Probability Distribution of Multiple-Access Interference in Chaotic Spreading Codes based on DS-CDMA Communication System

A. Litvinenko, E. Bekeris
Department of Fundamentals of Electronics, Riga Technical University, Azenes str. 12, Riga, Latvia, phone:+37126491589, e-mails: anna.litvinenko@rtu.lv, elmars.bekeris@rtu.lv

crossref http://dx.doi.org/10.5755/j01.eee.123.7.2380

Introduction

Chaotic binary sequences are considered to be promising for spread-spectrum communications. There is a possibility to generate a significantly higher number of chaotic spreading sequences with good cross correlation properties than m-sequences and Gold sequences of the same length \( N \). In order to prove this, it is mentioned in [1]: “... there is only one communication problem known so far where the application of chaos is beneficial: the generation of spreading codes for classical CDMA systems”. In recent years, a great effort has been devoted to the investigation of the performance of CDMA systems using chaotic spreading sequences [3–7].

Estimation of BER in multiuser situation has been carried out by a computer simulation [7] or calculated theoretically assuming Gaussian approximation of the multiple-access interference [4, 5, 8]. At the same time, it is not clear whether it is always correct to consider the multiple-access interference as Gaussian random variable.

In [2] it is demonstrated that relatively short spreading sequences generated by the logistic map (length \( N = 7, 15, 31 \)) have better correlation properties than m-sequences. Therefore, it is reasonable to test the correspondence of the multiple-access interference to the Gaussian random variable. This article deals with a computer simulation of the multiuser DS-CDMA system aimed at estimating the probability density of inter-user interference. The paper is organized in the following way: the next section describes the block diagram of the system under simulation, the results of which are presented in the third section. Finally, an overview of the obtained results is shown in conclusions.

DS-CDMA system using chaotic spreading sequences

DC-CDMA system with \( K \) transmitters and one receiver is presented in Fig. 1. The modeling parameters are as follows: \( b_i(t) \) – information bits NRZ sequence at the input of \( i \)-th transmitter; \( c_i(t) \) – chaotic spreading NRZ sequence generated for \( i \)-th transmitter; \( \tau_i \) – time delay observed by the receiver on the signal from \( i \)-th transmitter. Because of the simulation which is aimed on the investigation of the multiple-access interference, the channel noise is neglected. It is supposed that the correlation receiver is matched to the \((K + 1)\)-th transmitter spreading sequence \( c_{K+1} \).

Therefore, the multiple-access interference is caused by the input signal \( r(t) \):

\[
 r(t) = \sum_{i=1}^{K} b_i(t) \cdot c_i(t) \cdot U_m \cos(\omega t + \alpha \tau_i) \quad (1)
\]

Simulation was performed for the following parameters:
- \( b_i(t) \) – binary information NRZ sequence (+1; -1) with the duration of \( B_i T_b \), where \( T_b \) – the duration of one bit, \( B_i \leq (100; 400) \), a definite value of \( B_i \) is randomly selected from the set of \((0,100)\) or \((0,400)\) with an
equal probability; generation of this sequence starts at the moment \( T'_1 = nT_c \), where \( T_c \) – the chip duration of spreading sequence;

- \( c(t) \) – a chaotic spreading sequence of \( L_c \) chips generated on the basis of the logistic map: \( x_{n+1} = 4x_n (1 - x_n) \), and \( c_i = 1 \) if \( x_n \geq 0.5 \); \( c_i = -1 \) if \( x_n < 0.5 \);

- Initial values \( x'_0 = y_i \) for generation of \( c(t) \) are selected from another logistic map: \( y_{n+1} = \beta y_n (1 - y_n) \); the length of spreading sequences is selected \( L_c = 7, 31, 63 \), the number of transmitters causing the multiple-access interference correspondingly: \( K = (3, 8, 17), (11, 32, 65), (64, 129) \).

The example of the transmission structure for simulation is given in Fig. 2. Therefore, the following modeling parameters are presented: binary information sequence maximal length \( B_i = 5 \), the length of spreading sequence \( L_c = 5 \), the number of transmitters \( K = 3 \).

![Fig. 2. Example of the transmission structure for simulation](image)

At the output of correlation receiver the following values will be obtained at the time moments of \( jT_b \):

\[
U_j = A \int_0^{2\pi} \sum_{i=1}^{K} b_i(t)c_i(t)\cos(\omega\tau_i) c_{K+1}(t)dt .
\]

where \( A \) – a coefficient which depends on the parameters of correlation receiver elements.

**The results of simulation**

The goal of simulation was to test the default hypothesis for relatively short spreading sequences: the sample in the multiple-access interference \( U_j \) comes from the distribution in the normal family. The simulation was carried out for the following situations:

- All time delays \( \tau_i \) are equal, and a perfect synchronization takes place (as it is assumed in the theoretical BER calculations in [4, 8]);
- Time delays are different.

Spreading sequences \( c(t) \) were generated for various initial values \( x'_0 = y_i \) selected from the logistic map \( y_{n+1} = \beta y_n (1 - y_n) \) and according to the following rules:

**A.** Consecutive values \( y_1, y_2, y_3, ..., \) for three initial values: \( y'_0 = 0.81472369, y'_0 = 0.90579194, y'_0 = 0.12699682 \);

**B.** Every 15-th value for the same initial values as in A: \( x'_0 = y_1, x'_0 = y_15, x'_0 = y_30, ... \). Some histograms for (2) of a perfect synchronization are presented in Fig. 3.

The results of tests for normality according to the following three criteria: chi-square, Kolmogorov, Lilliefors test are presented in Tab. 1 for a perfect synchronization case, and in Tab. 2 - for different time delays where a definite \( \tau_i \) value is randomly selected from the set of \( (0, 2\pi) \).

**Table 1. Results of tests for normality: perfect synchronization**

<table>
<thead>
<tr>
<th>( K )</th>
<th>( y_0 )</th>
<th>( B_i = 100 )</th>
<th>( B_i = 400 )</th>
<th>( L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( y'_0 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>( y'_0 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>31</td>
</tr>
<tr>
<td>17</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td>63</td>
</tr>
<tr>
<td>32</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>1 1 1 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>1 1 1 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>1 1 1 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>129</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>1 1 1 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Results of tests for normality: different time delays**

<table>
<thead>
<tr>
<th>( K )</th>
<th>( y_0 )</th>
<th>( B_i = 100 )</th>
<th>( B_i = 400 )</th>
<th>( L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( y'_0 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>( y'_0 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>31</td>
</tr>
<tr>
<td>17</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td>63</td>
</tr>
<tr>
<td>32</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td></td>
</tr>
<tr>
<td>129</td>
<td>( y'_0 )</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 1 1 1</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 3. Histogram representation of the multiple-access interference calculated according to (2). For all the histograms the initial values of spreading sequences are selected according to the rule $A$ and $y^0 = 0.90579194$. A normal distribution curve with the corresponding mean value and variance on every histogram is presented.
Thus, 0 means that the hypothesis of normality cannot be excluded; 1 means that the hypothesis of normality can be rejected with the significance level 0.05. Also the first symbol corresponds to the chi-square criterion, the second – Kolmogorov criterion, and the third – Lilliefors test.

Conclusions

1. The multiple-access interference in multi-user DS-CDMA communication systems with short chaotic spreading sequences generated on the basis of the logistic map and containing less than 60 chips in many cases does not come from the family with a normal distribution. Therefore, an application of theoretical bit error rate curves based on the consideration that interference is Gaussian random variable, and cannot be recommended for such spreading sequences.

2. In the case of chaotic spreading sequences with the length of more than 60 chips, the multiple-access interference in multi-user DS-CDMA communication systems can be considered as Gaussian random variable.

References


Received 2012 03 07
Accepted after revision 2012 05 12


This paper is devoted to the evaluation of the probability distribution of the multiple access interference in asynchronous Direct Sequence Code Division Multiple Access (DS CDMA) communication system based on chaotic spreading sequences. Statistical properties of such interference in a system with K users causing the multiple-access interference are obtained by a computer simulation. Simulation is carried out for the spreading sequences produced by the logistic map; the length of sequences: $N = 7, 31, 63$ and the number of users correspondingly: $K = (3, 8, 11); (11, 32, 65); (64, 129)$. Ill. 3, bibl. 8, tabl. 2 (in English; abstracts in English and Lithuanian).


Siekiant nustatyti daugkartinio priėjimo interferencijos tikimybės pasiskirstymo tankį asynchroninėse ryšio sistemoje su kodinio atskyrimo kanalais ir naudojant chaitines sekas išplėstų tiesioginio eiliškumo spektro. Naudojant kompiuterių modeliavimą gautos sistemos su K abonentų, sukeliančių trikdžius, statistinės interferencijos savybės. Chaitinėms išplėtimo sekmis sukurti generuoti panaudotas loginės lygtytės. Gauto chaitinio kodas ilgis $N = 7, 31, 63$; interferuojančiųjų abonentų skaičius atitinkamai $K = (3, 8, 11); (11, 32, 65); (64, 129)$. Il. 3, bibl. 8, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).