Computer Restoration of 2D Medical Diagnostic Signals with Noise Frequency Spectrum

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Introduction

In previous research articles of the authors interpolation of ECG signal was made [1–6]. The method used is the one from L. Aizenberg[1] for analytical continuation of finite functions. In connection with X-ray studies, CT scans and other application tasks require restoration of the Fourier spectrum, if its values are unknown at a certain interval. That task is to make interpolation of the spectrum in this interval. The article presents a calculation and experiment results, in a recovery of the spectrum of X-ray image by interpolation. Computing is done in MATLAB.

Theoretical formulation of the task to recover the spectrum of an image

The mathematical method developed by Aizenberg includes formulas for analytic continuation (extrapolation and interpolation) of finite functions. A brief description of the method is as follows:

If N_j is a limited sequence of different points in the halfplane \{z \in D: \text{Im } z > \sigma \} which has no points of density over its contour \(1 \leq j \leq n\), let us consider the following problem of restoration of function \(f(z)\) in \(D_{\sigma}\) where \(D_{\sigma} = \bigcap_{j=1}^{N} \{z \in D: \text{Im } z > \sigma\}\), in the set \(M=N_1 \times N_2 \times \ldots \times N_n\).

For \(N_l=\{x_lj\}\) is valid (1)

\[
\alpha_{(m, p), l} = \frac{x_{mp} - x_{lp} + 2\sigma}{u - x_{mp} + 2\sigma} \prod_{j \in p} (u - x_{mp} + 2\sigma)(x_{mp} - x_{lp})
\]

and \(f(z)=\lim_{m \to \infty} \sum_{k=1}^{m} \sum_{l=1}^{m} f(z) \prod_{j=1}^{m} \alpha(m, z, k, l),\) (2)

where \(H\) is a Hardy’s class of functions, \(\sigma>0\) is a parameter, \(\overline{x}\) is a conjugate complex value of \(x\).

The following formula, \(f(z)=\lim_{m \to \infty} \sum_{k=1}^{m} \sum_{l=1}^{m} f(z) \prod_{j=1}^{m} \alpha(m, z, k, l),\) (2)

\[
f(z)=\lim_{m \to \infty} \sum_{k=1}^{m} \sum_{l=1}^{m} f(z) \prod_{j=1}^{m} \alpha(m, z, k, l),
\]

where \(H\) is a Hardy’s class of functions, \(\sigma>0\) is a parameter, \(\overline{x}\) is a conjugate complex value of \(x\).

The formula (2) can be used for interpolation of function \(f(x)\) in Wiener class \(W^\alpha\)

\[
f(x)=\lim_{m \to \infty} \sum_{k=1}^{m} \sum_{l=1}^{m} f(z) \prod_{j=1}^{m} \alpha(m, z, k, l),
\]

(3)

Let us examine a spectrum with noise in a certain frequency range. The parameter \(m\) determines the number of points (spectral values) left and right of this interval, this is the number of known values for the restored signal. The image is a function of two real variables and the formula (3) can be used for interpolation of each row of the matrix spectrum of the image. Formula (3) includes the parameters \(m\) and \(\sigma\). From their values depends the accuracy of calculations. A MATLAB function that searches the optimal ratio between \(m\) and \(\sigma\) has been developed. It is determined by the minimum of the Mean square error (MSE) between the interpolated and actual values of the image. Once the initial \(m\) and \(\sigma\) for the first row of the matrix are determined, and then with those values interpolation for all other lines in it are carried. By assessing the accuracy of interpolation of the image in the time field the maximum absolute error is used. It is calculated with the following formula

\[
\varepsilon = \max(|S - I|),
\]

where \(S\) is the matrix of the original image, and \(I\) - the matrix of the interpolated image in the time field.

Computational experiments and results

The X-ray image in format. BMP (24-bit Bitmap) is imported in Matlab [2]. In Workspace it is recorded as three-dimensional array with a size 288x431x3, containing
values from the type uint8 (numbers in the interval [0..255]).

The color of each pixel is presented from the triplet RGB. Each of the three matrices in the massive with the dimension 288x431 includes saturation of the red (R-matrix), green (G-matrix) and blue (B-matrix) colors.

The overall appearance of the imported X-ray image is displayed with the command imagesc and is shown in Fig. 1.

![Fig. 1. Overview of the X-ray image](image)

For purposes of the computing experiment, proving the applicability of the method of Aizenberg, a simulation for noised spectrum and subsequent interpolation only R-matrix has been made, hereinafter briefly called image. The restoration of the rest of the signals could be done on the same way. Similarly, it is committed to restoring the values and the other two matrices. Fig. 2 shows the type of the image.

![Fig. 2. The image on which the calculation of the experiment is carried out](image)

The algorithm, which simulates the loss spectrum of the image and then its restoration according to the method of Aizenberg follows the steps in the block diagram of Fig. 3.

![Fig. 3. A summary scheme for the recovery of the algorithm of the image](image)

On Fig. 4 is displayed the spectrum of R-matrix.

![Fig. 4. The spectrum of the image](image)

The graph of the noised spectrum is shown in Fig. 5. 288 values in total (all of 11 column) from the spectrum of the matrix of the image are replaced with $10^5$.
The values of the parameters that the function performed interpolation of the image are $\sigma = 0.1$ and $m = 19$. As an assessment of the approximation which is made the absolute error between the actual matrices and interpolation images in the time domain are calculated. The value obtained for the maximum absolute error in the time domain is 9.9708.

Conclusions

The obtained results demonstrate the applicability of the method of Aizenberg for the recovery of medical images with incomplete spectrum. Better accuracy of the calculations could be obtained if for each row of the matrix of the image the optimal ratio between $m$ and $\sigma$ is calculated. For matrices with such large dimension the performance of the program will be reduced significantly. The optimal ratio between accuracy of the fast recovery performance of the algorithm, and the restoration of more lost values in the spectrum are the subject of future investigations.

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References

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An application of mathematical method of Aisenberg for restoration of low frequency medical diagnostic signals after influence of noise is described in the paper. The restoration of frequency spectrum of medical diagnostic signals has been done after preliminary analyses of frequency spectrum of signals with noise and disposition of frequency band of noise in the frequency band of information signals. Some experimental results obtained on the base of application of Aisenberg’s method for restoration of medical diagnostic signals are described in the paper. Ill. 9, bibl. 8 (in English; abstracts in English and Lithuanian).


Aprašomas Aisenbergo metodo taikymas signalui atkurti, kai iš medicininės diagnostinės sistemos perduodamas triukšmo paveiktas žemojo dažnio signalas. Šis signalas atkuriamasvykdomas po to, kai išaiškinamas triukšmo dažnio spektras. Pateikiami eksperimentiniai rezultatai, gauti taikant Aisenbergo metodą. Il. 9, bibl. 8 (anglų kalba; santraukos anglų ir lietuvių k.).