Modeling of Nonlinear Circuit using Volterra Series

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Introduction

Distortion is a key issue in the design of many types of circuits. In modern circuits linearity is a very important parameter, especially with the strict modern telecommunication system standards. The method of Volterra series is the most widely method to analyze the nonlinear circuits [1, 2]. In order to get a low signal distortion, a great focus is made on investigation of nonlinearities. The Volterra series method of distortion analysis is presented in the analysis of a common emitter circuit.

For analysis of nonlinear systems differential equations representing nonlinear, frequency and parametrical performances of the system are applied. Unlike numerical simulations which give no information about the source of the distortion, closed form expressions for distortion components in terms of circuit parameters can be found using Volterra series.

In this paper we modeling nonlinearities of a common emitter circuit, analyze their influence on signal distortions, obtain general solutions of Volterra kernels appropriate for engineering calculations. To determine nonlinear behaviour is used the Volterra series, the most widely used method to analyze the nonlinear behavior of analog circuits.

Model and analysis methodology

The Volterra series for a circuit is generally represented as a summation of operators [3]

\[ y(t) = \sum \int \cdots \int h_n(t_1, \ldots, t_n) x(t_1) \cdots x(t_n) dt_1 \cdots dt_n \]

or a view is as follows

\[ Y_n(p) = H_n(p)x(p), \]

where \( p \) – complex variable.

An output signal of the square subsystem is:

\[ y(t) = \sum \int h(t) x(t) dt \]

The Laplace transform of the Volterra kernel \( h_n(t_1, \ldots, t_n) \) is represented by \( H_n(p_1, \ldots, p_n) \), \( x(t) \) the input, \( y(t) \) the output.

Using the method of Volterra series of distortion analysis a nonlinear system is composed of linear, square, cube, etc. subsystems connected in parallel (Fig. 1).

Fig. 1. Model nonlinear system (H_i – appropriate kernel Volterra series)

The first subsystem is linear. The output signal of this system is the following:

\[ y_1 = \int h_1(t - \tau_1) x(\tau_1) d\tau_1 \]

or a view is as follows

\[ Y_1(p) = H_1(p)x(p), \]

where \( p \) – complex variable.

An output signal of the square subsystem is:
\[ y_2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(t - \tau_1, t - \tau_2) x(t_1, t_2) x(t_3) dt_1 dt_2 dt_3 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(t_1, t_2) x(t_1, t_2) dt_1 dt_2, \]
\[ Y_2(p_1, p_2) = H_2(p_1, p_2) \sum_{i=1}^{\infty} X(p_i). \]

An output signal of the cube subsystem is:
\[ y_3(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_3(t - \tau_1, t - \tau_2, t - \tau_3) x(t_1, t_2, t_3) dt_1 dt_2 dt_3 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_3(t_1, t_2, t_3) x(t_1, t_2, t_3) dt_1 dt_2 dt_3, \]
\[ Y_3(p_1, p_2, p_3) = H_3(p_1, p_2, p_3) \sum_{i=1}^{\infty} X(p_i). \]

Only three harmonics will be evaluated as already third harmonic in amplifier is very low.

If an amplifier receives the following signal
\[ x(t) = U \cos \alpha t = \frac{1}{2} U \left( e^{i \alpha t} + e^{-i \alpha t} \right), \]
change constant component of an output signal is defined by analyzing a square system [4]
\[ i_2(t) = \frac{1}{2} U^2 \text{Re} \left( H_2(j w, -j w) \right) + \frac{1}{2} U^2 \left( H_2(j w, j w) \cos(2w t + \phi_2) \right), \]
where \( \phi_2 \) – phase of the second harmonic.

Change in the first harmonic is figured out by analyzing a cube system
\[ i_3(t) = \frac{3}{4} U^3 H_3(j w, j w, -j w) \cos(2w t + \phi_3 w) + \frac{1}{4} U^3 H_3(j w, j w, j w) \cos(3w t + \phi_3 w), \]
where \( \phi_3 \) – phase of the third harmonic.

**Analytic solutions**

The first step before calculating a Volterra series is to expand the circuit nonlinearities in Taylor series.

The main nonlinearities that determine the signal distortion of amplifier are nonlinear junction currents of an emitter and collector and their nonlinear capacitances [4].

Fig. 2 presents an equivalent circuit for signal distortion analysis of amplifier.

Most analytical amplifier models [5] express the emitter current following
\[ i_e = I_{e_0} \exp(\gamma_i u_e) - 1, \]
where \( u_e \) – junction voltage, \( \gamma_i \) – temperature potential, \( \alpha \) – collector current transmission coefficient, \( i_k \) – current of collector, \( I_{e_1} \) – heat current.

**Fig. 2.** Equivalent circuit for distortion analysis of amplifier

After expanding (12) in a Taylor series, the following is obtained
\[ i_e = I_{e_0} + \lambda_1 u_e + \lambda_2 u_e^2 + \lambda_3 u_e^3, \]
where \( I_{e_0} \) – current constant component;
\[ \lambda_1 = G_e; \lambda_2 = \frac{1}{2} \frac{G_e^2}{I_{e_0}}; \lambda_3 = \frac{1}{6} \frac{G_e^3}{I_{e_0}^2}. \]

Nonlinear generator current flowing through the junction capacitance of an emitter is as follows:
\[ i_{e_0} = \beta_1 du_e / dt + \beta_2 du_e^2 / dt + \beta_3 du_e^3 / dt, \]
\[ \left\{ \begin{array}{l} \beta_1 = \frac{\tau_{at} G_e}{\sqrt{2}} \frac{v(\varphi_T - U)}{\sqrt{2}}, \\
\beta_2 = \frac{\tau_{at} G_e^2}{2 \sqrt{2}} \frac{1}{4} \frac{v(\varphi_T - U)}{\sqrt{2}}, \\
\beta_3 = \frac{\tau_{at} G_e^3}{6 \sqrt{2}} \frac{1}{8} \frac{v(\varphi_T - U)}{\sqrt{2}}, \end{array} \right. \]
where \( \tau_{at}, v_e \) – physical parameters of integrated transistor.

Nonlinear generator current flowing through the collector capacitance
\[ i_{C_K} = \gamma_1 du_k / dt + \gamma_2 du_k^2 / dt + \gamma_3 du_k^3 / dt, \]
where \( U_{K} \) – junction voltage of a collector;
\[ \gamma_1 = V_{K} U_{K}^{-3/2}; \gamma_2 = \frac{1}{2} \frac{4}{3} V_{K} U_{K}^{-3/2}; \gamma_3 = \frac{2}{5} \frac{7}{27} V_{K} U_{K}^{-3/2}, \]
where \( V_{K} \) – constant.

Variable component of an output current is
\[ i_K = (L_4 - p D_1) u_e + (L_2 - p D_2) u_e^2 + (L_3 - p D_3) u_e^3, \]
where \( L_4, L_2, L_3 \) and \( D_1, D_2, D_3 \) – coefficients dependent on circuit parameters and frequency.

After evaluation of amplifier nonlinearities for equivalent circuit formed equations system:
where $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ – appropriate note potentials, $Z_{IN}$ – input impedance, $Z_L$ – load impedance, $Z_e$ – emitter impedance, $C_{KB}$ – feedback capacitance, $C_K$ – output capacitance, $r_b$ – base impedance, $Z_K$ – output impedance.

Addressing the equations system by method of Volterra series first found Volterra kernels of linear subsystem, then of square subsystem, after evaluation of linear kernels, afterwards of cube subsystem, after evaluation kernels of linear, squar subsystems.

Kernels of a linear system will be referred to as $H_{11}(p)$, $H_{12}(p)$, $H_{13}(p)$ and $H_{14}(p)$. They correspond to the units of an equivalent circuit diagram and are figured out from a matrix equation

$$
\begin{pmatrix}
H_{11}(p) \\
H_{12}(p) \\
H_{13}(p) \\
H_{14}(p)
\end{pmatrix} = W^{-1}(p)
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} \times
\begin{pmatrix}
1/Z_{IN} \\
0 \\
0 \\
0
\end{pmatrix},
$$

(21)

where $W(p)$ – conductance matrix.

$$
W(p) =
\begin{pmatrix}
\frac{1}{Z_{IN}} & -\frac{1}{r_b} & 0 & -\frac{1}{r_b} \\
1 - \frac{1}{r_b} & \gamma_1 p & \frac{1}{r_b} & -\frac{1}{r_b} \\
\frac{1}{r_b} & \gamma_1 p & \frac{1}{r_b} & -\frac{1}{r_b} \\
0 & \frac{1}{r_b} & \frac{1}{r_b} & -\frac{1}{r_b}
\end{pmatrix}.
$$

(22)

Two-dimensional kernels of IT diagram units will be marked respectively $H_{21}(p_1, p_2)$, $H_{22}(p_1, p_2)$, $H_{23}(p_1, p_2)$ and $H_{24}(p_1, p_2)$, and obtained from the (23).

After figuring out two-dimensional kernel $H_{23}(p_1, p_2)$ from (23), dependences of a constant component of an output current in a frequency range shall be established taking into account both the parameters of an equivalent diagram and outer elements.

Fig. 3 presents dependences change a constant component of a output current after figuring out a two-dimensional kernel $H_{23}(j\omega, -j\omega)$, according to (10), is the following

$$
\Delta I_{K_0} = \frac{U^2 H_{23}(j\omega, -j\omega)}{2|L_e + r_b|}.
$$

(24)

Thus, application of the method of Volterra series allows defining dependences of a output current on the input signal parameters and evaluating the influence of parameters of a amplifier itself and outer elements on the distortion of the signals transferred.

Similarly, change first harmonic output current depending on nonlinearities in a amplifier is defined. The first harmonic is observed at the output of linear and cube systems. In order to establish the first harmonic at the output of a cube system, three-dimensional kernels shall be calculated. A matrix equation that is used to figure out the aforementioned kernels is the following
components. The output signal constant component and first harmonic variations in dependence on the parameters of the input signal were presented. Ill. 4, bibl. 5 (in English; abstracts in English and Lithuanian).

After calculating three-dimensional kernel $H_{33}(p_1, p_2, p_3)$ from (11), change output current first harmonic may be calculated

$$\Delta I_{K1} = \frac{3}{4} (H_{33}(j\omega_1, j\omega_2, j\omega_3) \|Z_K + Z_L\|).$$

Fig. 4 presents dependences of change the first harmonic output current $\gamma_{K1} = \Delta I_{K1}/I_{K1}$ on magnitude and frequency on an input signal $m = U/\varphi_T$.

Conclusions

For modeling nonlinearities of nonlinear circuit the method of Volterra series has been offered. Unlike numerical simulations, closed form expressions for distortion components in terms of common emitter circuit parameters has been found. The mathematical model established here let to calculate Volterra kernels of the respective linear, square and cubic systems and presents dependences of change output current component on magnitude of an input signal within a frequency range.

References


Volterra series method was used to analyze signal distortions. During application of this method there is no need to form differential equations, which characterize parametric and frequency properties of non-linear circuits. Volterra series method was applied to analyze the nonlinearities of common emitter circuit. Modeling of nonlinearities was performed. Fundamental components of signal distortions were evaluated. Equation system in operator form was written for non-linear circuit, after solving which, the analytical expressions of Volterra series kernels were obtained. Laplace transform of kernels was used to determine the magnitudes of signal distortion components. The output signal constant component and first harmonic variations in dependence on the parameters of the input signal over a frequency range were presented. Ill. 4, bibl. 5 (in English; abstracts in English and Lithuanian).


Netiesinei grandinei modeliuoti taikomas Volterra eilučių metodas, kuris, palyginti su kita timedais, analizuojančiais netiesines sistemas su kintamais parametrais, sutiekia daugiau informacijos apie iškraipymų šaltinius; nereikia spręsti sudėtingų diferencialinių lygčių. Bendro emiterio schemos iškraipymų analizei priimatinumas Volterra eilučių metodas. Įvertinus pagrindinius netiesiškumus, ekivalentinio stiprintuvo schemai sudaryta operacinės formos lygčių sistema. Gautos analitinės išrašos Volterra eilučių branduoliams skaičiuoti. Pateikta išėjimo signalo pastovios dedamosios ir pirmissos harmonikos pokyčių priklausomybės nuo jėjimo signalo dydžio dažnių diapazono. Ill. 4, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).