

A Novel Design Technique for Variable Non-recursive Digital Filter based on FrFT

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Introduction

Finite Impulse Response (FIR) filters with variable magnitude and phase characteristics in the pass-band (also called tunable digital filters), are often required in several signal processing applications. They are needed to perform a slightly modified task in accordance with the need that arises in real time processing. One such application is sample rate converter which employs various sample rates depending on the required signal quality and the available bandwidth of the channel and the data rate of the interfaces. If one applies the existing traditional constant filter design techniques, then for each rate change factor a new filter must be designed to satisfy the new desired characteristics. This results in higher computational complexity. Further the phase response of a window-based filter is directly proportional to the length of the filter at a particular frequency. Thus, filter length must be increased to tune the phase response which in turn increases complexity. To overcome this, a new method based on fractional Fourier transform is proposed here in order to modify the magnitude and phase characteristics of a filter. By analysing the behaviour of desired impulse response of FIR filter in various fractional Fourier domain, it is experiential that pass-band width and phase response can be made dependent on the changeable fractional order parameter.

We, here put forward a novel application of fractional Fourier transform in tuning frequency characteristics of window-based low-pass FIR digital filters. In this paper we analyze the behaviour of their frequency response in several fractional Fourier domains.

The design and implementation of digital filters with variable characteristics has been a constant area of interest for several researchers. Schuessler and Winkelnkemper[1] were the first to propose a transform approach in 1970. Georgi Stoyanov and Masayuki Kawamata [2] suggested a new method for design of variable IIR digital filters. Zarour and Fahmy [3] proposed the spectral parameter approximation method. Yeremeyev[4] designed a new cascaded structural realization of high selective filters implemented without multiplications. Sharma et al. [5] have introduced an alternate methodology to tune transition-width of window-based FIR digital filter with FrFT. Kumar et al. [6] have developed a new mathematical model for obtaining the fractional Fourier transforms of Dirichlet and Generalized “Hammimg” window functions and established the direct dependence of their FrFT on fractional angle α which can control the main-lobe width and stop-band attenuation of the resulting window function with the adjustable fractional parameter.

Review of fractional Fourier transform

The fractional Fourier transform(FrFT) was first defined mathematically by Namias [7] and by Mc-Bride and Kerr. FrFT has received much attention in recent years [8–10]. It is the generalization of ordinary Fourier transform [1] that depends on a parameter α and can be interpreted as a counterclockwise rotation by an angle α in the time-frequency plane. It is represented by R^α where $\alpha = a\pi/2$, α is the angle of rotation and a is the fractional order parameter.

The FrFT of the signal $f(u')$ represented along time axis denoted by u' , with angular parameter α is computed as

$$F_{\alpha}(u) = \int_{-\infty}^{\infty} f(u') K_{\alpha}(u', u) du' , \quad (1)$$

where the transform kernel $K_{\alpha}(u', u)$ of the FrFT is given by

$$K_{\alpha}(u', u) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}} \exp\left\{j\frac{u'^2+u^2}{2}\cot\alpha - juu'\csc\alpha\right\}, & \text{if } \alpha \text{ is not a} \\ & \text{multiple of } \pi, \\ \delta(u'-u), & \text{if } \alpha \text{ is a multiple of } 2\pi, \\ \delta(u'+u), & \text{if } \alpha + \pi \text{ is a multiple of } 2\pi, \end{cases} \quad (2)$$

where α indicates the rotation angle of the transformed signal for the FrFT.

It can transform a signal into an intermediate domain between time and frequency. The FrFT holds all the properties of Fourier transform and therefore has tremendous potential for improvement in the areas of signal processing and optics. FrFT gives one more degree of freedom while designing signal processing tools compared to Fourier transform.

FRFT has many applications in time-varying signal analysis. Signals of non-stationary spectral characteristics can be analyzed using FrFT with superior performance compared to Fourier transform. Fractional Fourier domain filtering [11] is particularly advantageous when the distortion or noise is of chirped nature. Such situations are encountered in several real life applications. One such example is SAR (Synthetic aperture radar which employs chirps as transmitted pulses). Other applications are holography, sonar signal processing, watermarking, encryption etc. It can be used in almost all applications where Fourier transforms were used.

Proposed tunable methodology

The impulse response function of ideal low-pass filter is given by

$$h_{LP}(F_c) = \left\{ K \frac{\sin[\pi F_c (\frac{N}{2} - k + 1)]}{\pi (\frac{N}{2} - k + 1)} \right\}, \quad k=1, 2, \dots, N+1 \quad (3)$$

where N is relatively large even number, $F_c = f_c/f_N$ is the normalized frequency, where f_c is the design low-pass cut-off frequency, f_N is the Nyquist frequency ($=f_s/2$), f_s being the sampling frequency, and K is a gain factor.

In window-based FIR filter, the multiplication of the window function $w(n)$ with $h_d(n)$ is equivalent to convolution of $H_d(w)$ with $W(w)$:

$$H(w) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(v) W(w-v) dv, \quad (4)$$

$$H(w) = H_d(w) * W(w), \quad (5)$$

where '*' denotes the convolution operator for ordinary Fourier transform; $W(w)$ is the frequency domain representation of the window function, $H(w)$ is the actual frequency response of filter and $H_d(w)$ is desired or ideal frequency response. Thus the convolution of $H_d(w)$ with $W(w)$ yields the frequency response of truncated FIR filter.

From the fractional Fourier domain analysis of desired impulse response in several fractional domains, it can be observed that the band-edge characteristics and the phase response of filter can be varied by changing FrFT order 'a'.

Proposed Novel Technique. The proposed method for tuning filter frequency characteristics includes the following steps:

1. Compute the desired impulse response coefficients $h_d(n)$ of low-pass FIR filter of specified length and initial cut-off frequency;
2. Take the a th fractional Fourier transform of $h_d(n)$ to obtain $H_a(w)$. Thus the desired frequency response is parameterized in passband-width with FrFT angle. When 'a'=1, the desired magnitude response can be obtained in ordinary Fourier domain;
3. Convolve $H_a(w)$ with the magnitude response of appropriate window function $W(w)$ to obtain adjustable magnitude and phase response of the filter.

In the proposed methodology, the desired impulse response coefficients $h_d(n)$ corresponding to the initial filtering requirements are computed at first. Then the desired magnitude response of filter is obtained by taking FrFT order $a=1$, and is convolved with the window response to obtain actual frequency response $H(w)$, which is stored to perform filtering. If required, the passband-edge characteristics of filter can be varied by changing FrFT order, and thus modifying $H(w)$ to $H_a(w)$. Thus, in the proposed work filter coefficients need not to be recomputed and tuning can be achieved by exploiting the additional degree of freedom coming from the fractional order parameter 'a'. Also the cost of computing the fractional Fourier transform is not more expensive than computing the ordinary Fourier transform. Also the computational resources required for achieving variable filter characteristics using proposed method will remain same.

Simulation results

To validate the proposed approach, we have simulated a low-pass FIR filter with cut-off frequency = 0.5π radians and length = 16. We analyzed the magnitude response and phase response of low-pass filter versus normalized frequency for different fractional order 'a' and also measured its performance parameters. The MATLAB code developed by Candan, Kutay and Ozaktas [12] is used for computing DFrFT of desired impulse response which closely matches with the continuous transform i.e. the transform results of DFrFT are similar to those of the continuous FrFT. If the similarity condition is obeyed, the continuous signal processing algorithms derived in continuous fractional Fourier domains can be directly

modified into the digital signal cases by replacing continuous FrFT with DFrFT. The variable filter frequency response of low-pass FIR filter (*Kaiser window response convolved with FrFT of desired impulse response*) for several values of fractional order parameter is shown in Fig. 1.

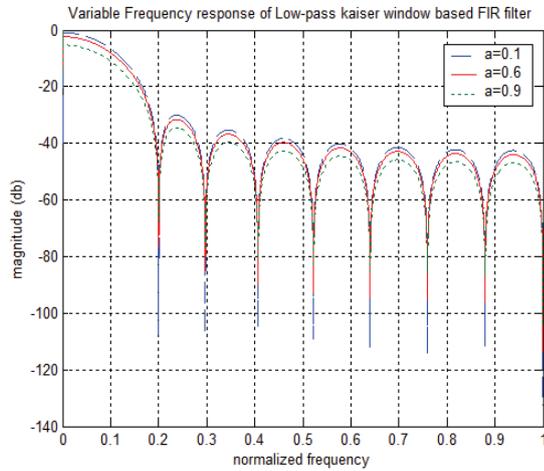


Fig. 1. Variable band-edge low-pass FIR filter for fractional order 'a'=0.1, 0.6, 0.9

Table 1. Performance features of tunable passband Kaiser window-based (with variable parameter beta=3.5) FIR lowpass filter

| Fractional order 'a' | Passband-width |
|----------------------|----------------|
| 0.1 | 0.18 |
| 0.6 | 0.17 |
| 0.9 | 0.16 |

The phase response of FIR filter is also analysed in fractional Fourier domain and is shown in Fig. 2. The phase response of filter is directly proportional to the length of filter at a particular frequency. Thus it can be tuned by increasing filter length which increases complexity. The above result show that by varying the FrFT order 'a', we can also tune the phase response for application in sample rate conversion [10]. We have taken another example in Fig. 3 to show output of proposed methodology by convolving the hamming window response with FrFT of desired impulse response.

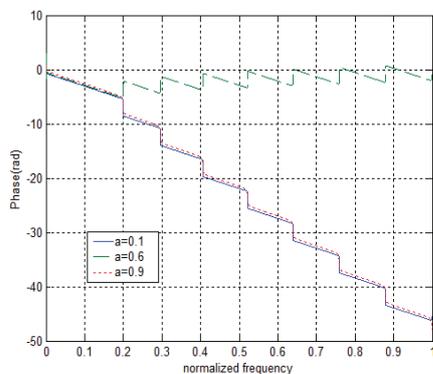


Fig. 2. Variable phase response (unwrap) of kaiser window (with variable parameter 3.5) based low-pass FIR filter for different fractional order

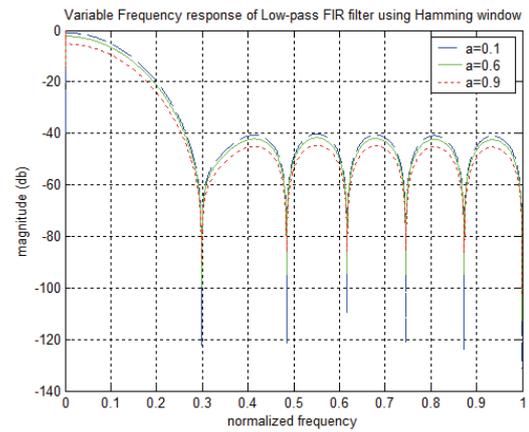


Fig. 3. Variable band-edge low-pass FIR filter frequency response using Hamming window convolved with impulse response for different fractional orders on db scale

Table 2. Performance features of tunable pass-band Hamming window-based FIR lowpass filter

| Fractional order 'a' | Passband-width |
|----------------------|----------------|
| 0.7 | 0.134 |
| 0.75 | 0.13 |
| 0.8 | 0.12 |
| 0.9 | 0.1 |

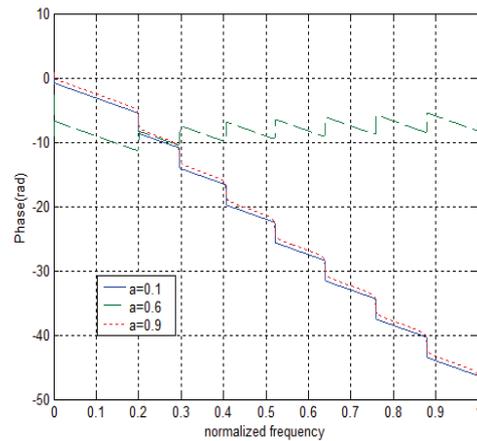


Fig. 4. Variable phase response (unwrap) of hamming window based low-pass FIR filter for different fractional order

The above simulated results and the performance features measured in Table. 1 and Table. 2, clearly show that by varying FrFT order 'a', passband-width can be made varied online without the need to redesign a new filter off-line, however tuning range is small. Also, it is observed from Fig. 2 and Fig. 4, that the phase response can also be adjusted on-line as the FrFT order 'a' is reduced but linearity can not be guaranteed.

Conclusions

It is concluded in the work that using fractional order 'a' as tuning parameter, variable band-edge characteristics can be achieved by convolving the DFrFT of impulse response of low-pass FIR filter with the magnitude response of desired window function without the need to redesign a new filter. In addition phase response can also

be varied using FrFT. Thus implementation complexity and computational cost is reduced as band-width can be increased on-line by simply reducing the FrFT order.

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A novel technique is introduced through this paper to tune the frequency characteristics of a low-pass finite impulse response (FIR) filter. The work is based on fractional Fourier transform (FrFT) which can be used in modifying the magnitude and phase characteristics of a filter with adjustable FrFT angle as tuning parameter. The analysis of desired impulse response of filter in fractional Fourier domain establishes the dependence of its frequency response on the changeable fractional order parameter. The essence of using FrFT in variable digital filter design is that we need not to re-compute impulse response coefficients and redesign a new filter in order to bring changes in the filter characteristics. Thus it is projected in the work that fractional Fourier transform can be used in tuning window-based FIR digital filters. Ill. 4, bibl. 2, tabl. 12 (in English; abstracts in English and Lithuanian).

P. Mohindru, R. Khanna, S. S. Bhatia. Originalus metodas kintamam nerekursiniam skaitmeniniam filtrui projektuoti FrFT pagrindu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 5(121). – P. 89–92.

Siūlomas originalus metodas žemų dažnių baigtinio impulsinio atsako filtro dažninėms charakteristikoms derinti. Tyrimas pagrįstas frakcine Furjė transformacija, kuri gali būti naudojama filtro charakteristikoms keisti kaip keitimo parametru naudojant FrFT kampą. Gauta filtro dažnio priklausomybė nuo keičiamo frakcinės eilės parametro. FrFT naudojimo filtrams projektuoti esmė yra ta, kad, norint pakeisti filtro charakteristikas, nereikia perskaičiuoti impulsinio atsako koeficientų ir perprojektuoti filtro. Il. 4, bibl. 2, lent. 12 (anglų kalba; santraukos anglų ir lietuvių k.).