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Fractional Channel Estimation and Equalization for MIMO Systems

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***Abstract*—In this paper, a novel method for joint channel estimation and equalization based on fractional Fourier transform is presented for block faded MIMO systems. The channel response is obtained at the receiver by sending known training symbols along with the data. The channel state information (CSI) obtained from the received training symbols is used for equalization. Therefore, the equalization process depends on the quality of channel estimation. The proposed technique performs the channel estimation and equalization in the optimum fractional domain outperforming the conventional time domain estimation and equalization technique. For a 2x2 MIMO system the proposed fractional domain technique gives SNR advantage of 1.67 dB over the existing technique.**

***Index Terms*— Channel estimation, equalization, FRFT, MIMO.**

I. INTRODUCTION

Over the last decade, the popularity of wireless applications has risen tremendously, and there is an ever increasing demand for higher data rates. Multiple-input multiple-output (MIMO) systems have recently emerged as one of the significant technical breakthroughs in modern communication because of its ability to provide high spectral efficiency without the need of additional bandwidth [1]. However, MIMO system relies upon the knowledge of the channel response at the receiver for data detection and decoding. In practice, however, the channel state information (CSI) is never known to the receiver apriori and some form of channel estimation technique has to be used to estimate the channel response [2], [3]. A simple way to estimate the channel is to transmit known training symbols to the receiver. The two common methods used for training based channel estimation are least squares (LS) and minimum mean square (MMSE) estimation. The channel response estimate obtained by channel estimation is used for equalization. In this paper, the MMSE receiver is considered for equalization as it is the optimum linear receiver [4]. Since, equalization process involves the CSI, it can be understood that the output depends on the quality of channel estimation. In [5], an improved MMSE receiver based on fractional Fourier transform was proposed. It was shown that the performance of MMSE receiver can be improved by performing the equalization in the fractional domain. In this improved MMSE receiver, the channel response was assumed to be perfectly known to the receiver, which is not practical. In this paper, we propose a novel joint channel estimation and equalization technique where the channel estimation and equalization is performed in the optimum fractional domain rather than the conventional time domain.

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II. SYSTEM MODEL

In this article, it is assumed that the channel remains constant within one transmission block and changes completely and independently for the next block and then remains constant for the duration of that block. Most of the analysis of MIMO receivers in literature assumes that the channel is perfectly known at the receiver. This assumption is not valid in practice and a reasonable estimate of the channel response needs to be obtained at the receiver for exploiting MIMO systems. In the case of a block fading channel, the estimation is done by transmitting a training sequence at the start of the transmission block. The channel response estimated by the training sequence is valid for the entire length of the transmission block and a new estimate needs to be obtained after that.

We consider a MIMO system with N_t transmit antennas and N_r receive antennas, where $N_t \le N_r$. Within one block of L symbols, the MIMO model is

$$\mathbf{Y} = \sqrt{\frac{\rho}{N_t}}\mathbf{H}\mathbf{X} + \mathbf{N}\ , \qquad (1)$$

where ρ denotes the signal-to-noise ratio (SNR) for a single receive antenna. LDPC codes of rate 1/2 are used for the forward error correction (FEC) of the received data bits. LDPC codes have been selected as the channel coding scheme because they achieve better performance than other block codes for various fading channels [6], [7]. The coded bits are modulated using QPSK.

III. CHANNEL ESTIMATION AND EQUALIZATION

The MIMO technology promises to meet the data rate demands of the future generation without needing additional bandwidth. Although the prospects of MIMO systems look bright but the predictions have been made on the assumption that channel is perfectly known at the receiver. Practically, this is never the case and some kind of estimation needs to be performed at the receiver to obtain the channel estimate. Estimation can be done by using training sequences or by using blind estimation methods which do not use any kind of training. Although the blind estimation methods do not impose any bandwidth penalty on the system, their performance is inferior to the techniques that use pilot symbols. In this paper, only the training based channel estimation techniques are considered. For a block fading channel, the channel is estimated only once within one block. The training symbols are transmitted at the beginning of the block followed by the information (data) symbols. The transmit symbol matrix $\mathbf{X}$ can be decomposed into $\mathbf{X}=\mathbf{X}_T+\mathbf{X}_d$ and the receive symbol matrix into $\mathbf{Y}=\mathbf{Y}_T+\mathbf{Y}_d$ where $\mathbf{X}_T,\mathbf{Y}_T$ and $\mathbf{X}_d,\mathbf{Y}_d$ are the training symbols and data symbols respectively. The data symbols X_d are encoded using the LDPC parity check matrix for FEC at the receiver. The block length L can be further decomposed into $\mathbf{L}=\mathbf{L}_T+\mathbf{L}_d$, where $\mathbf{L}_T$ is the length of training symbols in a block and $\mathbf{L}_d$ is the length of data symbols in a block.

Let us consider normalized matrix $\overline{\mathbf{X}}_T$ which contains the training symbols only. The optimum training symbol design is based on the orthogonality condition, where the training symbols have to be orthogonal to each other in time and space. The orthogonality condition is given by

$$\overline{\mathbf{X}}_T\overline{\mathbf{X}}_T^H = const.\mathbf{I}_{L_T}, \tag{2}$$

where const denotes an arbitrary, real, non zero factor. The orthogonality condition minimizes the variance of estimation error.

Once the training symbols are received at the receiver, the channel state information can be extracted from them in various methods. Two common techniques used for obtaining channel estimates are LS and MMSE:

$$\hat{\mathbf{H}}_{LS} = (\mathbf{X}_T^H\mathbf{X}_T)^{-1}\mathbf{X}_T^H\mathbf{Y}_T, \tag{3}$$

$$\hat{\mathbf{H}}_{MMSE} = (\mathbf{X}_T^H\mathbf{X}_T + \sigma_n^2\mathbf{I}_{N_t})^{-1}\mathbf{X}_T^H\mathbf{Y}_T, \tag{4}$$

where $\hat{\mathbf{H}}_{LS}$ is the least squares estimate and $\hat{\mathbf{H}}_{MMSE}$ are the channel estimates obtained by MMSE method. After estimation the channel matrix can be decomposed into

$$\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}. \tag{5}$$

After the channel estimate $\hat{\mathbf{H}}$ is obtained, it is used for equalization. In this paper, the equalization is performed using MMSE receiver which is considered to be the optimum linear receiver because of its good performance at low and high SNR [4]. The MMSE receiver tries to minimize the mean square error (MSE) between the received signal Y and the desired signal X by multiplying the received signal by an optimal weight vector

$$\mathbf{X}_k = \boldsymbol{w}_k^H\mathbf{Y}_k, \qquad 1 \le k \le N_r. \tag{6}$$

The weight vector is given by

$$\boldsymbol{w} = \mathbf{R}_{YY}^{-1}\mathbf{R}_{YX}, \tag{7}$$

where $\mathbf{R}_{YY}$ is the auto covariance matrix of the received signal and $\mathbf{R}_{YX}$ is the cross covariance matrix of the desired signal and the received signal:

$$\mathbf{R}_{YY} = E[\mathbf{Y}\mathbf{Y}^H] = \hat{\mathbf{H}}\mathbf{R}_{XX}\hat{\mathbf{H}}^H + \mathbf{R}_{NN} \tag{8}$$

$$\mathbf{R}_{YX} = \mathbf{R}_{XX}\hat{\mathbf{H}}^H \tag{9}$$

where $\mathbf{R}_{NN} = E[\mathbf{N}\mathbf{N}^H] = \sigma_N^2.\mathbf{I}$ and $\mathbf{R}_{XX} = E[\mathbf{X}\mathbf{X}^H] = \sigma_X^2.\mathbf{I}$.

The MMSE receiver is optimal in the sense of minimizing the mean squared error. In [5], an improved MMSE receiver based on equalization in the fractional Fourier domain was proposed. It was shown that the MMSE receiver in the optimum fractional Fourier domain outperforms the conventional MMSE receiver in time domain. The received signal and the desired signal are converted to the ‘ath’ fractional domain by using fractional Fourier transform (FRFT) and the equalization process is performed in that domain. After equalization the output is converted back to the original domain by inverse fractional Fourier transform (IFRFT) and the MSE of the output and the desired signal is calculated. The domain in which the MSE is minimum is selected as the optimum domain. For a detailed description of FRFT and its application to wireless communication, refer to [8]–[12]. A comparison of the conventional MMSE receiver and the FRFT based MMSE receiver for 2 transmit and 2 receive antennas without any forward error correction (FEC) and assuming perfect CSI is shown below.

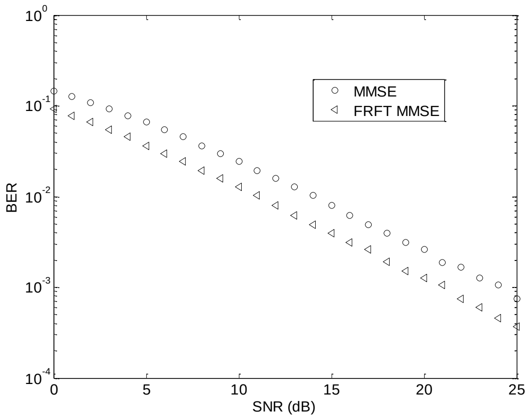

Fig. 1. BER comparison of time domain and FRFT domain MMSE receiver for 2x2 MIMO system.

Although the FRFT based MMSE outperforms the conventional MMSE receiver in terms of bit error rate (BER) [8], it assumes perfect channel knowledge at the receiver which in practice is never known apriori. Therefore, for practical applications, some kind of estimation technique needs to be used. In this paper, we propose a novel joint channel estimation and equalization method in the fractional

Fourier domain.

IV. CHANNEL ESTIMATION AND EQUALIZATION IN FRACTIONAL DOMAIN

The transmitted and received vectors **X** and **Y** are transformed from time domain to the fractional Fourier domain by using the transformation kernel $\mathbf{K}_a$ or $\mathbf{K}_a$ [10]

$$\mathbf{Y}_a = F^a\{\mathbf{Y}\},\ \mathbf{X}_a = F^a\{\mathbf{X}\}, \tag{10}$$

where F^a denotes the a^{th} order FRFT:

$$\begin{cases} \mathbf{Y}_a = \mathbf{Y}_{T_a} + \mathbf{Y}_{d_a}, \\ \mathbf{X}_a = \mathbf{X}_{T_a} + \mathbf{X}_{d_a}. \end{cases} \tag{11}$$

In the training phase we consider $\mathbf{X}_{T_a}$ and $\mathbf{Y}_{T_a}$ and during the information phase we consider $\mathbf{X}_{d_a}$ and $\mathbf{Y}_{d_a}$. The channel estimation in this work is done using the MMSE method given by eq. (4). Channel estimation in the a^{th} domain is given by

$$\hat{\mathbf{H}}_a = \left(\mathbf{X}_{T_a}^H \mathbf{X}_{T_a} + \sigma_n^2 \mathbf{I}_{N_t}\right)^{-1} \mathbf{X}_{T_a}^H \mathbf{Y}_{T_a}, \tag{12}$$

where $\hat{\mathbf{H}}_a$ is the channel gain estimate in the ath fractional domain. The estimate obtained in the a^{th} domain is used for equalization using the Wiener filtering technique [4], [13]. The optimal Wiener solution in the fractional domain is given by

$$\boldsymbol{w}_a = \boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{Y}_{d_a}}^{-1} \boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{Y}_{d_a}}, \tag{13}$$

where $\boldsymbol{w}_a$ is the weight vector in the ath domain, $\boldsymbol{R}_{\mathbf{Y}_a\mathbf{Y}_a}$ is the auto covariance of the vector **Y** in the ath domain and $\boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{Y}_{d_a}}$ is the cross covariance of **Y** and **X** in the a^{th} domain. $\boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{Y}_{d_a}}$ and $\boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{X}_{d_a}}$ are given by:

$$\boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{X}_{d_a}} = \boldsymbol{R}_{\mathbf{X}_{d_a}\mathbf{X}_{d_a}} \hat{\mathbf{H}}_a^H, \tag{14}$$

$$\boldsymbol{R}_{\mathbf{Y}_{d_a}\mathbf{Y}_{d_a}} = \hat{\mathbf{H}}_a \boldsymbol{R}_{\mathbf{X}_{d_a}\mathbf{X}_{d_a}} \hat{\mathbf{H}}_a^H + \boldsymbol{R}_{\mathbf{N}_{d_a}\mathbf{N}_{d_a}}, \tag{15}$$

where $\mathbf{N}_{d_a}$ denotes the noise in the a^{th} domain during the data transmission phase. The Wiener solution given in eq. (13) reduces to time domain wiener solution for a=0 and frequency domain wiener solution for a=1. The recovered signal is described by

$$\hat{\mathrm{X}}_k = F^{-a}\left\{\boldsymbol{w}_{a(k)}^H \mathbf{Y}_{d_a(k)}\right\}, \quad 1 \le k \le N_r, \tag{16}$$

where F^{-a} is the inverse fractional Fourier transform.. The optimum value of 'a' is simply found by calculating the mean squared error (MSE) for sufficiently closely spaced discrete values of 'a' $\in [-1,+1]$ and choosing the one which minimizes the MSE. The MSE gives the difference between the desired and estimated signal and can be calculated by

$$MSE(\boldsymbol{w}_a) = E\left\{\left\|\mathbf{X} - F^{-a}\left\{\boldsymbol{w}_a^H \mathbf{Y}_{d_a}\right\}\right\|^2\right\}. \tag{17}$$

Once the optimum value of 'a' is calculated, it can be used for arbitrary many realizations of that signal and noise statistics. The optimum domain only needs to be recalculated when the statistics of the signal and noise change. After processing the signal in the fractional domain and converting it back to time domain, it is demodulated and decoded to obtain the original transmitted data.

Computational Complexity of FRFT. For time-invariant degradation models and stationary signals and noise, the classical Fourier domain Wiener filter can be implemented in $O(N \log_2 N)$ time, where N is the temporal length of the signal. The filtering process in fractional Fourier domain, which enables significant improvement in the estimation and equalization process, can also be implemented in $O(N \log_2 N)$ time. Thus, an improved performance is achieved at no additional cost [9], [11], [14].

V. SIMULATION DETAILS

Total number of bits is 10^5. Block length (L) is taken to 50 and the total number of blocks is 2000. The first part of the block consists of training symbols and the second part consists of data symbols which are encoded by irregular LDPC coes of rate 1/2. For simplicity and loss of generality, we consider QPSK modulation. The channel is assumed to obey the block fading law in which the channel remains constant over the full duration of a transmission block and changes completely and independently for the next block. The fading is considered to be frequency flat and is assumed to follow Rayleigh distribution. Both the fading channel and noise are comprised of i.i.d complex Gaussian random variables ***CN*****(0,1)**. The value of FRFT order 'a' is varied from -1 to +1 with a step size of 0.1. A total of 21 values of 'a' are investigated, selecting the one which gives the minimum MSE. It can be observed from the results that the proposed receiver clearly outperforms the existing estimation and equalization technique. The MSE vs. 'a' graph shows the optimum value of 'a' to be -0.4 at SNR=10 dB. The analysis in this paper is done for N_t=N_r=2 but it is applicable to higher antenna configurations without loss of generality.

It can be seen from Fig. 2 and 3 that the proposed receiver has better error performance (in terms of BER and MSE) than the existing receiver.

Table I gives the comparison of the two receivers at a BER of 10^{-3} and it is seen that the proposed receiver gives an SNR advtantage of 1.67 dB over the existing receiver.

TABLE I. SNR COMPARISON AT A BER OF 10^{-3}.

FRFT domain estimation and equalization	16.43 dB
Time domain estimation and equalization	18.10 dB
SNR Advantage	1.67 dB

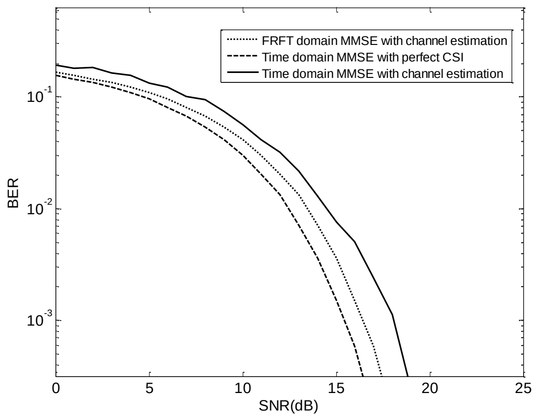

Fig. 2. BER comparison of the existing and the proposed technique for 2x2 LDPC coded MIMO system.

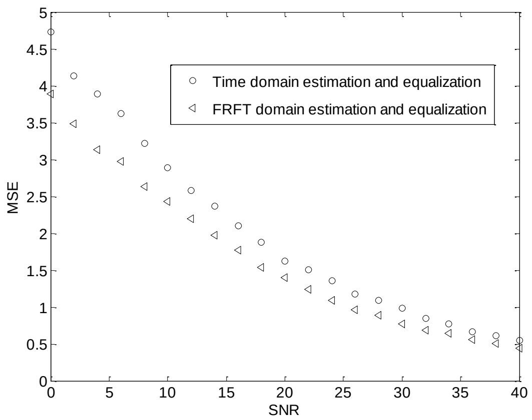

Fig. 3. MSE vs. SNR comparison of existing and proposed receiver for 2x2 MIMO system.

VI. CONCLUSIONS

A novel FRFT based joint channel estimation and equalization technique has been proposed in this paper. From the results it is clearly seen that the proposed technique outperforms the existing technique. The signal which appears scattered in time domain is filtered in the optimum domain where it appears to be compact. Moreover, this performance improvement comes at no additional cost since the fractional Fourier transform has an O ($N \log_2 N$) algorithm for time invariant degradation models which is the same as classical Fourier transform. The further study prospects involve the validation of the technique in the presence of correlated channels.

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