

## Vector Approach of Collision Prognosis of Mobile Robots

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### Introduction

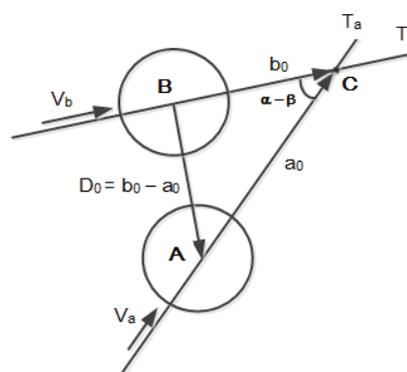
If several mobile robots are operating in the same environment, it is necessary to create a control system in order to avoid not only obstacles but also inter-collisions. Usually, paths of robots are designed, to ensure movement without collisions with obstacles and each other [1,2,3], but it also restricts the "freedom" of movement of robots, is more appropriate to resolve conflicts dynamically. It is necessary to predict the potential conflicts between robots for this purpose. Assume that the robot can "see" the other robots, and to determine direction and speed of their movement, or is controlled by supervision system which "see" all the robots, and can "communicate" with them. Capability to predict collisions is necessary in both cases. The method to predict possible collisions is described in this paper, i.e. the method and program of a robot or supervisor actions to achieve this result is proposed, but there is no deal with ways to avoid them.

### Assumptions

The trajectory of a mobile robot may be quite complicated curve, with respect of dynamic features of it, aims of operation and so on. Therefore, it is necessary to approximate traces before analyzing possibility of collision. It seems quite acceptable to use polygon line, consisting of straight sections, the more so that the design of the future trajectory is often found only in the characteristic, such as turning, points [4]. This paper assumes that robots move along straight lines, which are extrapolated sections of the fragments, Fig. 1.

The size of a robot, especially in transportation, is not steady, but in the initial analysis seems useful to assume that each of the robots, "fit" circle with a radius  $R$ , while the robot trajectories are straight lines intersecting at point  $C$  (Fig. 1). So, they will examine whether these circles not intersects.

Robot dimensions involving circles may be different radius, but, as we shall see below, the expressions are obtained using the minimum distance between centers of the circles, corresponding robots, i.e. the amount of distance of two circles  $2R$ , and it can be concluded from the two different circles.



**Fig. 1.** Robots A and B, moving speeds  $v_a$  and  $v_b$ , their tracks intersect at point C

### The collision prediction

Moving one of the robots in their first, for example,  $A$  (Fig.1), will come to a point of intersection of the trajectories, then it is clear that the other one ( $B$ ) must be removed from the center of the latter more than in the  $2R$ .

Assume that the robots move by  $v_a$  and  $v_b$ , speeds, respectively, and their distances from the point of intersection of the trajectories are vectors  $a$  and  $b$ . Denote the initial distances to the point  $C$  are  $a_0$  and  $b_0$ , respectively. Then, applied a cosine theorem, we obtain that the skin contact of robots describes the equation

$$(a_0 - v_a t)^2 + (b_0 - v_b t)^2 + 2(a_0 - v_a t) \cdot (b_0 - v_b t) \cdot \cos(\alpha - \beta) = 4R^2. \quad (1)$$

If speeds are different, there may be cases that skin contacts between two robots are when:

1. Still did not come, or both have passed the point C;
2. The point C point is passed, only by one robot.

These two cases differs by sign of  $\cos(\alpha - \beta)$ , but sign of the product of the distances changes too, so that (1) terms for all positions of the robots.

In general, the distance between the centers of robots, analogous to the (1) expression can be described as follows

$$d^2(t) = (a_0 - v_a t)^2 + (b_0 - v_b t)^2 - 2(a_0 - v_a t) \cdot (b_0 - v_b t) \cdot \cos(\alpha - \beta). \quad (2)$$

Let us find the time change from start moment, when the distance between the robots will be minimal. At extremum point derivative

$$\frac{\partial d^2(t)}{\partial t} = 0$$

$$-a_0 v_a - b_0 v_b + (v_a^2 + v_b^2) \cdot t_{extr} + \cos(\alpha - \beta) \cdot (-v_a b_0 - v_b a_0 + 2v_a v_b t_{extr}) = 0 \quad (3)$$

or

$$t_{extr} = \frac{a_0 v_a + b_0 v_b - \cos(\alpha - \beta) \cdot (v_a b_0 + v_b a_0)}{(v_a^2 + v_b^2) - 2 \cdot \cos(\alpha - \beta) \cdot v_a \cdot v_b}. \quad (4)$$

Robots do not come into collision if

$$d(\Delta t_{extr}) > 2R. \quad (5)$$

When the robot speeds are equal ( $v_a = v_b = v$ ), extremum time is

$$t_{extr} = \frac{a_0 + b_0}{2v}. \quad (6)$$

Then

$$d^2(t_{extr}) = \frac{(b_0 - a_0)^2}{2} [1 + \cos(\alpha - \beta)], \quad (7)$$

i. e. robots pass without conflict, if following condition is fulfilled:

Obtained with (8) condition is sufficient, when speeds of both robots are equal, i. e.  $v_a = v_b$ , otherwise we have to use (4) (2) and (5) expressions. Favorable is the vector interpretation of passing condition

$$|b_0 - a_0| > \frac{2\sqrt{2} \cdot R}{\sqrt{1 + \cos(\alpha - \beta)}} = \frac{2R}{\cos\left(\frac{\alpha - \beta}{2}\right)}. \quad (8)$$

Let the centers of robots, at collision time is forecast at points A and B (Fig. 2). Let us treat the distances from robots to crossing point C as vectors, along the trajectories of robots  $T_a$  and  $T_b$ , preserving the same directions as appropriate velocity vectors (Fig. 2). Than passing without collision condition can be written as follows

$$\left| (\vec{b}_0 - \vec{v}_b t) - (\vec{a}_0 - \vec{v}_a t) \right| \geq 2R \quad (9)$$

or

$$\left| (\vec{b}_0 - \vec{a}_0) - (\vec{v}_b - \vec{v}_a) \cdot t \right| \geq 2R. \quad (10)$$

Then, from Fig. 2 can be seen that (10) inequality is guaranteed to be satisfied, if  $|\vec{b}_0 - \vec{a}_0| > 2R$ , and the vector  $(\vec{v}_b - \vec{v}_a) \cdot t$  drawn from point L will not cross the circle of radius  $2R$ , i. e., if the vector is present in a limited sector with vectors  $\vec{BM}$  and  $\vec{BN}$ . These vectors are tangent to the circle. In other words, the angle between the vectors  $(\vec{b}_0 - \vec{a}_0)$  and  $-(\vec{v}_b - \vec{v}_a) \cdot \delta$  must be

$$\delta = \left| \arccos \left[ \frac{(\vec{b}_0 - \vec{a}_0) \cdot (\vec{v}_b - \vec{v}_a)}{|\vec{b}_0 - \vec{a}_0| \cdot |\vec{v}_b - \vec{v}_a|} \right] \right| \geq \rho, \quad (12)$$

here

$$\rho = \arcsin \frac{2R}{|\vec{b}_0 - \vec{a}_0|}. \quad (13)$$

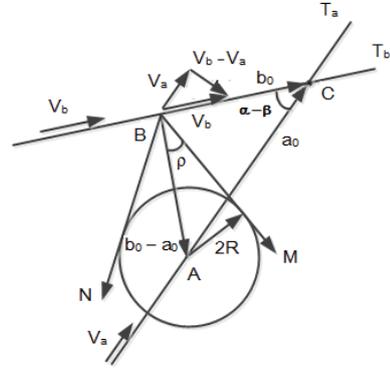


Fig. 2. The graphical interpretation of the passing terms

The (12) expression is valid for any robot speed. When modules of speeds are same (Fig. 3), from the sinus theorem, we get

$$\sin \theta = \frac{|a_0 - b_0|}{|\vec{b}_0 - \vec{a}_0|} \sin \gamma = \frac{|a_0 - b_0|}{|\vec{b}_0 - \vec{a}_0|} \cos \frac{\alpha - \beta}{2}, \quad (14)$$

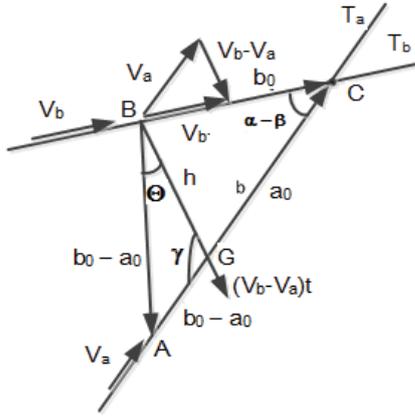
because  $\gamma = \frac{\pi + (\alpha - \beta)}{2}$ .

Robot safely pass then  $\theta > \rho$ , i. e.

$$\arcsin \left( \frac{|a_0 - b_0|}{|\vec{b}_0 - \vec{a}_0|} \cos \frac{\alpha - \beta}{2} \right) > \arcsin \left( \frac{2R}{|\vec{b}_0 - \vec{a}_0|} \right). \quad (15)$$

It is obvious that this condition is identical to (8) expression.

Real robot trajectory is not linear. It is the polygon line made up of straight sections. Therefore, it may happen that at least one of the robots will turn without reaching a potential crash point, if the extrapolation of these straight sections determines the robots crash yet. Then it becomes important to determine the collision (skin contact) place. Again, let the local coordinates of the conflict remains hypothetical robot distances to the point of intersection of the trajectories.



**Fig. 3.** The passing geometry of the robot when speed modules are the same

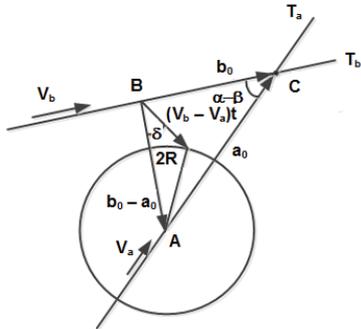
Robots conflict when the angle  $\delta < \rho$ . Then the vector  $(\vec{v}_b - \vec{v}_a) \cdot t$  cross the circle whose radius is  $2R$  (Fig. 4).

The time value corresponding to the intersection can be found using the cosine theorem

$$\begin{aligned} & |\vec{v}_b - \vec{v}_a|^2 t^2 + |\vec{b}_0 - \vec{a}_0|^2 - \\ & - 2|\vec{v}_b - \vec{v}_a| \cdot t \cdot |\vec{b}_0 - \vec{a}_0| \cos \delta = 4R^2. \end{aligned} \quad (16)$$

The smaller root of the equation is  $t_c$ . Denote

$$t_D = \frac{|\vec{b}_0 - \vec{a}_0|}{|\vec{v}_b - \vec{v}_a|}. \quad (17)$$



**Fig. 4.** Detection of the skin contact time

Then (16) equation we can rewrite as follows

$$\frac{t^2}{t_D^2} - 2 \frac{t}{t_D} \cos \delta + 1 - \frac{4R^2}{|\vec{b}_0 - \vec{a}_0|^2} = 0. \quad (18)$$

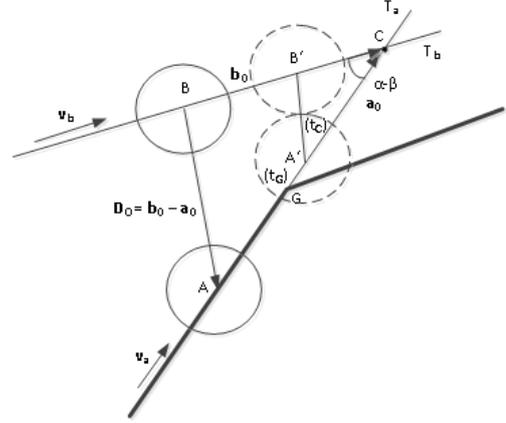
The collision time is the root of the equation (18)

$$t_C = t_D \left[ \cos \delta - \sqrt{\cos^2 \delta - 1 + \frac{4R^2}{|\vec{b}_0 - \vec{a}_0|^2}} \right]. \quad (19)$$

It remains to compare the time, when the robots will come to the positions  $A'$  and  $B'$  (Fig. 5), with the time when the robot comes to a turning point in  $G$ ,  $t_G$  and, if

$$t_c > t_G = \frac{AG}{v_a}, \quad (20)$$

conflict is absent.



**Fig. 5.** Turning point is reached before conflict

In order to perform the control of potential collisions is useful to streamline the analysis as screening for potential collisions in pairs, the number of robots is a  $n$  need to make  $0,5 \cdot n \cdot (n-1)$  checks.

From the (19) expression shows that

$$t_C < t_D, \text{ because } |\vec{b}_0 - \vec{a}_0| > 2R. \quad (21)$$

Moreover, the upper time limit of the range of conflict is:

$$t_H = t_D \cos \delta \geq t_C. \quad (22)$$

The lower limit  $-t_C$ , as can be seen from Fig.4, is:

$$t_L = \frac{|\vec{b}_0 - \vec{a}_0| - 2R}{|\vec{v}_b - \vec{v}_a|} = t_D (1 - \sin \delta). \quad (23)$$

If the time  $t_G$  which spend up to the next turn point in robot trajectory is  $t_G < t_L$ , conflict will not arise before the turn point and the conflict time  $t_C$  is not need to calculate.

Then it is sufficient to look up for new potential collisions only in start position, and when any robot reaches a Turn point on its. The pseudo-code of the algorithm to forecast collisions, when potential collisions are recorded in "bump" array, could be:

```

loop
  if start or robot in turning point then
    i := 1; / robot number
    repeat
      for k := i+1 to n do /n - number of robots
        if the k-th and i-th robot can bump,
          (15-th expression) then
            if any of the robots
              (k-th and the i-th) have turn
              point before the trajectories
              the intersection point of C then
  
```

```

if not reach a turn point
before the collision, (24 –th
expresion)then
    Observed collision in
array "bump";
else
    Observed collision in array "bump";
    i := i+1;
until i = n;
end loop

```

## Conclusions

Two robots, moving towards to the point of intersection of the straight paths, will collide, if the angle between the vector of difference of velocities and the vector difference between the distances to the intersection point, is less than the critical angle, i.e. half of the angle in which the circle of diameter equal to the sum of diameters of circles surrounding robots, placed in the position of another robot, is visible.

It is sufficient to check possibility of collision of robots only in start position of robots and when one of them reaches a turning point of the trajectory

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**S. Bartkevičius, Z. Jakas, K. Sarkauskas. Vector Approach of Collision Prognosis of Mobile Robots // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 5(121). – P. 75–78.**

A method to detect potential collision of moving robots, operating in the same environment, is discussed. Criteria of collision is formulated – two robots will collide, if the angle between vector of difference of speeds and difference of vectorial distances of robots to crossing point of linear traces is less than half of the visibility angle, from position of one of robots, of the circle with diameter equal to sum of diameters of circles involving robots, placed in position of other robot. It is determined, that usage of this criteria allows to check possible collisions only in start position of robots or when one of them changes direction of movement. Ill. 4, bibl. 4 (in English; abstracts in English and Lithuanian).

**S. Bartkevičius, Ž. Jakas, K. Šarkauskas. Vektorinė mobiliųjų robotų potencialių susidūrimų prognozė // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 5(121). – P. 75–78.**

Nagrinėjamas kelių vienoje aplinkoje judančių robotų potencialių konfliktų nustatymo būdas. Suformuluotas kriterijus nustatyti, kada du robotai susidurs, atsižvelgiant į jų greičių skirtumo ir vektorinių atstumų iki trajektorijų susikirtimo skirtumo tarpusavio kampą. Konflikto nebus, jeigu šis kampas bus didesnis už pusę kampo, kuriuo iš vieno roboto pozicijos „matomas“ antrojo roboto pozicijoje esantis skritulys, kurio skersmuo yra robotus apgaubiančių skritulių skersmenų suma. Nustatyta, kad taikant šį kriterijų susidūrimo galimybę reikia tikrinti tik robotams esant pradinėje pozicijoje arba kai kuris nors iš robotų keičia judėjimo kryptį. Il. 4, bibl. 4 (anglų kalba; santraukos anglų ir lietuvių k.).