Linearization Approach for Symmetric Hysteresis Loop Modelling and Core Loss Prediction

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Abstract—First, the paper proposes the method for interpolation of any experimentally obtained symmetric hysteresis loop curve (SHLC) with accuracy and computation efficiency at discrete Fourier transformation (DFT) level. Second, the method has been further developed so that, based on the family of the properly chosen and measured SHLCs, it reliably and accurately predicts an arbitrary inner SHLC. Sinusoidal magnetic flux, along with applied zero crossing sampling system, allows for the introduction of the pure linearization approach. The novelty of this approach is a direct transformation of a cosine polynomial (CP) interpolating of one SHLC over the set of equidistant nodes in the electric angle (EA) domain to the algebraic polynomial (AP) interpolating the same SHLC over the set of nonequidistant Chebyshev nodes in the magnetic flux (MF) domain, with the accuracy remaining unchanged. Based on the results of the interpolation error analyses, the SHLC measurement has been proposed for nonequidistant values of magnetic flux at the loop tip, matching the Chebyshev nodes of the second kind. This is the second novelty which enables a successful prediction of an arbitrary inner SHLC.

Index Terms—Accuracy; error estimation; interpolation; magnetic hysteresis; magnetic losses; transformer cores.

I. INTRODUCTION

The usage of a properly selected numerical fitting method is a common characteristic of the majority of phenomenological approaches to the magnetic curves analysis and modelling. This applies to the earlier works treating some particular magnetic curves (Widger [1], Rivas, Zamarro, Martin, and Pereira [2]), over the flexible Takacs model [3] numerically parameterized by Meszaros [4] and de la Barriere, Ragusa, Appino and Fiorillo [5] to the most comprehensive and usable models recently developed by Zirka, Moroz, Harrison and Chieza [6], Zirka, Moroz, Harrison, Chieza and Hoidalen [7], [8], and Alonso, Jazebi, and de Leon [9]. It should be noted that the works [6]–[8] are based on the continuous earlier research, referenced in [6], and a careful analysis of certain drawbacks in practical usage of the Jiles-Atherton [10], [11] and Preisach model [12], [13].

Different numerical fitting methods, applied to the different models of magnetic curves, affect error distribution in very specific ways. The authors have generally avoided treating this difficult problem explicitly, and mainly resorted to giving graphical presentation of error distribution in particular cases, and offering a remark regarding the good correlation between the experimental and theoretical results. Therefore, there are no consistent means for the comparative estimation of the accuracy that the proposed methods can offer.

On the other hand, the improved flexibility of the proposed models [6]–[8] usually has as a consequence a very tedious preparation for the numerical computations. As an alternative approach, the interpolation method has been proposed in the symmetric hysteresis loop (SHL) modelling by M. Hoffman and H.-G. Herzog [14]. Unfortunately, this remarkable work does not provide an explicitly derived error estimation of a chosen set of the interpolation nodes for the measured SHLs. Secondly, the very interpolation procedure for an arbitrary SHL prediction was omitted. Given that the base set of the experimentally obtained SHLs is the milestone for the second and higher order reversal curves prediction [6], [7], [14], the accuracy of the interpolated SHLs is an important issue.

The above mentioned works [6]–[9], [14] treat the so-called inverse hysteresis models. The first step in each of the proposed models [6], [7], [14] is getting a certain number of SHLs by the measurement at some equidistant values of the magnetic flux or magnetic flux density at the loop tip. This work proposes the SHL measurement at nonequidistant values of magnetic flux at the loop tip matching the Chebyshev nodes of the second kind. Since the treated SHLs are symmetrical, their ascending or descending curves only need to be considered.

The symmetric hysteresis loop curves (SHLC) are measured under sinusoidal magnetic flux (MF) regime and
then represented by using the cosine polynomials (CP) in the electrical angle (EA) domain \([-\pi, 0]\) with an achieved accuracy at the discrete Fourier transform (DFT) level. Actually, sinusoidal MF allows for the introduction of the pure linearization approach, which in turn allows for the direct transformation of the CP of the form

\[ p_n(\theta) = \sum_{k=0}^{n} c_k \cos(k\theta), \quad (1) \]

over the EA domain \( \theta \in [-\pi, 0] \), to the algebraic one of the form

\[ \Pi_n(\phi) = \sum_{k=0}^{n} c_k T_k(\phi/\Phi_m), \quad (2) \]

with the remaining \( c_k \) coefficients and accuracy unchanged. The polynomial (2) represents the same SHLC, but now in the MF domain \( \phi \in [-\Phi_m, \Phi_m] \). Terms \( T_k(\phi/\Phi_m) \) in (2) denote the k degree Chebyshev polynomials of the first kind, whereas \( \phi \) represents the instantaneous MF values, and \( \Phi_m \) denotes the maximal value of MF at the loop tip. In Section II the linearization approach will be briefly presented and the existence of the discrete sets of samples showing how the method can be applied will be considered. An estimation of the approximation error, as well as the testing of the accuracy of the method by considering distribution of the approximation error for experimentally obtained SHLC, is presented in Section III. In Section IV, this method is further developed for the interpolative prediction of an arbitrary inner SHLC, while the discussion and conclusion is given in Section V and Section VI respectively.

II. THEORY OF LINEARIZATION APPROACH AND EXPERIMENTAL RESULTS

A. Theoretical and Experimental Requirements for the Validity of the Method

Let us assume that the MF in magnetic circuit of an Epstein frame (toroidal specimen) or a single phase transformer at industrial frequency \( f = 50 \text{ Hz} \) or \( f = 60 \text{ Hz} \) is sinusoidal. A generally accepted view in this case is that the extremes of the same sign of the excitation current and the MF occur simultaneously at the moments when the induced electromotive force in the primary and secondary windings achieves zero values (Fitzgerald, Kingsly, Umans [15]).

The previous assumption implies the use of a dual-channel zero crossing system for synchronous sampling. For this purpose, the authors used two Agilent 3458 multimeters in the zero crossing mode with a sampling rate of 10 kHz. One channel was used for the sampling of the excitation current \( i_0(t) \) and another for the sampling of the induced electromotive force in the secondary winding \( e_0(t) \). The \( e_0(t) \) was used as a trigger zero crossing signal, while the power source was the sinusoidal voltage source Fluke 6100 A with \( f = 50 \text{ Hz} \) mains frequency. For this purpose, a single phase transformer 220 V/57.73 V, \( S_n = 100 \text{ VA} \) was used as a measuring object. Along with sample sets of the \( i_0(t) \) and \( e_0(t) \) obtained by the measurement, the MF samples were obtained by the numerical integration from the \( e_0(t) \) samples. The measurements were done for the next series of 13 supply voltages matching the Chebyshev nodes of the second kind. These values are represented in Table I.

| Table I. RMS SUPPLY VOLTAGE VALUES MATCHING CHEBYSHEV NODES OF THE SECOND KIND. |
|---------------------------------|---------------------------------|
| 13.81 V, 41.22 V, 67.98 V, 93.67 V, 117.88 V, 140.23 V, 160.37 V, 177.98 V, 192.78 V, 204.55 V, 213.08 V, 218.26 V, 220.00 V |

The above mentioned sampling system enables a set of 200 samples of the excitation current \( i_0(t) \) and the electromotive force \( e_0(t) \) for an entire period \( T = 0.02 \text{ s} \), \( f = 50 \text{ Hz} \). The sampling process is synchronized with regard to the zero values of the \( e_0(t) \) signal, which is equal to the synchronization with regard to the extremes of the signals \( \phi(t) \) and \( i_0(t) \). Therefore, from the set of 200 samples for an entire period \( T \), and by using software processing, a set of 101 samples was selected that belong to the ascending part of the signals \( \phi(t) \) and \( i_0(t) \) on the EA domain \( \theta \in [-\pi, 0] \), \( \theta = 2\pi n / T \), resulting in \( i_0(-\pi) = -I_m \), \( i_0(0) = I_m \), \( \phi(-\pi) = -\Phi_m \), \( \phi(0) = \Phi_m \). In other words, the first sample from this set that corresponds to the negative extreme, for both the MF and the excitation current, was indexed with the index value 0. Then, the last sample from this set that corresponds to the positive extreme for both the MF and the excitation current was indexed with the index value 100. The synchronized sets of 101 samples that belong to the ascending part of the signals \( \phi(t) \) and \( i_0(t) \) are graphically shown in Fig. 1.

![Fig. 1. Waveforms of excitation current (grey line) and magnetic flux (black line) synchronized on the electric angle segment \([-\pi, 0]\) (a); Excitation current mapping into the ascending hysteresis loop curve (b).](image)

The synchronized set of sinusoidal MF samples actually represents the values of cosine function \( \phi(t) = \Phi_m \cos \theta \) into the EA domain \( \theta \in [-\pi, 0] \). By mapping the abscissae domain \( \theta \in [-\pi, 0] \) by means of the function \( \Phi_m \cos \theta \) onto abscissae domain \( \phi \in [-\Phi_m, \Phi_m] \), with the ordinates remaining unchanged, the MF samples are perfectly linearly distributed along a straight line of the form \( y = x \). As a consequence of the new MF distribution, the excitation current samples will be distributed along the ascending
B. Existence of the Subsets of Samples Which Represent the Chebyshev Nodes of the First and the Second Kind

Now the sample index is denoted by \( k \). Then the Chebyshev nodes of the first kind (CHN_I nodes) are determined by the expression [17]:

\[
x_{k\text{CHI}} = \cos\theta_{k\text{CHI}},
\]

\[
\theta_{k\text{CHI}} = -\left(2n_{\text{CHI}} - 2k + 1\right)\pi / 2n_{\text{CHI}},
\]

where \( k = 1, 2, \ldots, n_{\text{CHI}} \lt 100 \), and the Chebyshev nodes of the second kind (CHN_II nodes) are determined by the expression [17]:

\[
x_{k\text{CHII}} = \cos\theta_{k\text{CHII}},
\]

\[
\theta_{k\text{CHII}} = (k / (n_{\text{CHII}} - 1) - 1)\pi,
\]

where \( k = 0, 1, \ldots, n_{\text{CHII}} - 1 \). By means of the substitution \( \Delta \theta = \pi / 100 \), where \( \Delta \theta \) represents the sampling step, expressions (3)–(4) and (5)–(6) get the next form:

\[
x_{k\text{CHI}} = \cos\left\{-\frac{\left(2n_{\text{CHI}} - 2k + 1\right)\times 100 \times \Delta \theta}{2n_{\text{CHI}}}\right\},
\]

where \( k = 1, 2, \ldots, n_{\text{CHI}} \).

\[
x_{k\text{CHII}} = \cos\left\{\frac{k \times 100}{n_{\text{CHII}} - 1} - 100\right\} \Delta \theta,
\]

where \( k = 0, 1, \ldots, n_{\text{CHII}} - 1 \). Based on (7), the expression for determining index values of the CHN_I nodes is derived as

\[
k_{\text{CHI}} = 100 - \frac{\left(2n_{\text{CHI}} - 2k + 1\right)\times 100}{2n_{\text{CHI}}},
\]

(9)

where \( k = 1, 2, \ldots, n_{\text{CHI}} \). The above expression allows only those values for \( n_{\text{CHI}} \) that produce the integer values for \( k_{\text{CHI}} \). Therefore, \( n_{\text{CHI}} \) can take only the values from the set \{50, 25, 10, 5, 2, 1\}. Similarly, from (8) it is obtained

\[
k_{\text{CHII}} = k \times 100 / (n_{\text{CHII}} - 1),
\]

(10)

where \( k = 0, 1, \ldots, n_{\text{CHII}} - 1 \) and integer values for \( k_{\text{CHII}} \) will be obtained only if \( n_{\text{CHII}} \) takes the values from the set \{101, 51, 26, 11, 6, 5, 3, 2\}.

All possible index values obtained from the expressions (9) and (10) are represented in Table II, except for the values for \( n_{\text{CHI}} \leq 5 \) and \( n_{\text{CHII}} \leq 6 \). The reason for this is that an expected number of excitation current harmonics is certainly higher than 5. In addition, the index values for \( n_{\text{CHII}} = 101 \) are excluded from the Table II. In the case of \( n_{\text{CHII}} = 101 \), the entire set of samples would be included, making the analysis of the error distribution virtually impossible.

### Table II: Identification of the Sample Subsets that Represent Chebyshev Nodes

<table>
<thead>
<tr>
<th>( n_{\text{CHI}} )</th>
<th>( n_{\text{CHII}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100</td>
</tr>
</tbody>
</table>

C. Generating a SHLC Interpolation Polynomial over the Chebyshev Nodes of the First and the Second Kind

The interpolation polynomial over \( n_{\text{CHI}} = n + 1 \) CHN_I nodes from normalized MF domain \( x \in [-1, 1] \), \( x = \phi / \Phi_m \) has the following form [17]

\[
P_{n\text{CHI}}(x) = \frac{1}{2} c_0 \times T_0(x) + \sum_{k=1}^{n_{\text{CHI}}} c_k \times T_k(x),
\]

(11)

where \( x \in [-1, 1] \). Term \( T_k(x) \) in the above expression denotes a \( k \) degree Chebyshev polynomial of the first kind.

By virtue of the discrete orthogonality [17], [18] of the \( T_k(x) \) Chebyshev polynomials over CHN_I nodes, the \( c_k \) coefficients are computed by two equivalent expressions:

\[
c_k = \frac{2}{(n + 1)} \sum_{j=1}^{n_{\text{CHI}}} i_0(x_j) \times T_k(x_j),
\]

(12)

\[
c_k = \frac{2}{(n + 1)} \sum_{j=1}^{n_{\text{CHI}}} i_0(\theta_{j\text{CHI}}) \times \cos(k \times \theta_{j\text{CHI}}),
\]

(13)

where \( k = 0, \ldots, n_{\text{CHI}} - 1 \), \( j = 0, \ldots, n_{\text{CHI}} - 1 \), \( x_j = \cos(\theta_{j\text{CHI}}) \), and \( \theta_{j\text{CHI}} \) is determined by (4).

In accordance with (11), (12) and (13) a cosine interpolation polynomial over \( n + 1 \) CHN_I nodes in the EA domain \( \theta \in [-\pi, 0] \) has the following form [17]

\[
P_{n\text{CHI}}(\theta) = \frac{1}{2} c_0 + \sum_{k=1}^{n_{\text{CHI}}-1} c_k \times \cos k\theta,
\]

(14)

where \( \theta \in [-\pi, 0] \). An example of generating an
interpolation polynomial by means of (12)–(14) is represented for the set of \( n_{ChI} = 10 \) Chebyshev nodes of the first kind in tabulated form in Table III and in graphical form in Fig. 2. This polynomial interpolates the ascending SHLC of the most outer hysteresis loop obtained at 220 V supply.

![Graph of the cosine interpolation polynomial over 11 CHN_I nodes in the EA domain \( \theta \in [-\pi, 0] \) (a); Graph of the algebraic interpolation polynomial over 10 CHN_I nodes in the normalized MF domain \( x \in [-1, 1] \) (b).](Image 42x554 to 286x709)

**Table III. Chebyshev Nodes of the First Kind and the Coefficients of the Corresponding Interpolation Polynomial.**

<table>
<thead>
<tr>
<th>( n_{ChI} ) node index</th>
<th>sample ( \phi ) (index) ([\text{mWb}])</th>
<th>sample ( \phi ) (index)</th>
<th>sample ( \mu ) (index) ([\text{mA}])</th>
<th>coefficients for Chebyshev polynomial basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.343223</td>
<td>-0.98768834</td>
<td>117.73383</td>
<td>( c_0 ) = 0.0276080973</td>
</tr>
<tr>
<td>15</td>
<td>-1.214427</td>
<td>-0.891008652</td>
<td>-71.73744</td>
<td>( c_1 ) = 0.0897163468</td>
</tr>
<tr>
<td>25</td>
<td>-0.966170</td>
<td>-0.70710678</td>
<td>-20.84362</td>
<td>( c_2 ) = -0.0095927426</td>
</tr>
<tr>
<td>35</td>
<td>-0.621415</td>
<td>-0.45399050</td>
<td>6.71507</td>
<td>( c_3 ) = 0.0301421501</td>
</tr>
<tr>
<td>45</td>
<td>-0.214354</td>
<td>-0.15643446</td>
<td>16.62675</td>
<td>( c_4 ) = -0.0048764490</td>
</tr>
<tr>
<td>55</td>
<td>0.214354</td>
<td>0.15643446</td>
<td>21.98335</td>
<td>( c_5 ) = 0.0037755316</td>
</tr>
<tr>
<td>65</td>
<td>0.621415</td>
<td>0.45399050</td>
<td>33.87069</td>
<td>( c_6 ) = -0.0003212031</td>
</tr>
<tr>
<td>75</td>
<td>0.966170</td>
<td>0.70710678</td>
<td>59.07249</td>
<td>( c_7 ) = 0.0005555546</td>
</tr>
<tr>
<td>85</td>
<td>1.214427</td>
<td>0.891008652</td>
<td>90.99124</td>
<td>( c_8 ) = 0.0004339397</td>
</tr>
<tr>
<td>95</td>
<td>1.343223</td>
<td>0.98768834</td>
<td>119.09578</td>
<td>( c_9 ) = 0.0001550182</td>
</tr>
</tbody>
</table>

Similarly, by virtue of the discrete orthogonality of the \( T_k(x) \) Chebyshev polynomials over \( n_{ChII} = n + 1 \) Chebyshev nodes of the second kind [17], the following expressions hold

\[
P_{n_{ChII}}(x) = \frac{1}{2} c_0 T_0(x) + \sum_{k=1}^{n} c_k T_k(x) + \frac{1}{2} c_n T_n(x),
\]

where \( x \in [-1,1] \).

\[
c_k = \frac{2}{n} \left( \frac{1}{2} i_0(x_0) T_k(x_0) + \sum_{j=1}^{n-1} i_0(x_j) T_k(x_j) + \frac{1}{2} i_0(x_n) T_k(x_n) \right),
\]

\[
c_k = \frac{2}{n} \left( \frac{1}{2} i_0(\theta_0) T_k(\theta_0) + \sum_{j=1}^{n-1} i_0(\theta_j) \cos(k \theta_j) + \frac{1}{2} i_0(\theta_n) \cos(k \theta_n) \right),
\]

\[
(16)
\]

To generate an AP over the domain of actual values of MF \( \phi \in [-\Phi_m, \Phi_m] \), it is necessary to transform an AP \( P_\phi(x) \) from the basis of the Chebyshev polynomials of the first kind \( T_k(x) \) into the basis of monomial \( x^k \)

\[
P_\phi(x) = \sum_{k=0}^{n} c_k T_k(x) = \sum_{k=0}^{n} c_k x^k.
\]

By using the substitution \( x = \phi / \Phi_m \) into (19) we obtain an AP over the MF domain \( \phi \in [-\Phi_m, \Phi_m] \) of the form

\[
(17)
\]

Fig. 2. Graph of the cosine interpolation polynomial over 10 CHN_I nodes in the EA domain \( \theta \in [-\pi, 0] \) (a); Graph of the algebraic interpolation polynomial over 10 CHN_I nodes in the normalized MF domain \( x \in [-1, 1] \) (b).

**Table IV. Chebyshev Nodes of the Second Kind and the Coefficients of the Corresponding Interpolation Polynomial.**

<table>
<thead>
<tr>
<th>( n_{ChII} ) node index</th>
<th>sample ( \phi ) (index) ([\text{mWb}])</th>
<th>sample ( \phi ) (index)</th>
<th>sample ( \mu ) (index) ([\text{mA}])</th>
<th>coefficients for Chebyshev polynomial basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.359462</td>
<td>-1.000000000</td>
<td>-123.795498</td>
<td>( c_0 ) = 0.0276971868</td>
</tr>
<tr>
<td>10</td>
<td>-1.294644</td>
<td>-0.951056516</td>
<td>-98.412061</td>
<td>( c_1 ) = 0.0896541475</td>
</tr>
<tr>
<td>20</td>
<td>-1.104122</td>
<td>-0.890916994</td>
<td>-44.543009</td>
<td>( c_2 ) = -0.009541213</td>
</tr>
<tr>
<td>30</td>
<td>-0.803888</td>
<td>-0.587785252</td>
<td>-3.735111</td>
<td>( c_3 ) = 0.0300749038</td>
</tr>
<tr>
<td>40</td>
<td>-0.423263</td>
<td>-0.309016994</td>
<td>13.030176</td>
<td>( c_4 ) = -0.0047526785</td>
</tr>
<tr>
<td>50</td>
<td>0.000000</td>
<td>0.000000000</td>
<td>19.154967</td>
<td>( c_5 ) = 0.0034839057</td>
</tr>
<tr>
<td>60</td>
<td>0.425263</td>
<td>0.309016994</td>
<td>26.337257</td>
<td>( c_6 ) = -0.0000148624</td>
</tr>
<tr>
<td>70</td>
<td>0.803888</td>
<td>0.587785252</td>
<td>45.335193</td>
<td>( c_7 ) = 0.0004456793</td>
</tr>
<tr>
<td>80</td>
<td>1.104122</td>
<td>0.890916994</td>
<td>74.312542</td>
<td>( c_8 ) = 0.0004815687</td>
</tr>
<tr>
<td>90</td>
<td>1.294644</td>
<td>0.951056516</td>
<td>107.005978</td>
<td>( c_9 ) = 0.0000286386</td>
</tr>
<tr>
<td>100</td>
<td>1.359462</td>
<td>1.000000000</td>
<td>123.795498</td>
<td>( c_{10} ) = -0.0000424001</td>
</tr>
</tbody>
</table>

Fig. 3. Graph of the cosine interpolation polynomial over 11 CHN_II nodes in the EA domain \( \theta \in [-\pi, 0] \) (a); Graph of the algebraic interpolation polynomial over 11 CHN_II nodes in the normalized MF domain \( x \in [-1, 1] \) (b).
\[ \Pi_n(\phi) = \sum_{k=0}^{n} K_k \phi^k. \]  

(20)

### D. Determination of Core Losses by Usage of Interpolation Polynomials: \( P_n(x) \) versus \( \Pi_n(\phi) \)

Figure 4 depicts the closed symmetrical hysteresis loop over the domain \( x \in [-1,1] \) that consists of an AP \( P_n(x) \) of the form (19) for an ascending SHLC and \(-P_n(-x)\), the centrally symmetric to the origin \( \phi = 0 \), the descending SHLC. The polynomial \( P_n(x) \) has the form of (11) for CHN_I nodes, or (15) for CHN_II nodes. The same figure also depicts a closed symmetrical hysteresis loop over the MF domain \( \phi \in [-\Phi_m, \Phi_m] \) consisting of polynomials \( \Pi_n(\phi) \) (20) and \(-\Pi_n(-\phi)\).

Core losses can be determined by calculating the area of the closed hysteresis loop. Although it seems that this calculation requires the use of the AP form (20)

\[ S_{H\phi} = \int_{-\Phi_m}^{\Phi_m} (\Pi_n(\phi) + \Pi_n(-\phi)) d\phi, \]  

(21)

the integration gives as a result

\[ S_{H\phi} = 4 \Phi_m \sum_{i=0}^{n/2} \left( c_{2i}/(2i-1)(2i+1) \right). \]  

(22)

In particular, for \( n+1=10 \) CHN_I nodes, expression (22) gives

\[ S_{H\phi} = 4 \Phi_m \left\{ \frac{c_0}{2} - \left( \frac{c_2}{1 \times 3} + \frac{c_4}{3 \times 5} + \frac{c_6}{5 \times 7} + \frac{c_8}{7 \times 9} \right) \right\}, \]  

(23)

and for \( n+1=11 \) CHN_II nodes it gives

\[ S_{H\phi} = 4 \Phi_m \times \left\{ \frac{c_0}{2} - \left( \frac{c_2}{1 \times 3} + \frac{c_4}{3 \times 5} + \frac{c_6}{5 \times 7} + \frac{c_8}{7 \times 9} - \frac{1}{2} \times \frac{c_{10}}{9 \times 11} \right) \right\}. \]  

(24)

The expressions (22), (23) and (24) point out an important fact regarding numerical calculations, namely, that an area of the closed symmetrical hysteresis loop can be calculated directly on the basis of \( c_n \) coefficients and the \( \Phi_m \) value without the need for generating an AP of the form (20).

### III. ANALYSES OF ERROR BEHAVIOUR AND ERROR DISTRIBUTION

#### A. CHN_I Nodes versus CHN_II Nodes

After transition from the EA domain (time domain) to the algebraic domain (domain of MF), CPs (14) and (18) will be transformed into the APs (11) and (15). This transformation makes possible the distribution function of approximation error

\[ e_{n_{ChI}}(x_i) = i_0(x_i) - P_{n_{ChI}}(x_i), \]  

(25)

where \( i = 0,...,100, x_i \in [-1,1] \), based on the application of Rolle's theorem for the CHN_I nodes, to be expressed in the form

\[ e_{n_{ChII}}(x) = i_0(x) - P_{n_{ChII}}(x) = i_0(\Phi_m)(\Phi) \cdot \frac{T_{n_{ChII}}(x)}{2^n_{ChII} \times n_{ChII}}, \]  

(26)

where \( \min(\Phi_0,\ldots,\Phi_{n_{ChII}-1}) < \zeta < \max(\Phi_0,\ldots,\Phi_{n_{ChII}-1}) \).

Terms \( T_n(x) \) and \( U_n(x) \) in the expressions (26) and (28) represent the Chebyshev polynomials of the first and second kind of degree \( n \) [17].

Since the analytical form of the function \( i_0(x) \) is unknown, the expressions (26) and (28) enable the comparison of the quality of approximation for the case when \( n_{ChI} = n_{ChII} = n \). In such case, the next expressions hold:

\[ \max_{x\in[-1,1]} \left| i_0^{(n)}(x) \right| \leq M, \]  

(29)

\[ \left| e_{n_{ChII}}(x) \right| \leq \left| i_0^n(x) \right| \left| T_n(x) \right| \frac{M}{2^n!} \leq \frac{M}{2^n!}, \]  

(30)

\[ \left| e_{n_{ChII}}(x) \right| \leq \left| i_0^n(x) \right| \left| U_{n-2}(x) \right| \frac{M}{2^{n-1}!} \leq \frac{M}{2^n!}. \]  

(31)

The term \( i_0^{(n)}(x) \) represents the \( n \)-th derivative of the \( i_0(x) \) function.

For the practical application, when the interpolation nodes are formed from the set of samples on the basis of (7) and (8) (Table I), will be \( n_{ChII} = n_{ChI} + 1 \). Then, the impact of
the modulation polynomial \((x^2-1)U_{n_{ChII}}^{-2}(x)\) on the distribution function \(e_{n_{ChII}}(x)\) will be favourable toward the impact of the modulation polynomial \(T_{n_{ChII}}(x)\) on the distribution function \(e_{n_{ChII}}(x)\). This is represented in Fig. 5 for the \(n_{ChI} = 10\) and \(n_{ChII} = 11\) case.

Fig. 5 Graphs of the error distribution functions produced by interpolation polynomials obtained for 10 CHN_I and 11 CHN_II nodes (a); Graphs of the corresponding modulation polynomials for 10 CHN_I and 11 CHN_II nodes (b).

As an illustration of the convergence behaviour, Fig. 6 represents the graphics of the experimentally obtained ascending hysteresis loop curve \(i_0(t)\) and its interpolation polynomials for \(n_{ChI} = 10\), \(n_{ChII} = 11\), \(n_{ChII} = 21\), \(n_{ChII} = 25\) and \(n_{ChII} = 26\) nodes, together with the corresponding error distribution functions.

Fig. 6. Graphs of the error distribution functions: (a) for 10 CHN_I – gray and for 11 CHN_II nodes black line; (b) for 21 CHN_II nodes – gray dotted, 25 CHN_I nodes – black dotted and for 26 CHN_II nodes – black solid line.

B. Accuracy Considerations Regarding the Choice of the Kind and the Number of Chebyshev Nodes and Comparison with the DFT Accuracy

Since the analysed SHLC belongs to the most outer hysteresis loop, obtained at 220 V supply, it represents the worst case scenario with regard to the error magnitude. The complete analysis of the all error aspects exceeds the frame of this work and demands an article entirely devoted to this very topic. Instead, the most important facts for interpolation as an approach for the reliable and efficient prediction of an arbitrary inner SHLC will be highlighted here. Two questions arise here about the further interpolation method development: which one of the two kinds of Chebyshev nodes is more suitable for the SHLC application and which number of nodes is needed to obtain a reliable accuracy?

With regard to the first question, there are three good reasons for the choice of the Chebyshev nodes of the second kind. The first is the above analysed impact of the modulation polynomial \((x^2-1)U_{n_{ChII}}^{-2}(x)\) on the error distribution function \(e_{n_{ChII}}(x)\), when the interpolation nodes are formed from the set of samples. The second is that the measured loop tips belong to the set of CHN_II nodes. And, finally, it has been shown in [19] that for CHN_II nodes usage, the accuracy achieved by using the cosine polynomial is at the level of DFT method proposed in [20].

For an estimation of the accuracy dependence upon the number of CHN_II nodes, the next two aspects will be considered: the accuracy of the closed hysteresis loop area determination and the accuracy of the SHLC instantaneous values determination.

The determination of the closed hysteresis loop area, based on the expressions (21)–(24), gives the results represented in Table V.

Concerning the second aspect, since the relative errors for the all approximated and measured instantaneous values fall below 1 %, whatever number of CHN_II nodes is used, all of the above treated sets of CHN_II nodes could be employed with regard to the achieved accuracy.

**TABLE V. RELATIVE ERRORS OF THE CLOSED HYSTERESIS LOOP AREA DETERMINED BY POLYNOMIAL TOWARD THE MEASURED.**

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>Measured area</th>
<th>Area obtained by polynomial</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_{ChII} = 11)</td>
<td>9.42364E-05</td>
<td>9.42528E-05</td>
<td>0.017</td>
</tr>
<tr>
<td>(n_{ChII} = 21)</td>
<td>9.42364E-05</td>
<td>9.42541E-05</td>
<td>0.019</td>
</tr>
<tr>
<td>(n_{ChII} = 25)</td>
<td>9.42364E-05</td>
<td>9.42541E-05</td>
<td>0.019</td>
</tr>
<tr>
<td>(n_{ChII} = 26)</td>
<td>9.42364E-05</td>
<td>9.42509E-05</td>
<td>0.015</td>
</tr>
<tr>
<td>(n_{ChII} = 50)</td>
<td>9.42364E-05</td>
<td>9.42513E-05</td>
<td>0.016</td>
</tr>
<tr>
<td>(n_{ChII} = 51)</td>
<td>9.42364E-05</td>
<td>9.42524E-05</td>
<td>0.017</td>
</tr>
</tbody>
</table>

IV. BRIEF EXPLANATION OF THE PROCEDURE FOR AN ARBITRARY INNER SHLC INTERPOLATION

A. Decomposition of a Measured SHLC into its Hysteric and Anhysteretic Components

As the first step in the interpolation of an arbitrary inner hysteresis loop, it is necessary to decompose each of the measured SHLC to its hysteric and anhysteretic components

\[
i_0(\theta) = i_{anhyst}(\theta) + i_{hyst}(\theta),
\]

over EA domain \(\theta \in [-\pi,0]\) by using the next two expressions:

\[
i_{anhyst}(\theta) = 0.5 \times (i_0(\theta) - i_0(-\theta + \pi)),
\]

\[
i_{hyst}(\theta) = 0.5 \times (i_0(\theta) + i_0(-\theta + \pi)),
\]

or in normalized MF domain \(x \in [-1,1]\) by using:

\[
i_{anhyst}(x) = 0.5 \times (i_0(x) - i_0(-x)),
\]

\[
i_{hyst}(x) = 0.5 \times (i_0(x) + i_0(-x)).
\]
This decomposition has been proposed in [19], [21] and is represented in Fig. 7 for the most outer ascending SHLC measured at 220 V supply. The hysteretic components of all 13 measured ascending SHLCs are represented in Fig. 8.

C. Choice of the Number of CHN_II Nodes over the domain \( x \in [-1, 1] \)

The next step is the choice of \( k_{CHII} \) CHN_II nodes over the domain \( x \in [-1, 1] \). These nodes are represented by vertical lines in Fig. 8. For each of these \( k_{CHII} \) nodes, the instantaneous value of the function \( i_{hyst}(x_k) \) must be determined for an arbitrary chosen value of the supply voltage for the range from 0 V to 220 V. This task is accomplished by using the interpolation polynomials generated at each of \( x_i \) nodes, over 13 CHN_II nodes matching the instantaneous values of the measured hysteretic SHLC components. This situation is represented in Fig. 8. Since each of these 13 values is positive, the above mentioned polynomials could be treated as odd, and they are represented by the coefficients with the odd indexes only.

Choice of \( k_{CHII} = 21 \) instead of \( k_{CHII} = 26 \) CHN_II nodes in \( x \in [-1,1] \) domain decreases computational costs without significant influence on accuracy (Fig. 6). Taking into account that hysteretic component of SHLC is, by definition (36), an even function, the actual number of computations is decreased from 21 to 10. This means that an interpolative prediction of an arbitrary inner hysteretic curve, in this case, resolves 10 vectors consisting of 13 coefficients for each one. These 10 vectors are represented in Table VI.

D. Interpolative Prediction of a Hysteric Curve for the Given Supply Voltage Value

For an arbitrary supply voltage rms value from the range \( U \in [0 \text{ V}, 220 \text{ V}] \) it is necessary to get its corresponding normalized value \( x \) by simple dividing \( x = U/220 \text{ V} \). Then, this normalized value \( x \) is to be submitted to each of the 10 interpolation polynomials represented by its coefficients in Table VI. As a result, 10 interpolated instantaneous values of an inner hysteretic curve \( i_{hyst}(x) \) are obtained at 10 of 21 CHN_II nodes. These nodes correspond to the samples with the indexes \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\} (Table I and Table VI). The instantaneous values at the first (index 0) and the last (index 100) node take the zero value, by definition of the function \( i_{hyst}(x) \). Finally, given that the function \( i_{hyst}(x) \) is even, the remaining 9 nodes from the range \([0, 1]\) take the same values as their corresponding nodes from the range \([-1, 0] \). This correspondence is represented by the following series of the index pairs \{5, 95\}, \{10, 90\}, \{15, 85\}, \{20, 80\}, \{25, 75\}, \{30, 70\}, \{35, 65\}, \{40, 60\}, \{45, 55\}.

Based on knowing 21 CHN_II nodes, the hysteretic curve \( i_{hyst}(x) \) can be interpolated by using the polynomial of the form (13), or after the transformation \( \theta = \arccos(x) \) by the polynomial of the form (16). The represented interpolation procedure is applied to 7 values of the supply voltage \{150 V, 160 V, 170 V, 180 V, 190 V, 200 V, 210 V\} and the interpolated hysteretic curves are represented in Fig. 8. The relative errors obtained by direct measurement of the core losses, and predicted by using the hysteretic curves, are below 0.2%.
TABLE VI  COEFFICIENTS OF THE POLYNOMIAL INTERPOLATING THE INSTANTANEOUS VALUES OF THE ANHYSTERETIC CURVES AT 10 NODES.

<table>
<thead>
<tr>
<th>coeff.</th>
<th>node index 5</th>
<th>node index 10</th>
<th>node index 15</th>
<th>node index 20</th>
<th>node index 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>6.3833E-04</td>
<td>3.4546E-03</td>
<td>7.9391E-01</td>
<td>1.2756E-02</td>
<td>1.6602E-02</td>
</tr>
<tr>
<td>c2</td>
<td>1.9670E-04</td>
<td>1.0753E-03</td>
<td>2.0113E-03</td>
<td>2.5881E-03</td>
<td>2.6902E-03</td>
</tr>
<tr>
<td>c3</td>
<td>2.1268E-05</td>
<td>5.2613E-04</td>
<td>6.6462E-04</td>
<td>7.1131E-04</td>
<td>9.5985E-04</td>
</tr>
<tr>
<td>c4</td>
<td>8.8017E-05</td>
<td>6.5178E-05</td>
<td>8.3940E-05</td>
<td>8.6819E-05</td>
<td>2.6178E-04</td>
</tr>
<tr>
<td>c5</td>
<td>-2.0495E-05</td>
<td>-1.1058E-04</td>
<td>-1.9030E-04</td>
<td>-2.9603E-04</td>
<td>4.0002E-04</td>
</tr>
<tr>
<td>c6</td>
<td>-4.1350E-05</td>
<td>-1.5731E-04</td>
<td>7.9399E-05</td>
<td>1.7102E-04</td>
<td>1.5289E-04</td>
</tr>
<tr>
<td>c7</td>
<td>-1.2483E-04</td>
<td>-1.8550E-04</td>
<td>-1.8922E-04</td>
<td>-1.3610E-04</td>
<td>9.0908E-05</td>
</tr>
<tr>
<td>c10</td>
<td>7.6067E-05</td>
<td>1.6202E-04</td>
<td>2.2437E-04</td>
<td>2.3772E-04</td>
<td>2.1392E-04</td>
</tr>
<tr>
<td>c21</td>
<td>9.6771E-05</td>
<td>2.0077E-04</td>
<td>2.5467E-04</td>
<td>2.5607E-04</td>
<td>2.1167E-04</td>
</tr>
<tr>
<td>c22</td>
<td>9.1052E-06</td>
<td>2.9877E-06</td>
<td>-1.0400E-05</td>
<td>-1.6869E-05</td>
<td>-1.5025E-05</td>
</tr>
</tbody>
</table>

V. DISCUSSION

Herein all steps of the proposed work will be summarized and the main contributions will be briefly discussed.

The first step is an identification of the sample subsets that represent Chebyshev nodes of the first and the second kind dependent of a given sampling frequency. Expressions (9) and (10) (Section II B) show explicit solution. Generation of the corresponding SHLC approximation polynomial is performed using (11)-(14) and (15)-(18) (Section II C). Described linearization approach gives the first important result – the degree of an approximation polynomial should be equal to the number of significant harmonics of an excitation current or slightly greater, to achieve expected DFT accuracy level.

The successful interpolative prediction of an inner SHLC requires usage of CHN II nodes, along with decomposition of all measured SHLCs to its hysteretic and anhysteretic components. These issues, along with the choice of optimal subsets of CHN II nodes, were analysed and resolved through Section III and Section IV.

The interpolative prediction of an arbitrary inner SHLC was tested through prediction of hysteretic SHLC components and its essence is presented in Section IV D. Since the hysteretic component of one SHLC and their interpolation polynomial are both even functions, core losses are predicted directly using (24).

VI. CONCLUSIONS

The linearization approach provides simple and efficient procedure of any magnetic curve approximation by generating an accurate interpolation polynomial. The level of accuracy of these polynomials can be adjusted choosing proper number of Chebyshev nodes, and the level of accuracy can be directly traced and observed. The situation with the fitting functions usage is mainly different, but this important topic exceeds the frame of this work.

The interpolative prediction of an inner SHLC requires that all measured SHLCs have to be decomposed to its hysteretic and anhysteretic component. In that case, the interpolative procedure must be performed over either normalized MF or EA domain. Moreover, generated interpolation polynomial can be transformed into the corresponding one from an actual MF domain without any loss of accuracy.

Finally, sum of both hysteretic (even) and its corresponding anhysteretic (odd) SHLC interpolation polynomial gives the actual SHLC interpolation polynomial.

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http://dx.doi.org/10.1109/TMAG.2014.2353579


