and 7T in the Fig. 5). The BC plays an important role in the formation of the overall structure of the bifurcation diagram. Though, in this case collisions with the border (5) do not destabilize periodic regimes, but lead to very uncommon phenomenon, when stable periodic orbits disappear at BC point (see points 1–3 in the Fig. 5). This causes very abrupt changes in the dynamics of the system to appear.

Thus it has been shown that the introduction of compensation ramp noticeably changes the dynamics of boost converter, leading to the appearance of uncommon nonlinear phenomena and regimes that should be avoided.

IV. CONCLUSIONS

In this paper we have investigated various types of instabilities and chaotification scenarios in boost SPC under current mode control.

The significance of detection and continuation of unstable periodic regimes, as well as possible practical applications of obtained results, have been demonstrated, constructing complete bifurcation diagram and unstable manifolds of periodic regimes for the SPC without compensation ramp.

It has been observed that the introduction of compensation ramp signal significantly increases the region of existence of stable P1 operation. On the other hand, it changes the whole division of the parameter space into the regions of stability of subharmonic and chaotic regimes, excluding the possibility of existence of the region with robust chaotic operation. Therefore, it is not permitted to use the compensation ramp in the current mode controlled SPC, if the improvement of the EMC by means of spread spectrum technique is under consideration.

It should be noted that in the proposed model (1)–(5) all switching events are assumed to be ideal, so some unavoidable practical nonidealites (such as switching delays voltage ripples) are disregarded. The investigation provided in [15] shows that the mentioned phenomena may significantly affect the dynamics of the system, underscoring the necessity of development of more complete discrete-time models, which is the first task for the future work.

The second task is a consolidation of existing results of complete bifurcation analysis into a set of practically-oriented techniques that can be directly applied to solve design problems, providing accurate stability information.

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