Neural 3-D Smith Chart

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Introduction

Smith chart is a graphical tool for solving transmission line problems which was developed by P. H. Smith [1]. Every book related on microwave theory or engineering field has had detailed information, solved problems and various applications after its ease of utilization improved on transmission lines [2]. Although analysis and design of microwave circuits are generally tedious with their complicated equations, Smith chart provides a very useful graphical tool or calculator which was improved by computer programmers for these types of the problems with its numerous applications. A microwave engineer having whole concept of the Smith chart in his mind has an ability to picture probable matching solutions for complex problems which includes extensive computation cost.

In the literature, early studies on the computerized Smith chart have been taken place between 1992 and 1995 by Prasad and her group in [3–5]. New generation RF/Microwave circuit designers use sophisticated computer-aided design (CAD) tools to decrease the computation time as much as possible. However, the developments in CAD tools do not eliminate the usage Smith chart in design problems. Especially, designers need to consider stability, gain and noise figure circles on a Smith chart for optimum solution options while designing matching networks. Famous design software packages serve design parameters to be figured in a Smith chart plane. Moreover, network analyzers have provided graphical outputs on a Smith chart (e.g. s-parameters applications).

When negative impedances are involved in design, visualizing the Smith chart as a three-dimensional (3-D) sphere rather than a 2-D circle the Smith chart can perform greater insight [6]. Thus, all possible impedances can be performed on the 3-D Smith chart. The power of the 3-D Smith chart has been strengthened with recent studies which are relevant to the theory of the spherical generalized omnipotent Smith charts [8–10]. Besides this, fractional Smith chart is a novel theory to represent fractional order circuit elements as well [11].

The conventional circular Smith chart on a flat 2-D plane has been modeled with Artificial Neural Networks (ANNs) and its impedance matching application in low-noise amplifier (LNA) has been presented [12-13].

In this paper, the 3-D Smith chart has been modeled with ANN to be utilized on the microwave circuitry with the purpose of enough accuracy having fast and practical in-use. Firstly, the mathematical basics of the 3-D Smith chart would have been explained in next section. Then the transformation of the conventional 2-D Smith in order to 3-D spherical Smith chart using ANNs would have been expressed.

The 3-D Smith Chart

In this section, mathematical analysis of spherical 3-D Smith chart has been indicated briefly to get transformation 2-D Smith chart to 3-D Smith chart and numerical computations [8–10]. Initially, the transformation rule between rectangular normalized Z-plane and polar Γ-plane, that is a one-to-one mapping of the resistance and reactance circles as seen in Fig. 1. The transformation rule has been presented as below

\[ z = \frac{1 + \Gamma_j}{1 - \Gamma_j} \Leftrightarrow r + jx = \frac{(1 + \Gamma_j)(1 - \Gamma_j)}{1 - \Gamma_j} \]

As seen in Fig. 2, two 2-D Smith charts standing side-by-side provides the designer to easily monitor all possible solutions including positive and negative reel part of impedance applications [6, 7].
Fig. 1. Resistance \((0<r<\infty)\) and reactance \((-\infty<x<\infty)\) contours in the impedance and reflection coefficient planes [10]

Fig. 2. Side-by-side 2-D Smith chart including resistance \((-\infty<r<\infty)\) and reactance \((-\infty<x<\infty)\) circles

The main idea of joining left-hand side and the right-hand side charts is necessity of overlapping the same circles (also points) like \(r=0\) circles, as given in Fig. 2. Because they are identical and duplication has not been allowed. If the overlapping operation takes place on the 2-D plane, there will be ultimate complexity via mixing positive and negative real parts.

Zelley has proposed a comprehensible transition way of transforming conventional 2-D Smith chart to 3-D spherical Smith chart without mathematical expressions [6]. Y. Wu et al. have improved mathematical analysis of the 3-D spherical Smith chart [8, 9]. They have introduced two groups of transitions parameters to denote points that generate the resistance and reactance circles on the surface of the unit sphere. The representation of a data point on a resistance or a reactance circle can be developed briefly as follows.

\[
\Gamma_x, \Gamma_y \text{ and } \Gamma_z \text{ are the axes of the 3-D Cartesian coordinate system as illustrated in Fig. 3. The points O and A are the centers of the unit sphere and an arbitrary circle on a plane having angular position to } \Gamma_x-\Gamma_y \text{ plane, respectively. The points B, C and D are on the surface of unit sphere and on the circle, too. So the } |BD| \text{ line defines the diameter of the circle and it’s perpendicular to the } |OA| \text{ line. The two transition parameters are angles which can be defined as } \square ABO = \phi \text{ and } \square BAC = \theta \text{. The length of the line } |OB|=1 \text{ and the radius of the arbitrary circle is } |AC|=|AB|=|OB|\cos \phi = \cos \phi. \text{ The } \Gamma_y \text{ axis value of the point } C \text{ that is a random selected point on the circle can be obtained as below}
\]

\[
\Gamma_{yc} = |AC| \sin \theta = \cos \phi \sin \theta. \tag{2}
\]

The value of \(\Gamma_z\) axis of point \(C\) can be obtained via the projection of \(|AC|\) to \(|AB|\) which is \(|AC|_{AB}\):

\[
\begin{align*}
\Gamma_{zc} &= (|AB| - |AC|_{AB}) \sin \phi, \\
\Gamma_{zc} &= (\cos \phi - \cos \phi \cos \theta) \sin \phi, \\
\Gamma_{zc} &= \sin \phi \cos \phi - \sin \phi \cos \phi \cos \theta. \tag{3}
\end{align*}
\]

Similar way to get the \(\Gamma_x\) axis value of \(C\) can be tracked:

\[
\begin{align*}
\Gamma_{xc} &= |OB| - (|AB| - |AC|_{AB}) \cos \phi, \\
\Gamma_{xc} &= 1 - (\cos \phi - \cos \phi \cos \theta) \cos \phi, \\
\Gamma_{xc} &= 1 - \cos^2 \phi + \cos^2 \phi \cos \theta = \sin^2 \phi + \cos^2 \phi \cos \theta. \tag{4}
\end{align*}
\]

All possible points on the surface of the unit sphere can be denoted by using transition parameters which are exactly providing angular intervals in radians as \(-\pi<\phi<\pi\) and \(-\pi/2<\theta<\pi/2\). Thus, whole surface of the unit sphere can be scanned by an infinite number of circles.
The points that generates resistance and the reactance circles of the spherical 3-D Smith chart is obtained via renaming coordinates $\Gamma_x \rightarrow \Gamma_i$, $\Gamma_y \rightarrow \Gamma_r$, $\Gamma_z \rightarrow \Gamma$ for resistance circles and $\Gamma_x \rightarrow \Gamma_d$, $\Gamma_y \rightarrow \Gamma_i$, $\Gamma_z \rightarrow \Gamma_i$ for reactance circles, respectively. So the coordinates of the $r$ (resistance) points and the $x$ (reactance) points on the surface of the spherical 3-D Smith chart can be defined as follows in (1) and (2), (3), (4) respectively:

$$
\begin{align*}
\Gamma_r &= \sin^2 \phi_r + \cos^2 \phi_r \cos \theta_r , \\
\Gamma_i &= \cos \phi_r \sin \theta_r , \\
\Gamma_z &= \sin \phi_r \cos \phi_r - \sin \phi_r \cos \phi_r \cos \theta_r , \\
\Gamma_x &= \sin \phi_x \cos \phi_x - \sin \phi_x \cos \phi_x \cos \phi_x \\
\Gamma_y &= - \cos \phi_x \sin \theta_x 
\end{align*}
$$

(5)

Some $r$ and $x$ circles have been generated in Fig. 4 using (5) and (6) with $-\pi < \phi_r \leq \pi$ and $-\pi/2 \leq \phi_x \leq \pi/2$ intervals which blue circles are for $x$ points and red circles are for $r$ points.

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\Gamma_i &= \cos \phi_r \sin \theta_r , \\
\Gamma_z &= \sin \phi_r \cos \phi_r - \sin \phi_r \cos \phi_r \cos \theta_r , \\
\Gamma_x &= \sin \phi_x \cos \phi_x - \sin \phi_x \cos \phi_x \cos \phi_x \\
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At the moment, the most important point is how the relationship can be obtained between conventional 2-D Smith and 3-D Smith chart coordinate system. If Fig. 5 has been examined, the projections of the $r$ and $x$ circles (red and blue ones) of the 3-D Smith chart would have not been overlap the $r$ and $x$ circles of the 2-D conventional Smith chart on the $\Gamma_r-\Gamma_i$ plane. So, there is a nonlinear transformation between normalized impedance $z=r+jx$ and transition parameters $\phi_r$, $\phi_x$, $\theta$, and $\theta$. The projection of the diameter $|BD|$ equals to two times of the radius of the concerned resistance circle of the conventional Smith chart on the $\Gamma_r-\Gamma_i$ plane as seen in Fig. 3.

$$
\left( \frac{\Gamma_r - r}{r+1} \right)^2 + \Gamma_i^2 = \left( \frac{1}{r+1} \right)^2.
$$

(7)

Evaluating this rule and using (7) which has been used for plotting or defining $r$ circles on the 2-D Smith chart the nonlinear relationship between $r$ and $\phi_r$ has been obtained as below:

$$
|BD| \cos \phi_r = 2 \frac{1}{r+1} \Rightarrow 2 \cos \phi_r \cos \phi_r = 2 \frac{1}{r+1},
\Rightarrow r = \tan^2 \phi_r.
$$

(8)

The $-r$ (negative $r$) circles are all on the lower hemisphere, so the relationship can be generalized between all possible $r$ circles and $\phi_r$ angles as follows:

$$
\frac{r}{r+1} \Gamma_r - 1 = \left( \frac{\Gamma_i}{r+1} \right) = \left( \frac{1}{x} \right)^2.
$$

(10)

Using similar angular relationships in Fig. 6, the radius of a random oblique circle ($P$ is the center point) on the surface of the spherical Smith chart has been obtained as

$$
|KP| = |BP| = |BO| \sin \phi_x = \sin \phi_x.
$$

(11)

The diameter of corresponding $x$ circle $|BR|$ on the conventional Smith chart has been presented as follows:

$$
\frac{2}{x} \frac{2 \cos \phi_x}{\sin \phi_x}.
$$

(12)
The nonlinear relationship between all possible $x$ circles and $\varphi_x$ angles can be obtained

$$x = \tan \varphi_x$$  \hfill (13)

and inverse relationships of (9), (12) and (13) can be obtained as follows:

$$\varphi_x = \frac{1}{2} \text{sign}(r) \arccos \left( \frac{1-|r|}{1+|r|} \right),$$  \hfill (14)

$$\varphi_x = \arctan x.$$  \hfill (15)

If a point on the surface of the 3-D Smith chart is necessary to be sketched, the parameters $\theta_r$ and $\theta_x$ would have been needed. It is known that a $z=r+jx$ point has been sketched via $r$ and $x$ circle intersection on the conventional 2-D Smith chart. Consequently, $\Gamma_r$, $\Gamma_x$, $\Gamma_x$ of $r$ and $x$ circles have been equaled using (5) and (6):

$$\Gamma_r(\varphi_r, \theta_r) = \Gamma_x(\varphi_x, \theta_x),$$  \hfill (16)

$$\Rightarrow \sin^2 \varphi_r + \cos^2 \varphi_r \cos \theta_r = \sin^2 \varphi_x + \cos^2 \varphi_x \cos \theta_x,$$  \hfill (17)

$$\Gamma_x(\varphi_r, \theta_r) = \Gamma_x(\varphi_x, \theta_x),$$  \hfill (18)

$$\Rightarrow \cos \phi_x \sin \theta_r = \cos \phi_x \cos \phi_x \sin \phi_x - \sin \phi_x \cos \phi_x \cos \theta_x.$$  \hfill (19)

By omitting $\cos \theta_x$ in (16) and (17) it has been obtained as

$$\theta_r = 2 \arctan \left( \frac{1}{\tan \varphi_x \cos \varphi_r} \right)$$  \hfill (19)

and by similar approach and omitting $\cos \theta_r$ in (16) and (17) it has been obtained as below

$$\theta_x = -2 \arctan \left( \frac{1}{\tan \varphi_r \cos \varphi_x} \right).$$  \hfill (20)

As a result, all possible $r$ and $x$ circles from 2-D Smith chart can be transformed to related $r$ and $x$ circles on the 3-D Smith chart via equations (14), (15), (19) and (20). The inverse transformation has been possible via equations (9), (12) and (13).

### ANN Model of 3-D Smith Chart

The advances in the computational sciences have made nonlinear learning machines possible, which enable to generalize discrete data into the continuous data domain. ANNs are fast and accurate nonlinear learning machines in their matured forms and capable of the parallel processing. Thus, they have found too wide applications in areas of science and engineering. Neural networks are also universal function approximators allowing reuse of the same modeling technology for both linear and nonlinear problems at both device and circuit levels. Neural network models are simple and model evaluation is very fast.
data. Unless the testing results satisfy, different types of
the ANN structures with varied hidden sizes and training
algorithms have been tried. A loop has been created to
get these trials for best test results.

The Levenberg-Marquardt (LM) back-propagation
algorithm for the smallest testing error and four layered
network with the minimum number of neuron for faster
training have been performed with the MLPNN. The
performance function of this network is the Mean Squared
Error (MSE). The training convergence of the algorithm is
taken place in Fig. 8.

Results

The proposed two hidden layer MLPNN has provided
performance with the minimum training error which has
been achieved after nearly 400 epochs as 2.45x10⁻⁵ and
average testing error as 4.45x10⁻⁴. The MLPNN and the
computed (target) output values of the parameters for
different r (red holes and stars) and x (blue holes and stars)
circles have been given in Fig. 9(a) and Fig. 9(b). The
values of the examples have been chosen within the testing
data set.

![Fig. 8. Training performance of LM backpropagation algorithm](image)

Likewise, an infinite number of arbitrary r and x
circles are able to be plotted, but the resolution of any figure
type doesn’t allow. The Tab. 1 has been presented to
compensate this leakage. When it has examined, the
selected values which are characteristic values of a 2-D
Smith chart (e.g. z=±50Ω) and others are extreme to show
proof of ANN learning ability satisfy to use “Neural 3-D
Smith Chart” in microwave circuitry.

By the way, all computations and manipulations of
ANN design have been performed by MATLAB®.

![Fig. 9. Comparison of ANN outputs and target data (a) and
zooming in of (a) for overlapped data points (b)](image)

Consequently, a “Neural 3-D Smith Chart” has been
formed by using the ANNs in the simple MLPNN
structures as the nonlinear learning machines from the
input space to the output space. This novel “Neural 3-D
Smith Chart” has been recommended to use in microwave
ingineering design, especially in some requirements for
negative resistances to be used, especially in active filter,
oscillator designs and etc. Future studies will involve
applications of microwave engineering designs via “Neural 3-D Smith Chart”.

References


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In this paper, the three-dimensional (3-D) Smith chart has been modeled with ANN to be utilized on the microwave circuitry with purpose of enough accuracy having fast and practical in-use. Firstly the theory of the 3-D Smith chart has been explained. Then the transformation of conventional 2-D Smith chart to 3-D spherical Smith chart using ANNs has been proposed. Results have shown that proposed ANN structure learns all possible points on the surface of the spherical 3-D Smith chart with enough approximation and accuracy. Consequently, a “Neural 3-D Smith Chart” has been formed by using the ANNs in the simple MLPNN structures as the nonlinear learning machines from the input space to the output space. Ill. 9, bibl. 13, tabl. 1 (in English; abstracts in English and Lithuanian).


Analizuojama trimatė Smito diagrama, sudaryta dirbtinių neuroninių tinklų pagrindu. Mikrobangų teorijoje tokia diagrama gali būti taikoma tikslumu padidinti. Pateikiamą informaciją aptie trimates Smito diagramas. Pasiūlytas būdas, kaip, taikant dirbtinius neuroninius tinklus, pererti į dvimatęs Smito diagramos į trimatę. Nustatyta, kad rezultatai pagerėja, kai mokomi visi galimi sferinio paviršiaus taškai. Ill. 9, bibl. 13, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).