http://dx.doi.org/10.5755/j01.eee.19.2.2348 ELEKTRONIKA IR ELEKTROTECHNIKA, ISSN1392-1215,VOL. 19, NO. 2, 2013

A New Method to Evidence the P300 Event-related Potential based on a Lifting Scheme

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***Abstract*—We explore the possibility of revealing the P300 event-related potential by means of a dedicated method that is both fast and reliable. It involves a lifting scheme based on a variant of Haar non-linear wavelet and a decision criterion based on a measure computed after the information linked with the P300 potential were obtained from the EEG signal. Some actual results are presented, for comparison purposes, involving the signals given in a well known database of EEG recordings.**

***Index Terms*—Electro-encephalographic signal, P300 event-related potential, brain-computer interface, lifting scheme, non-linear Haar wavelet.**

I. Introduction

Brain computer interfaces take advantage of the possibilities of feature extraction from the electro-encephalographic signals (EEG). One of these features is the P300 event-related potential. Paradigms based on the P300 event-related potential were involved in brain-computer interfaces mostly due to their simplicity and efficiency. There are also applications involving lie detection based on the same feature of the EEG signal. This is the reason why the detection of the P300 potential draws a lot of attention.

The essential parts of a brain-computer interface are the algorithms that permit the feature extraction from the EEG signal and the classifiers that actually reveal the user's intentions; without them any decision regarding the response of the subject to a certain stimulus is quite impossible. There are also a few requests that must be fulfilled by these components of the brain computer interface: they must be fast, efficient and reliable. Achieving this depends on the chosen paradigm, the method preferred for the detection of a certain feature that is present (or not) in the EEG signal as a subject's answer to an already known stimulus and the possibilities to classify accurately the results of the detection.

Different paradigms for brain computer interfaces were developed, both auditory and visual; among them the so called P300 speller is widely known due to its ease of implementation and to the type of response of the subject, that allows classifying the answer without further processing, as soon as the correct detection is achieved, [1].

Manuscript received May30, 2012; accepted August30, 2012.
This study was supported by the Romanian Ministry of Education, BCISIS project, under contract # 12115/2008-2011.

Recently new methods based on non-linear operators were introduced in the analysis of complex signals. It is already known the fact that the EEG falls in this category and this is one of the reasons its features extraction is not always a straightforward task. This paper will centre on a specific feature of the EEG signal, namely the P300 event-related potential that is used in several paradigms implementing brain computer interfaces, but is not limited to them, [2], [3].

Several methods were used in the P300 event-related potential detection. Among them are those that investigate the frequency content of the EEG signal with an elicited P300 [4]–[7], the time behaviour [8], the time-frequency dependent components [9], [10] or imply the wavelet transform to analyse the signal [11], [12].

In what follows we shall briefly remind some essential features regarding the P300 event-related potential; next the mathematical background on which our method of detection relies and the data set involved in our research will be presented. The subsequent part regards the actual results with a focus on our contributions, followed by a brief discussion; the last one is devoted to the conclusions and possible future research topics.

II. The Variability of P300 Event-Related Potential

The P300 event-related potential is a deflection found in the EEG signals acquired on specific zones of the scalp, elicited when a certain stimulus, with a rare probability of occurrence, is presented to the subject under investigation. Usually the involved zones are central - Cz, parietal - Pz, frontal - Fz, central-parietal- CPz and central C1-C9, named according to the international "10-20"electrodes system. The main problem that needs to be surpassed when trying to detect the P300 potential is the great variability of this feature of the EEG signal. Many factors may influence the rate of successful detection; among them there are natural, physical and induced ones, as stated almost exhaustively in [13] and [14]. The variability of the P300 led to different approaches for its detection, each one of them reflected by the methods and references mentioned before. It is worth noticing that in spite of its high variability, the P300 impulse has also two features that are supposed to be quite stable: the moment of appearance (as a mean value, approximately 300 milliseconds after the stimulus manifestation) and its basic shape, a positive deflection of the EEG signal. Our method of detection is based on these characteristics.

III. COMPUTATIONAL BACKGROUND

In what follows we shall present the mathematical background that we used for our method of P300 detection. It is based on an adapting lifting scheme first introduced by Sweldens in [15]–[17] and generalised by Goutsias and Heijmans under the name of general wavelet decomposition scheme in [18]–[21]. The main advantage of this option is the possibility to choose among a vast range of non-linear functions and therefore a good chance to offer a solid compromise between accuracy and processing speed. Because our goal is to detect the P300 event-related potential, we chose the simplest function that proved to give excellent detection results and that assured the fastest processing speed, since this is an important request for a brain computer interface.

Briefly, lifting may be used to develop wavelet like decompositions of a signal in the nonlinear case. The method is based on two kind of operators, named analysis and synthesis operators, that are capable of splitting a time dependent, discrete signal, into parts containing equal number of samples, according to a "rule" (in fact, a function that may be, but usually is not, linear). The splitting operation evidences either the details of the input signal or its global behaviour. This can be completed in several steps and each in depth step is justified by the information one can obtain versus the time needed to perform the calculations. The above observation led to the conclusion that the best suited function for the job is the one that is simple (and therefore fast when applying it) and accurate enough to promise a good detection rate of the P300 event-related potential. This can be achieved only in conjunction with an adequate decision criterion designed to conclude upon the absence or the presence of the potential.

We shall now follow reference [21] for a concise discussion of the algorithm used in our method. Let us consider a family of signal spaces U_j and three families of operators: signal analysis operators α_j and detail operators ω_j that map U_j into U_{j+1} and U_j into W_{j+1} , respectively. There are also synthesis operators σ_j that perform the reverse procedure, mapping $U_{j+1}\times W_{j+1}$ into U_j, so that the reconstruction is perfect, i.e. $\sigma_j(\alpha_j,\omega_j)=1_j$, where 1_j denotes the identity operator on U_{j+1}. These operators were outlined for the sake of completeness since our method does not involve any reconstruction of the signal. The analysis and the detail operators are the ones that deliver the features of interest of the signal. Our method is based only on the analysis operators since the P300 potential has a shape that is evidenced by the global behaviour during several hundreds of milliseconds and not by the details that are rather induced by noise. Each step of the algorithm splits the signal in two parts: one contains an approximation of the signal that reflects its essential features while the other includes the details. This approximation signal is used in our study for the P300 detection. If several steps are performed, one may obtain the following recursive scheme that decomposes the initial signal s_0 into approximation s_1 and detail d_1 signals, then the approximation signal s_1 is split into s_2 (approximation) and d_2 (detail) signals and so on, as presented in formula (1) – the "↔" sign denotes the possibility of decomposition/reconstruction of the signal

$$s_0 \leftrightarrow [s_1, d_1] \leftrightarrow [s_2, d_2, d_1] \leftrightarrow [s_3, d_3, d_2, d_1] \leftrightarrow \cdots [s_n, d_n, d_{n-1}, \cdots d_1] \cdots \tag{1}$$

The above sequence is generated applying the following operators: α (analysis), ω (detail) the last one, σ_n, being used to perform the reverse operation in case this is needed:

$$\begin{cases} s_{n+1} = \alpha_n(s_n) \in U_{n+1}, & n \geq 0, \\ d_{n+1} = \omega_n(s_n) \in W_{n+1}, & n \geq 0, \\ s_n = \sigma_n(s_{n+1}, d_{n+1}), & n = k-1, k-2, \cdots 0. \end{cases} \tag{2}$$

From the point of view of our method an adequate choice of the analysis operator is the one that may evidence the P300 event-related potential in its essential feature: the shape. It can be proved that if the signal analysis operator α is chosen as the maximum value between two consecutive samples of the signal

$$\alpha(s_n) = \max(s_{2n}, s_{2n+1}), \tag{3}$$

while ω, the detail analysis operator, is the difference between the same two consecutive samples

$$\omega(s_n) = s_{2n} - s_{2n+1}. \tag{4}$$

The perfect reconstruction is possible since all the conditions imposed, as mentioned in [20] and [21], are fulfilled.

It is worth noticing that in formula (3) it is possible to change the "maximum" operator with the "minimum" one

$$\alpha'(s_n) = \min(s'_{2n}, s'_{2n+1}) \tag{5}$$

and, as a result, one is able to obtain another recursive scheme like the one given by (1). This choice of functions is identical with the one known under the name of morphological Haar wavelet, [21], which is a variant of the Haar wavelet, [22].

Both functions are equally possible and the decision to use one of them (or both) depends on the task to be performed: while ours refers to the detection of a positive potential, we chose the mean value of the two signals obtained after the operations in (3) and (5) were performed.

IV. THE DATA SET

The data set used to illustrate our method was the one described in [23] and [24]. This choice was made because it was already used to validate the detection results obtained by other researchers and therefore could provide adequate feedback for our method. We will now remind, in brief, the paradigm and the conditions of the experiments that generated the data set.

The signals that were used in this study were collected by means of an international "10-20" system of electrodes, as described in [25], placed on the subject's scalp. A Farwell-Donchin paradigm, [1], [26], was used to collect the data:

the user was presented on the computer screen with a 6 by 6 matrix of characters; then, the subject was asked to focus his attention on characters in a word that was chosen in advance; each character of the word appeared on the screen in their initial succession. The rows and columns of the matrix were successively and randomly intensified at a rate of 5.7 Hz, and two out of 12 intensifications of rows or columns contained the desired character (one in a particular row and one in a particular column). The responses evoked by these infrequent stimuli are different from those generated by the ones that did not contain the desired character, and they are obviously the P300 responses. The sampling frequency used to acquire the EEG signals was 240 Hz, as also stated in [24].

V. THE DECISION CRITERION

To identify a P300 event-related potential, beyond the algorithm presented above, another element is needed to be certain if it was indeed elicited or not: a decision rule. It allows measuring the possibility of P300 appearance but also the confidence one may have in the obtained result. Choosing such a rule proved to be a challenging task because the EEG signals differ significantly according to the place they were acquired, the placement of the reference used in the experiment, the noise and the artefacts owed to the other physiological signals that interfere with them (most handy example: ocular artefacts) not to mention all the factors considered in [14]. We decided to normalise the signals obtained after the final step in which operators given in (3) and (5) were applied. We extracted the average value m of the initial EEG signal, computed for the entire recording from each signal s_{2k} that was obtained, so that each one of them should have approximately zero mean value (i.e. zero centred). In this manner the presence (or the absence) of the P300 potential is referenced to zero. Furthermore, a time interval (t_1, t_2) must be chosen as a possible time window where the event-related-potential is expected to appear. This was not so difficult to decide for t_1 since usually P300 was expected to be elicited usually not less than 300 milliseconds after the stimulus; for the upper limit we have considered 400 milliseconds as a value that should give significantly satisfactory results. It is worth mentioning the fact that small deviations of the real signal with an elicited P300 from the above time limits should not interfere with the detection if one computes the area A_{2k} delimited by the k-th decomposition of the initial signal s_{2k} and the time axis between the two already mentioned time limits

$$A_{2k} = \int_{smpl1}^{smpl2} (s_{2k} - m)dt. \qquad (6)$$

In (6) m denotes the average value computed for the whole EEG recording, while *smpl1* and *smpl2* are the corresponding samples of the signal for the initial time interval in the case of the k-th decomposition level.

Because the signal was already centred on zero, the presence of the potential is evidenced by a positive value of the area, while the missing one is indicated by a negative value.

It is also important to observe the need of the decomposed signal. The area given by (6) is computed using the sampled signal that is the electroencephalogram. Therefore computing the area in (6) with constant time divisions is reduced to summing the samples and, in the end, multiplying it with the time division. In fact, for uniform sampled signals acquired with the same sampling rate, comparison is possible summing just the samples of the EEG signal

$$A_{2k} \sim \sum_{smpl1}^{smpl2} (s_{2k} - m). \qquad (7)$$

To decide if the P300 was elicited or not, due to the relatively small time interval allowed (300 to 400 milliseconds), every sample that may be influenced by perturbations counts and this is why it is desirable to get a global behaviour that is evidenced by the approximation signal; this kind of signal is obtained when applying each level of decomposition of the lifting scheme.

VI. RESULTS

Let us recall the fact that for the data used in our study the sampling frequency was 240 Hz and the standard length of a recording was approximately one second. Therefore each recording was 240 samples long. Bearing in mind that we were interested only in the time interval t_1=300 ms and t_2=400 ms, transposed in samples, this time interval led to samples of interest between 72 and 96 in the case of the initial signal.

We shall focus just on one channel of the EEG signal to present our results, namely the Fz; we made that choice because the P300 event-related potential may be evidenced here, but this is not the most used channel (Pz and Cz are far more used and probably give better results). Nevertheless we shall prove that our method is suitable and provides excellent results even in this marginal case.

We used averaged signals both for the elicited and the missing P300 for all the 15 trials; an illustration of these signals, as they were initially recorded, may be seen in Fig. 1 and Fig. 2. Note the shape of the signal between samples 72 and 96: there is an obvious peak when P300 is present (Fig.1).

First, each signal was analysed using two levels of decomposition by means of the lifting scheme based on the morphological Haar wavelet with the “min” function given by (5). The first level in the case of the elicited P300 led to the components for the approximation signal and the detail signal that are presented in Fig. 3 and Fig. 4. It is worth remembering that for these signals the initial [300ms, 400ms] time interval corresponds to samples between 72/2=36 and 96/2=48 because we are dealing with the first level of decomposition. The second level is presented in Fig. 5 (for the approximation signal) and 6 (for the detail signal). Here the samples corresponding to the time interval of interest are between 18 and 24. In both cases the scales of amplitude for the detail components are significantly lower than those of the approximation ones.

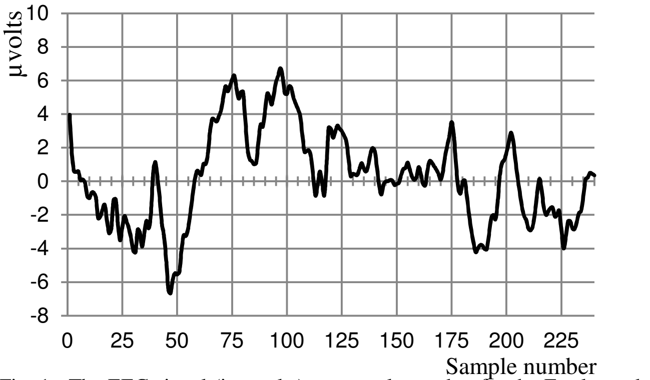

Fig. 1. The EEG signal (in µvolts) vs. sample number for the Fz electrode with elicited P300 event-related potential.

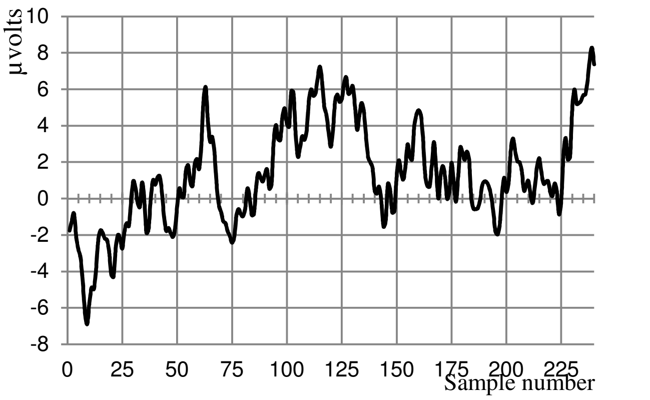

Fig. 2. The EEG signal (in µvolts) vs. sample number for the Fz electrode without elicited P300 event-related potential.

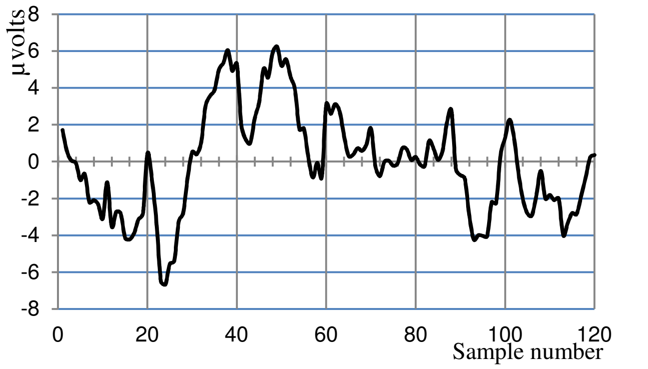

Fig. 3. The first level of the decomposition for the elicited P300: the approximation component (in µvolts) vs. sample number for the "min" function for the Fz electrode.

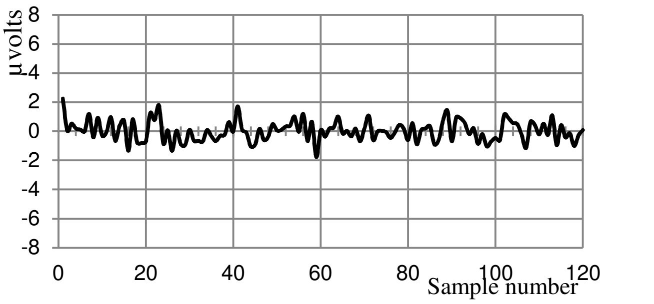

Fig. 4. The first level of the decomposition for the elicited P300: the detail component (in µvolts) vs. sample number for the "min" function for the Fz electrode.

The first attempts to apply the lifting scheme with the "min" function showed that our method gave good results in the case of non-elicited P300 but poor ones for the other case.

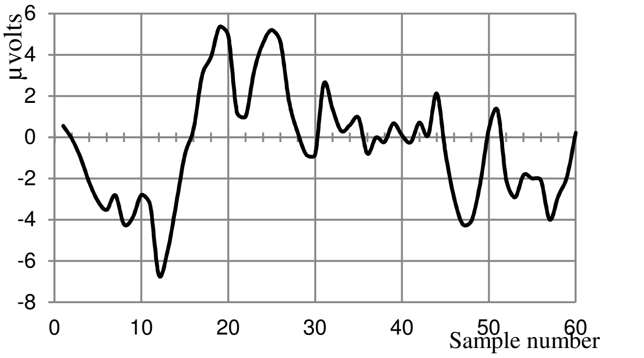

Fig. 5. The second level of the decomposition for the elicited P300: the approximation component (in µvolts) vs. sample number for the "min" function for the Fz electrode.

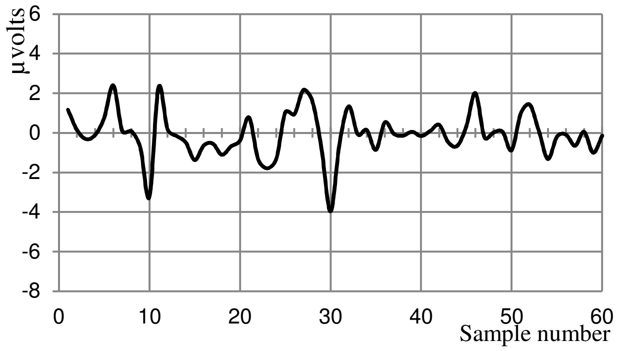

Fig. 6. The second level of the decomposition for the elicited P300: the detail component (in µvolts) vs. sample number for the "min" function for the Fz electrode.

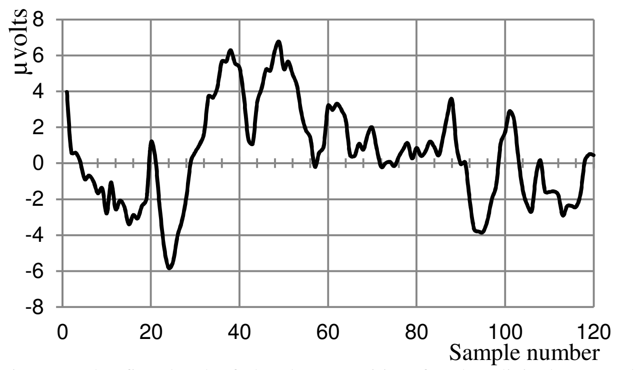

Fig. 7. The first level of the decomposition for the elicited P300: the approximation component (in µvolts) vs. sample number for the "max" function.

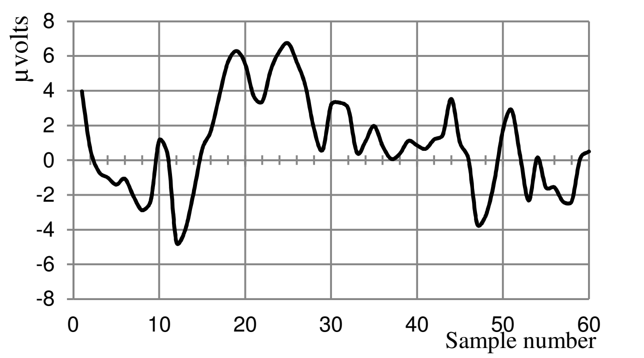

Fig. 8. The second level of the decomposition for the elicited P300: the approximation component (in µvolts) vs. sample number for the "max" function.

That led us to the decision of using the other function, “max”, given in (7), as shown below, in the case of the elicited P300, only for the approximation components for the first level, in Fig. 7, and the second one in Fig. 8.

The results showed the reverse situation: the detection was excellent when the P300 potential was present but rather poor in the case of its absence. To cure this we averaged the two components and with this choice we have obtained a rate of success of 100% for the detection of the presence (or the absence) of P300 event-related potential in the case of the averaged signals for all the electrodes that were usually involved in decision making. Figure 9 depicts the three signals (“min” function, “max” function and the averaged one) for the second level of decomposition in the case of the elicited P300.

The fundamental nature of this choice resides in the fact that the two initial functions “min” and “max” smoothed the recording of P300 both from beneath and above and applying the average of their result better revealed the elicited P300.

To be absolutely certain of a correct detection one may impose a threshold for the value of the computed area but obviously this option leads to more computations; nevertheless our results based on averaged signals showed no need for such a decision, at least for the recordings used to validate our research.

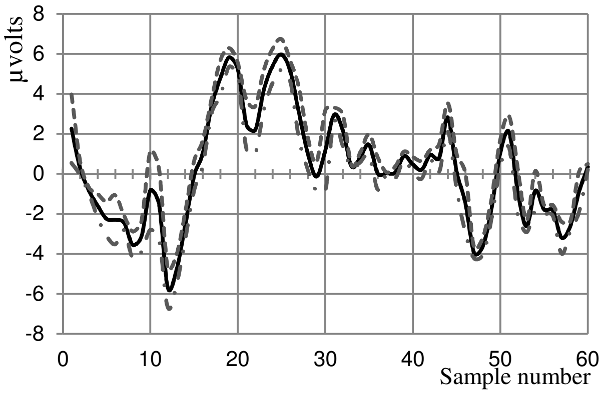

Fig. 9. The second level of the decomposition for the elicited P300: the approximation component, the “max” function (dashed), the “min” function (dash-dot) and the average (continuous).

VII. CONCLUSIONS

In spite of the fact that for the detection purpose usually an averaged signal is used, we tried to apply our method to recordings containing the average of 15 signals. Let us define the rate of success as the number of successful detections of the presence or the absence of the P300 event-related potential divided by the total number of attempts. This rate of success was 73% for the signals from Cz and Fz electrodes, 63% for CPz and Pz and 50% for Oz. It must be mentioned that these results were obtained with recordings that supposed to be of a certain type regarding the presence or the absence of the P300 potential, but they were not priory classified; it is well known the fact that the appearance of the P300 depends on many factors and not all the subjects always elicit one when the stimulus is present.

Another observation was that in the case of the wrong detection of an individual recording the average function acted upon the vicinity of zero and a certain amount of emphasising on the min function (in the case of the non-elicited P300) or max function (in the case of the elicited one) could led to far better results since more than 60% of the files were “nearly” misclassified (i.e. the computed area was in the [-0.5, 0.5] interval) when false detection occurred. This led to the conclusion that for individual recordings one may adjust the average function used in the detection according to the specification given by a certain subject and recording environments that influence the acquired signal, introducing both the “min” signal and the “max” signal in a weighted manner.

Our method must be compared to the ones that use the wavelet transform of the EEG signal; the main difference is the way in which the decomposed signal is obtained. Dealing only with comparisons between consecutive samples to obtain the higher levels of decomposition and by choosing the non linear function that “smoothes” the details, the proposed method is more efficient. The complexity of the computations is also lower than in the case of the wavelet transform. The decision criterion, based in the case of the classical wavelet transform on the comparison between two coefficients is replaced in our study with a comparison to zero of a quantity that consists in a sum of already known samples. Because the time interval of interest is rather short and the level of decomposition is no more than two, the computing time needed to perform the task is overall smaller than in the case of the linear wavelet transform, no matter the mother wavelet used.

We focused our efforts to develop a method for the detection of the P300 potential seen as a feature of the EEG and not to a whole BCI system (that would require an additional classifying algorithm, e.g. the one described in [27] and a strategy to choose the adequate electrodes; therefore the classifying part was not our concern). There are two ways in which the rate of success may be further improved: 1) averaging the EEG signals of the same type (with or without the P300 event-related potential) and 2) using a different percent mix (and not the average) of the signals obtained after the “min” and “max” functions were applied.

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