ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2012. No. 4(120)
ELEKTRONIKA IR ELEKTROTECHNIKA

ELECTRICAL ENGINEERING
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ELEKTROS INŽINERIJA

Amplify-and-Forward Relay Transmission System over Mixed Rayleigh and Hoyt Fading Channels

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crossref http://dx.doi.org/10.5755/j01.eee.120.4.1445

Introduction

Wireless systems in the future will be envisaged to offer ubiquotious high data-rate coverage in large areas. A promising solution for providing broader and more efficient coverage in both modern (ad-hoc, WLAN) and traditional (bent pipe satellites) communications networks, are multihop relaying technology communications. Multihop transmission is a communication technique that arises in many applications: to attain broader coverage without the need to use large power at the transmitter; to communicate via ad hoc networks where nodes are communicating without the aid of central control/infrastructure; to combat the impairment of the wireless channel through spatial/multiuser diversity. The main idea of multy and dualhop transmission is that communication is achieved by relaying the information from the source to the destination thought a number of intermediate terminals, when the direct link between the source and destination is in deep fade, so the signals to the destination propagate through two or more hops/links in series [1]. Multipath fading can seriously degrade system performances of wireless communications. Few statistical models are used to describe fading in wireless environments in communications systems analysis. The most frequently used distributions are Nakagami-*m*, Rice, Hoyt, Rayleigh, α-μ and Weibull. Several studies have shown that the Hoyt fading model provides a very accurate fit to experimental channel measurements in a various communication applications, like mobile satellite propagation channels and spans the range of the fading figure from the one-sided Gaussian to the Rayleigh distribution. Similarly, the Hoyt distribution can be considered as an accurate fading model for satellite links with strong ionospheric scintillation

Dual-hop transmission systems can be classified into two main categories, depending on the nature and complexity of the relays: 1) regenerative systems 2) nonregenerative systems. In the case of regenerative systems (Decode-and-Forward), the relay fully decodes the signal that went through the first hop and retransmits the decoded version into the second hop. Nonregenerative systems use less complex relays that just amplify and forward the incoming signal without performing any sort of decoding (Amplify-and-Forward).

Relays in nonregenerative systems can in their turn be classified into two subcategories, namely, 1) “blind” relays and 2) channel state information (CSI)-assisted relays. Systems with “blind” relays employ at the relays amplifiers with fixed gains and consequently result in a signal with variable power at the relay output. On the other hand, nonregenerative systems with CSI-assisted relays use instantaneous CSI of the first hop to control the gain introduced by the relay and as a result fix the power of the retransmitted signal.

Several papers have studied the performance of amplify-and-forward (AF) relay systems using performance measures such as the outage probability and the average bit error probability (ABEP) [2]–[3]. In [2] lower bounds for the performance of a two hop channel state information (CSI) assisted AF relay system over non identically distributed generalized gamma fading channels have been presented. [3] studies the ABEP of two hop systems with AF relaying over Rayleigh fading channels. AF relay system operating in an Rayleigh-Ricean fading environment was studied in [4].

However, due to the various objects in the environment a base station link cannot always have strong LOS (line-of-sight) component, so more common case is when link experiences different type of fading. Moreover to our best knowledge, there is no case reported in literature, when this link is considered over the influence of Hoyt fading.

The main contributions of this paper are outage probability closed form expressions for the exact and lower bounds which become tight for high signal-to-noise ratios (SNRs). Numerical results are graphically presented in order to discuss the effects of fading parameters on overall system performance. Also another important performance measure, an average bit error probability over different modulation schemes is analyzed.

System model

In the dual hop transmission technique communication from the source S to the destination D via a relay R takes place in two time slots. In the first time slot, S sends its signal to R, while in the second time slot, R first amplifies the received signal by a gain factor G and then forwards the resultant signal to D. It is assumed that there is no direct path between S and D. The instantaneous end-to-end SNR, γ_{eq}, at the destination can be defined as

$$\gamma_{eq}=\frac{(P_1/N_0)|h_{SR}|^2(P_2/N_0)|h_{RD}|^2}{(P_2/N_0)|h_{RD}|^2+\left(1/G^2N_0\right)}, \qquad (1)$$

here $|h_{SR}|$ and $|h_{RD}|$ are denoting the wireless channels fading amplitudes for the $S-R$ and $R-D$ links, P_1 and P_2 are denoting the transmitted powers at S and R respectively, and N_0 denotes the power of the additive white Gaussian noise (AWGN) component. If G is selected according to the instantaneous CSI assisted relay gain then γ_{eq} can be re-expressed as [2]

$$\gamma_{eq}=\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+c} \qquad (3)$$

with $\gamma_1=|h_{SR}|^2P_1/N_0$ and $\gamma_2=|h_{RD}|^2P_2/N_0$ being the SNRs per hop. Exact γ_{eq} is given by substituting $c=1$. Also in some cases, γ_{eq} can be approximated at some level by substituting $c=0$.

An asymmetric scenario for the fading distributions of the $S-R$ and $R-D$ links will be studied, namely: the $S-R$ link is subject to Rayleigh fading and the $R-D$ link is subject to Hoyt fading.

$S-R$ link experiences Rayleigh fading, so γ_1 is distributed with probability density function (PDF) given by [5]

$$p_{\gamma_1}(\gamma_1)=\frac{1}{\bar{\gamma}_1}\exp\left(\frac{\gamma_1}{\bar{\gamma}_1}\right), \qquad (4)$$

where the average SNR per hop of the $S-R$ channel is denoted by $\bar{\gamma}_1=E\{|h_{SR}|^2\}P_1/N_0$ and $E\{\cdot\}$ is the statistical expectation.

$R-D$ link experiences Hoyt fading, where γ_2 is being distributed according to [5]

$$p_{\gamma_2}(\gamma_2)=\frac{1+q^2}{2q\bar{\gamma}_2}\exp\left(-\frac{(1-q^2)^2\gamma_2}{4q^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q^4)\gamma_2}{4q^2\bar{\gamma}_2}\right), \qquad (5)$$

the average per hop SNR of the $S-R$ channel is denoted with with $\bar{\gamma}_2=E\{|h_{RD}|^2\}P_2/N_0$; $I_0(x)$ denotes the zero-th order modified first kind Bessel function and $0\le q\le 1$ representing the Hoyt fading severity parameter.

Outage probability

Standard performance criterion for communication systems operating over fading channels used to help the designers of wireless communications system's to meet the QoS and grade of service (GoS) demands is outage probability, P_{out}. Mathematically, P_{out} is defined as the probability that the instantaneous end-to-end SNR falls below a threshold γ_{th}, also known as a protection ratio. Therefore P_{out} can be defined as

$$P_{out}=F_{\gamma_{eq}}(\gamma_{th})=P_r\left[\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+c}<\gamma_{th}\right], \qquad (6)$$

where $F\gamma_{eq}(\gamma)$ denotes the cumulative distribution function (CDF) of the end-to-end SNR. After applying some algebraic manipulations, like in [4], $F\gamma_{eq}(\gamma_{th})$ can be re-expressed as

$$F_{\gamma_{eq}}(\gamma_{th})=1-\int_0^{\infty}C_{\gamma 1}\left[\gamma_{th}+\frac{\gamma_{th}^2+c\gamma_{th}}{w}\right]p_{\gamma 2}[\gamma_{th}+w]dw, \qquad (7)$$

with $C\gamma_i(\cdot)=1-F\gamma_i(\cdot)$ being the complementary CDF of γ_i After substituting (5) into (7), $F\gamma_{eq}(\gamma_{th})$ can be written as

$$F_{\gamma_{eq}}(\gamma_{th})=1-\int_0^{\infty}\exp\left(-\frac{\gamma_{th}+\frac{\gamma_{th}^2+c\gamma_{th}}{w}}{\bar{\gamma}_1}\right)\frac{1+q^2}{2q\bar{\gamma}_2}\times$$
$$\times\exp\left(-\frac{(1-q^2)^2(\gamma_{th}+w)}{4q^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q^4)(\gamma_{th}+w)}{4q^2\bar{\gamma}_2}\right)dw\ . \qquad (8)$$

Now, by applying [6, (3.471.9)]

$$\int_0^\infty x^{v-1}\exp\left(-\frac{\beta}{x}-\gamma x\right)dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{v}{2}} K_v\left(2\sqrt{\beta\gamma}\right), \quad (9)$$

with K_v (·) representing the v-order modified Bessel function of the second kind, finaly we can express outage probability in the form of :

$$F_{\gamma_{eq}}(\gamma_{th}) = 1-\left[\frac{1+q^2}{2q\bar{\gamma}_2}\exp\left(-\frac{(1+q^2)^2\gamma_{th}}{4q^2\bar{\gamma}_2}\right)\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right)\times\right.$$

$$\times\sum_{k=0}^{\infty}\sum_{n=0}^{2k}\binom{2k}{n}\frac{\gamma_{th}^{2k-n}}{(k!)^2}\left(\frac{1-q^4}{8q^2\bar{\gamma}_2}\right)^{2k} 2\left(\frac{\frac{\gamma_{th}^2+c\gamma_{th}}{\bar{\gamma}_1}}{\frac{(1+q^2)^2}{4q^2\bar{\gamma}_2}}\right)^{\frac{n+1}{2}}\times$$

$$\left.\times K_{n+1}\left(2\sqrt{\frac{\gamma_{th}^2+c\gamma_{th}}{\bar{\gamma}_1}\,\frac{(1+q^2)^2}{4q^2\bar{\gamma}_2}}\right)\right]. \quad (10)$$

Several recent papers have approximated γ_{eq} using an upper bound γ_b given by [4]

$$\gamma_b = \min(\gamma_1,\ \gamma_2). \quad (11)$$

Also, a closed-form lower bound which become tight for high signal-to-noise ratios (SNRs). to (11) can be given by

$$P_{out} = P_r[\min(\gamma_1\gamma_2) < \gamma_{th}] = 1 - C_{\gamma 1}(\gamma_{th})C_{\gamma 2}(\gamma_{th}). \quad (12)$$

After applying some mathematical manipulations (12) can be written as

$$P_{out} = 1-\exp\left(\frac{\gamma_{th}}{\bar{\gamma}_1}\right)\left(1-\sum_{k=0}^{\infty}\left(1-\frac{\Gamma\left(2k+1,\frac{(1+q^2)^2}{4q^2\bar{\gamma}_2}\right)}{\Gamma(2k+1)}\right)\frac{(1-q^4)^{2k}q}{(1+q^2)^{4k+1}2^{2k-1}}\right). \quad (13)$$

Average bit error probability

Another useful measure for evaluating the performance of wireless communication applications is the average bit error probability (ABEP). ABEP can be computed by determining the PDF of γ_{eq} and then averaging the conditional BEP in AWGN, $P_b(e|\gamma)$, over this PDF

$$P_b(e) = \int_0^\infty P_b(e|\gamma)\,p_{\gamma_{eq}}(\gamma)\,d\gamma \quad (14)$$

with

$$P_b(e|\gamma) = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{\beta\gamma}}^{\infty}\exp\left(-\frac{t^2}{2}\right)dt. \quad (15)$$

The following error performance derivations are related to well-known modulation schemes employed in communication systems such as BPSK ($\beta = 2$), QPSK ($\beta = 1$) and square/rectangular M-QAM.

Like it was shown in [4], ABEP of (15) can be written in the term of

$$P_e = \frac{1}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq}}\left(\frac{t^2}{\beta}\right)\exp\left(-\frac{t^2}{2}\right)dt \quad (16)$$

for the case of exact form solution from (10) and

$$P_e = \frac{1}{\sqrt{2\pi}}\int_0^\infty P_{out}\left(\frac{t^2}{\beta}\right)\exp\left(-\frac{t^2}{2}\right)dt \quad (17)$$

for the case of lower bound (13).

Table 1. Terms need to be summed in (10) to achieve accuracy at the 6th significant digit

	$\gamma_{\text{th}} =$ 0 dB		$\gamma_{\text{th}} =$ 10dB	
	c=1	c=0	c=1	c=0
q=0.4	14	16	18	20
q=0.5	9	11	11	12
q=0.6	7	8	9	9

Like it was stated earlier, aproximation at some level for $F\gamma_{\text{eq}}(\gamma_{\text{th}})$ can be achieved, by setting c parameter value at $c = 0$. Similar aproximation can be obtained for ABEP by substituting (10) into (16) can be written as:

$$P_e = \frac{1}{2}-\sqrt{\frac{\beta}{2}}\sum_{k=0}^{\infty}\sum_{n=0}^{2k}\binom{2k}{n}\frac{1+q^2}{2q\bar{\gamma}_2}\left(\frac{1-q^4}{8q^2\bar{\gamma}_2}\right)^{2k}\left(\frac{4q^2\bar{\gamma}_2}{(1+q^2)^2}\right)^{2k}\left(4\sqrt{\frac{(1+q^2)^2}{4q^2\bar{\gamma}_1\bar{\gamma}_2}}\right)^{2k}\times$$

$$\times\frac{\Gamma(2k+2n+\frac{7}{2})\Gamma(2k+\frac{3}{2})}{\bar{\gamma}_1^{n+1}(k!)^2\Gamma(2k+n+3)\left(\frac{\beta}{2}+\frac{(1+q^2)^2}{4q\bar{\gamma}_2}+\frac{1}{\bar{\gamma}_1}+2\sqrt{\frac{(1+q^2)^2}{4q^2\bar{\gamma}_1\bar{\gamma}_2}}\right)^{2k+2n+\frac{7}{2}}}\times$$

$$\times{}_2F_1\left(2k+2n+\frac{7}{2}, n+\frac{3}{2}; 2k+n+3; \frac{\frac{\beta}{2}+\frac{(1+q^2)^2}{4q\bar{\gamma}_2}+\frac{1}{\bar{\gamma}_1}-2\sqrt{\frac{(1+q^2)^2}{4q^2\bar{\gamma}_1\bar{\gamma}_2}}}{\frac{\beta}{2}+\frac{(1+q^2)^2}{4q\bar{\gamma}_2}+\frac{1}{\bar{\gamma}_1}+2\sqrt{\frac{(1+q^2)^2}{4q^2\bar{\gamma}_1\bar{\gamma}_2}}}\right). \quad (18)$$

Similary, for the case of lower bound expression, after sybstituting (13), expression (18) can be written as

$$P_e = \frac{1}{2}-\frac{1}{2\sqrt{\frac{2}{\beta\bar{\gamma}_1}}}\left(1-\sum_{k=0}^{\infty}\left(1-\frac{\Gamma\left(2k+1,\frac{(1+q^2)^2}{4q^2\bar{\gamma}_2}\right)}{\Gamma(2k+1)}\right)\frac{(1-q^4)^{2k}q}{(1+q^2)^{4k+1}2^{2k-1}}\right). \quad (19)$$

Numerical results

Outage probability in the case of balanced average SNRs per hop $\bar{\gamma}_1 = \bar{\gamma}_2$, for various values of γ_{th} and Hoyt fading severity parametar q is presented at Fig. 1 and Fig. 2. ABEP over BPSK modulation scheme for the case of $\bar{\gamma}_1 = \bar{\gamma}_2$, and various values of Hoyt fading severity parametar q is presented at Fig. 3

Results are presented graphically in order to discuss the effects of system parameters on overall performance.

From Fig. 2 we can see that outage probability increases for the higher values of defined threshold γ_{th}..

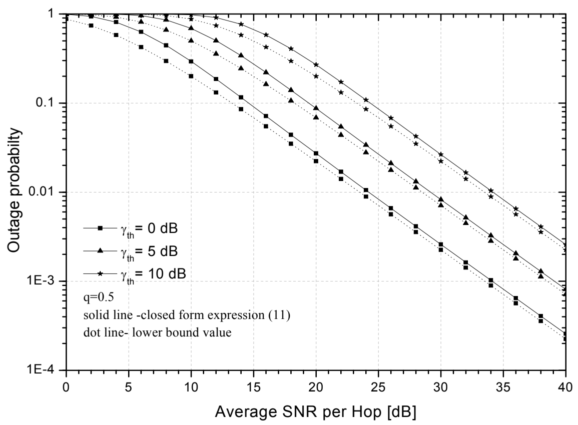

Fig. 1. Outage probability for various values of γ_{th} and the case of $\bar{\gamma}_1 = \bar{\gamma}_2$

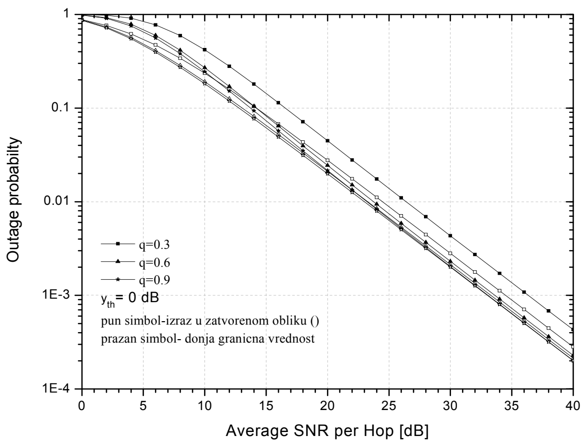

Fig. 2. Outage probability for various values of Hoyt fading parametar q and the case of $\bar{\gamma}_1 = \bar{\gamma}_2$

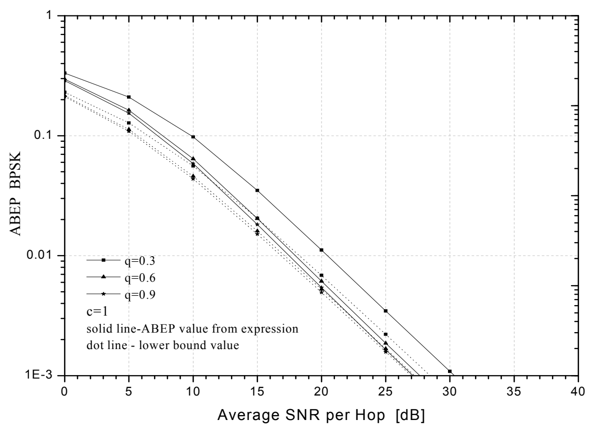

Fig. 3. ABEP for the case of BPSK modulation and various values of system parametars with $\bar{\gamma}_1 = \bar{\gamma}_2$

Also from Fig. 2 we can conclude that higher values of Hoyt fading severity parameter q lead to the decrease in outage probability.

Fig. 3 deals with ABEP. We can derive similar conclusions that increasing the value of Hoyt fading parameter q lead to providing better performances of relay transmitting system, because ABEP then tends to smaller values.

Conclusions

We have presented the numerical analysis of dual hop communication system with CSI assisted AF relay in the Rayleigh-Hoyt fading environment standard performance measures. Outage probability closed form expressions for the exact and lower bounds are derived. ABEP over different modulation schemes were also analyzed. Numerical results are graphically presented with the discussion on the influence of fading parameters on overall system performance.

Acknowledgement

This work has been funded by the Serbian Ministry of Science under the projects TR32023 and III 044006.

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Received 2010 12 04
Accepted after revision 2011 11 20

P. Spalevic, M. Stefanovic, S. Panic, S. Minic, Lj. Spalevic. Amplify-and-Forward Relay Transmission System over Mixed Rayleigh and Hoyt Fading Channels // Electronics and Electrical Engineering. – Kaunas: Technologija, 2012. – No. 4(120). – P. 21–25.

An approach to the performance analysis of dual hop amplify-and-forward relay transmission system, in the asymmetrical fading environment is presented. The main contributions of this paper are closed form expressions for the exact and lower bounds of important performance measure, an outage probability. In order to show the effects of system parameters on overall performance numerically obtained results are graphically presented. Also graphical analysis is provided for another performance measure, an average bit error probability. Ill. 3, bibl. 6, tabl. 1 (in English; abstracts in English and Lithuanian).

P. Spalevic, M. Stefanovic, S. Panic, S. Minic, Lj. Spalevic. Stiprinančiosios ir tiesioginės relinės sistemos signalui perduoti mišriais Reilėjaus ir Hoito slopinimo kanalais // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 4(120). – P. 21–25.

Pateikiamas požiūris į dvigubos stiprinančiosios ir tiesioginės relinės perdavimo sistemos našumo asimetrinėje slopinimo aplinkoje analizę. Pagrindiniai šio tyrimo rezultatai yra našumo vertinimo uždaros formos išraiškos. Siekiant parodyti sistemos parametrų poveikį, gauti rezultatai pateikti grafiškai. Taip pat pateikiama vidutinės bitų klaidos tikimybės grafinė analizė. Il. 3, bibl. 6, lent. 1 (anglų kalba; santraukos anglų ir lietuvių k.).