Efficiency of Electronic Devices for Unsteady Flow of Failures

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Introduction

In the paper, electronic devices are considered. At exploitation of devices, the control of serviceability state by the means of the built-in test equipment is stipulated. Also, periodic checks are executed. At the organization of exploitation, it is important to correctly choose characteristics of maintenance system [1]. The degree of rationality of the given decisions can be estimated by means of an index of efficiency of devices [2, 3].

At calculation of efficiency, it is necessary to consider properties of a flow of device failure. It is practically expedient to consider two kinds of flows of failure. The first kind is the Poisson flow, i.e. stationary flow of failure. The second kind of flow of failure is unsteady Poisson flow.

In the article, the method of calculation of efficiency of devices for unsteady Poisson flow of failure is analyzed. It is offered to evaluate the bottom and top limit for values of efficiency within a considered time interval of exploitation. The given evaluations are connected with the size of efficiency for the Poisson flow of failure. Also, a relative error in an evaluation of true value of efficiency of devices is considered.

Evaluation of efficiency index at Poisson flow of failures

Electronic devices, for which \( N \) controls of state of serviceability with periodicity \( T \) is stipulated in interval between periodic inspections, are analyzed. Failures of devices, depending on character of their display, shall be subdivided into two kinds. Failures of the first kind are identified directly according to external attributes. Failures of the second kind are found out at carrying out control of serviceability state and inspections.

In process model of devices behavior in exploitation, the following states (set of states \( E \)) are considered: \( Z_0 \) – restoration and extraordinary inspection; \( Z_{11}, Z_{12}, \ldots, Z_{1(N+1)} \) – serviceable at the intended use within interval of time after last inspections, the first, \( ..., N \) control of serviceability state (subset states \( E_1 \)); \( Z_{21}, Z_{22}, \ldots, Z_{2(N+1)} \) – non – serviceable, there is a failure of the second kind at the intended use within an interval of time after last inspection, the first, \( ..., N \) control of serviceability state (subset states \( E_2 \)); \( Z_{31}, Z_{32}, \ldots, Z_{3N} \) – non – serviceable, there is a failure of the second kind and the first, the second, \( ..., N \) control of serviceability state is carried out (subset states \( E_3 \)); \( Z_{41}, Z_{42}, \ldots, Z_{4N} \) – serviceable, and the first, the second, \( ..., N \) control of serviceability state is carried out (subset states \( E_4 \)); \( Z_{4(N+1)} \) – serviceable and periodic inspection is carried out.

Let's consider, that the effect of the use of the device is possible in cases when it is serviceable and is used as designed [4]. Then, the index of efficiency of use of the device can be calculated as follows [5]

\[
W = \sum_{j=1}^{N} W_j P_j ;
\]  

where \( P_j \) – stationary probability of \( j \) state of the device; \( W_j \) – efficiency of use the device in \( j \) state. Size \( W_j \) shall be defined as follows:

\[
W_j = \begin{cases} 
W_1, & j \in E_1, \\
0, & j \in E \setminus E_1
\end{cases},
\]  

Based on (2), we receive:

\[
W = \sum_{j \in E_1} W_j P_j = W_1 K ,
\]
\[ K = \sum_{j \in E} P_j. \]  

(4) The index \( K \) calculated by (4), represents utilization factor of the device. For its definition, the description of change of states of the device in exploitation shall be executed by means of semi – Markov casual process.

The given process is characterized by matrixes \( P_y(t) \) and \( P_y(t) \), is a matrix of transitive probabilities \( p_y \) \((i, j \in E)\). Matrix \( P_y(t) \) represents functions of distribution of conditional time in various states \( F_y(i, j \in E) \). The stationary value of utilization factor is therefore as follows [6–9]

\[ K = \sum_{i \in E} \pi_i a_i / \sum_{i \in E} \pi_i a_i, \]  

(5) where \( \pi_i \) – stationary probability of state \( Z_i \) of Markov chain; \( a_i \) – mean absolute time of a process in state \( Z_i \). Probabilities \( \pi_i \) can be defined by the following system of equations:

\[
\begin{align*}
\pi &= \pi p; (i \in E), \\
\pi &= 1.
\end{align*}
\]

(6) Time \( a_i \) is

\[ a_i = \sum_{j \in E} p_y \int_0^\infty dF_y. \]  

(7)

Let's consider a case of the Poisson flow of failures. Let's enter the following designations: \( \Lambda \) – failure rate of the device; \( \alpha \) – index describing a share of failure of the second kind; \( \alpha \) – probability of detection of failure of the second kind at the control of serviceability state \((0 < \alpha < 1)\); \( F(t) \) – distribution function of time of non-failure work of the device \( F(t) = 1 - \exp(-\Lambda t); F_i(t) \) – distribution function of work time of the device without failures of the first kind \( F_i(t) = 1 - \exp(-(1-\alpha)\Lambda t) \); \( T_0 \) – time between failures of the device, \( T_0 = 1/\Lambda; T_r \) – restoration mean time of the device; \( \tau_p \) – mean time of control of serviceability state; \( \tau_r \) – mean time of check.

Taking into account [8], it is reasonable:

\[
\begin{align*}
\alpha &= TF(T); i \in E, \\
\alpha &= F(T)T(1-\alpha); i \in E, \\
\alpha &= \tau_T; i \in E, \\
\alpha &= \tau; i \in E, \\
\alpha &= a = a = \tau, \\
\alpha &= T.
\end{align*}
\]  

(8) Utilization factor \( K \) shall be as follows [8]

\[ K = \left\{ 1 + T_r + \tau_r + \frac{1}{1 - p_1^{N+1}} \left[ \frac{1 - p_1^{N+1}}{1 - p_1} \right] \right\} \]  

(9) where \( B_1 = \frac{1 - p_3}{1 - \alpha} + \tau_k p_r; B_2 = \frac{1 - p_3}{1 - \alpha} + \tau_r p_3; \\
T_r = \frac{T_r}{T_0}; \tau_p = \frac{T_r}{T_0}; \tau_k = \frac{T_r}{T_0}; p_1 = 1 - F(T); \\
p_2 = \alpha F(T); p_3 = 1 - F_1(T); p_4 = 1 - \alpha.

Evaluation of efficiency index at unsteady flow of failure

The real flow of failure of electronic devices in exploitation can be closer to unsteady Poisson flow. The question of efficiency evaluation for the given case is analyzed further. Let's analyze process of exploitation within a time interval \([ t_1, t_2 ]\).

Let's consider that at \( t = t_1 \), value \( \Lambda(t_1) = \Lambda_0 = \Lambda_{\text{min}} \) is the lowest. At \( t = t_2 \), value \( \Lambda(t_2) = \Lambda_0 \theta_2(t_2) = \Lambda_{\text{max}} \) is the greatest on a considered interval of exploitation. Then at \( \Lambda(t) = \Lambda_{\text{min}} \), index \( K = K_{\text{max}} \), and at \( \Lambda(t) = \Lambda_{\text{max}} \), index \( K = K_{\text{min}} \).

Let's define the bottom and top limits for values of efficiency of the device. Based on (3), for this purpose, it is enough to define borders \( K = K_{\text{min}} \) and \( K = K_{\text{max}} \) for index \( K \).

Let's enter the following designations: \( K_0 \) – true unknown value of utilization factor at an unsteady flow of failures; \( K \) – value of utilization factor, estimated by (9) at the value of failure rate \( \Lambda \); \( \xi_s \) – relative error of estimation of \( K_0 \).

Failure rate \( \Lambda \) may be as follows

\[ \Lambda = \tau_r^{-1} \int_{t_1}^{t_2} \Lambda(t) dt, \]  

(10) where \( \tau_r \) – operating time of the device within an interval \([ t_1, t_2 ]\).

Relative error \( \xi_s \) shall be defined as follows

\[ \xi_s = \left| K_0 - K \right| / K \]  

(11) Considering that \( \Lambda_{\text{min}} = \Lambda = \Lambda_{\text{max}} \), it is reasonable that

\[ K_{\text{min}} < K_0 < K_{\text{max}} \]  

and

\[ K_{\text{min}} < K < K_{\text{max}}. \]  

(13)

From here, the size \( \xi \) defined by expression

\[ \xi = (K_{\text{max}} - K_{\text{min}})K^{-1}, \]  

(14) will be the greatest possible value for \( \xi_s \), that is \( \xi, \leq \xi_s \).
For $K_0$, the following could be written down
\[ K(1 - \xi) \leq K_0 \leq K(1 + \xi). \]  
(15)

Percentages values of error $\xi$ are listed in Table 1 and calculated for various values $\alpha$; $\alpha$; $\tau_{pr}$; $T_r$; $\tau_{ks}$; $N_r$; $T_r$. Values $N_r$ and $T_r$ are rational values of sizes $N$ and $T$.

Table 1. Dependence of relative error $\xi(\%)$ from $\phi(t_2)$ at various $\alpha$; $\tau_{pr}$; $T_r$; $\tau_{ks}$; $N_r$; $T_r$.

<table>
<thead>
<tr>
<th>Values of parameters</th>
<th>$\tau_{ks}$</th>
<th>$N_r$</th>
<th>$T_r/T_o$</th>
<th>$\phi(t_2) = 1.5$</th>
<th>$\phi(t_2) = 2.0$</th>
<th>$\phi(t_2) = 2.5$</th>
<th>$\phi(t_2) = 3.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.01$</td>
<td>0.0003</td>
<td>1</td>
<td>0.07</td>
<td>0.95</td>
<td>1.88</td>
<td>2.79</td>
<td>3.67</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.02$</td>
<td>0.0002</td>
<td>2</td>
<td>0.05</td>
<td>0.91</td>
<td>1.80</td>
<td>2.68</td>
<td>3.55</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.01$</td>
<td>0.0001</td>
<td>1327</td>
<td>0.02</td>
<td>0.94</td>
<td>1.85</td>
<td>2.73</td>
<td>3.60</td>
</tr>
<tr>
<td>$\alpha = 0.2$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.01$</td>
<td>0.0003</td>
<td>1</td>
<td>0.05</td>
<td>1.15</td>
<td>2.29</td>
<td>3.41</td>
<td>4.51</td>
</tr>
<tr>
<td>$\alpha = 0.2$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.02$</td>
<td>0.0002</td>
<td>2</td>
<td>0.04</td>
<td>1.15</td>
<td>2.29</td>
<td>3.42</td>
<td>4.52</td>
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<tr>
<td>$\alpha = 0.2$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.02$</td>
<td>0.0001</td>
<td>1868</td>
<td>0.01</td>
<td>0.98</td>
<td>1.95</td>
<td>2.90</td>
<td>3.83</td>
</tr>
<tr>
<td>$\alpha = 0.2$; $\alpha = 0.2$; $\tau_{pr} = 0.001$; $T_r = 0.01$</td>
<td>0.0003</td>
<td>1</td>
<td>0.05</td>
<td>1.61</td>
<td>3.22</td>
<td>4.81</td>
<td>6.38</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.4$; $\tau_{pr} = 0.01$; $T_r = 0.01$</td>
<td>0.0004</td>
<td>1</td>
<td>0.08</td>
<td>0.97</td>
<td>1.92</td>
<td>2.85</td>
<td>3.76</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.4$; $\tau_{pr} = 0.01$; $T_r = 0.02$</td>
<td>0.0002</td>
<td>653</td>
<td>0.03</td>
<td>0.86</td>
<td>1.69</td>
<td>2.52</td>
<td>3.33</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.4$; $\tau_{pr} = 0.02$; $T_r = 0.01$</td>
<td>0.0006</td>
<td>0</td>
<td>0.11</td>
<td>1.46</td>
<td>2.89</td>
<td>4.31</td>
<td>5.70</td>
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<tr>
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<td>5.65</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.4$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
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<td>653</td>
<td>0.03</td>
<td>1.33</td>
<td>2.64</td>
<td>3.94</td>
<td>5.23</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.01$</td>
<td>0.0006</td>
<td>1</td>
<td>0.10</td>
<td>1.13</td>
<td>2.22</td>
<td>3.28</td>
<td>4.31</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
<td>0.0004</td>
<td>2</td>
<td>0.08</td>
<td>1.13</td>
<td>2.23</td>
<td>3.30</td>
<td>4.34</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
<td>0.0002</td>
<td>961</td>
<td>0.02</td>
<td>0.98</td>
<td>1.93</td>
<td>2.86</td>
<td>3.77</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
<td>0.0006</td>
<td>1</td>
<td>0.10</td>
<td>1.59</td>
<td>3.15</td>
<td>4.68</td>
<td>6.18</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
<td>0.0004</td>
<td>2</td>
<td>0.08</td>
<td>1.60</td>
<td>3.16</td>
<td>4.70</td>
<td>6.21</td>
</tr>
<tr>
<td>$\alpha = 0.1$; $\alpha = 0.2$; $\tau_{pr} = 0.02$; $T_r = 0.02$</td>
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<td>961</td>
<td>0.02</td>
<td>1.45</td>
<td>2.88</td>
<td>4.27</td>
<td>5.65</td>
</tr>
</tbody>
</table>
Cited data show that the relative error $\xi$ at really possible values of parameters influencing index $K$ does not exceed 6.5%.

Conclusions

1. In the paper, electronic devices are considered. Failures of devices, depending on character of their display, have been subdivided into two kinds. Failures of the first kind are identified directly according to external attributes. Failures of the second kind are found out at carrying out control of serviceability state and inspections.
2. At an interval between periodic inspections of devices, $N$ controls of serviceability state with periodicity $T$ are implemented. The control of serviceability state is carried out by means of the built-in test equipment.
3. The method of calculation of efficiency of devices for a case of unsteady Poisson flow of failure is offered. The method is based on an estimation of the bottom and top limits for efficiency values on a considered interval of exploitation.
4. Definition of the bottom and top limits for efficiency demands calculation of utilization factors of devices by (9) at values of failure rate $\Lambda$, $\Lambda_{\min}$ and $\Lambda_{\max}$. Also, it is necessary to define by (14) the maximal relative error of estimation of true value of utilization factor $\xi$.
5. Data of computing experiment (Table 1) cover all range of really possible values of the parameters influencing utilization factor and efficiency of the device. Results in Table 1 testify that the maximal relative error of estimation of true value of utilization factor and index of efficiency does not exceed 6.5%.

References


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Electronic devices inspected by using built-in test equipment are analyzed. The method of calculation of efficiency of devices for a case of unsteady Poisson flow of failure is offered. The method is based on estimation of the bottom and top limits for values of efficiency on analyzed interval of exploitation. For this purpose, efficiency of devices for corresponding Poisson flows of failures is calculated. A relative error of definition of efficiency value is defined. As an estimation of an error, its maximal value is used. Computing experiment according to the given error is executed. Experiment considers really possible values of the parameters influencing efficiency of the device. Results of experiment show that the maximal relative error of estimation of a parameter of efficiency does not exceed 6.5 %. Bibl. 9, tabl. 1 (in English; abstracts in English and Lithuanian).


Analizuojami elektroniniai įtaisai, kurių darbingumo kontrolė atliekama vidiniais kontrolės įrangos. Siūlomas metodas įtaisų efektyvumui nustatyti esant nestacionariam Puasono gedimų srautui. Metodas grindžiamas efektyvumo viršutinių ir apatinų ribų analizuojamame eksploatacijos intervale vertinimu. Šiuo tikslu apskaičiuojamos tam tikrų Puasono srauto atitinkamos efektyvumo vertės. Nustatomos sančiokinė efektyvumo vertinimo paklaidos. Nagrinėjama sančiokinės efektyvumo nustatymo paklaidos maksimali vertė. Atliktas skaičiavimo eksperimentas, apimantis reašį efektyvumo vertėms įtakos turinčių parametro diapazoną. Eksperimento rezultatai rodo, kad sančiokinės efektyvumo nustatymo paklaidos maksimali vertė neviršija 6,5 %. Bibl. 9, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).