Analysis of Rejection Properties of Meander Systems

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Introduction

Meander-type structures are applied for retardation of electromagnetic waves in traveling-wave tubes, traveling wave cathode-ray tubes, delay lines and other electronic devices [1–3]. Models of meander-type lines are proposed and their properties are described in [3–7] and other monographs and papers. On the other hand, simplified models of the systems are usually used at analysis. The real meander systems are inhomogeneous structures. Inhomogeneities appear due to dielectric holders, bending of the conductor of the meander electrode, errors of manufacturing and for other reasons.

Periodic inhomogeneities of the wide-band transmission lines cause non-uniformities of their frequency characteristics. According to [8], the pass-band of such lines is limited because the stop-band appears at increase of the phase angle between voltages or currents in the period of inhomogeneities.

The influence of periodic inhomogeneities of meander systems that appear due to periodic variation of the width of strips of meander conductor, bending of the conductor at the sides of the meander electrode, dielectric holders of the electrode and other reasons are investigated in [9–11]. Unfortunately, at modeling of meander systems authors usually assume that elements of these systems are symmetrical with respect to the longitudinal plane, perpendicular to the central part of the system.

Besides inhomogeneities that are symmetrical with respect to the longitudinal plane, asymmetrical inhomogeneities can arise as a result of manufacturing errors and other reasons. As an example, a pattern of asymmetrical meander electrode is shown in Fig. 1. Due to errors at manufacturing (etching) the widths of narrow conductors at the peripheral parts of the electrode are different. According to [11], asymmetrical inhomogeneities can considerably reduce the pass-band of a meander system. On the other hand, the questions related to the influence of asymmetrical inhomogeneities on properties of meander systems are not solved.

In this paper we will consider influence of transverse asymmetry of meander systems on their frequency characteristics and rejection properties.

The model of the meander structure containing asymmetrical inhomogeneities

The model of the meander line containing transverse inhomogeneities is presented in Fig. 2. The model consists of three homogeneous sections of multiconductor lines. Their lengths and characteristic impedances can be different.

Using the quasi-TEM approximation and taking into account normal modes, we have the following expressions [3] for voltages and currents of the conductors in the multiconductor line

\[
U_{m,n}(x) = \left( A_m \sin kx + B_m \cos kx \right) e^{-jn\theta} + \left( C_m \sin kx + D_m \cos kx \right) e^{-jn(\theta+n\pi)},
\]

(1)

Fig. 1. The fragment of the asymmetrical meander electrode

Fig. 2. The model of the meander line: 1 – conductor of the multiconductor line; 2, 3 – shields
\[ L_m(x) = j Y_i(\theta) \left( A_1 \cos kx - B_1 \sin kx \right) e^{-j\theta_0} + j Y_i(\theta + \pi) \left( C_1 \cos kx - D_1 \sin kx \right) e^{-j\theta(\theta + \pi)} \]

where \( A_1, B_1, C_1, D_1 \) are coefficients, \( i \) is the number of the section of the meander line, \( n \) is the number of the conductor of the line, \( k = \omega / c_0 \) is the wave number, \( \omega \) is the angular frequency, \( \theta \) is the phase angle between the voltages on the adjacent conductors of the multiconductor line, \( Y(\theta) \) and \( Y(\theta + \pi) \) are characteristic admittances of the line.

The structure shown in Fig. 2 models the meander system if these boundary conditions are satisfied:

\[ U_{in}(0) = U_{in}^{(n-1)}(0), \quad I_{in}(0) = -I_{in}^{(n-1)}(0), \]
\[ U_{in}(x_i) = U_{in}^{2n}(x_i), \quad I_{in}(x_i) = I_{in}^{2n}(x_i), \]
\[ U_{2n}(x_2) = U_{2n}^{2n}(x_2), \quad I_{2n}(x_2) = I_{2n}^{2n}(x_2), \]
\[ U_{3n}(x_3) = U_{3n}^{2n}(x_3), \quad I_{3n}(x_3) = I_{3n}^{2n}(x_3), \]
\[ U_{4n}(x_4) = U_{4n}^{2n}(x_4), \quad I_{4n}(x_4) = I_{4n}^{2n}(x_4). \]

Substituting (1) and (2) into (3)–(8), we arrive at a set of algebraic equations. Using the principles of the matrix algebra and iterations we can find values of the retardation factor \( k_R \) and frequency \( f \) [7]:

\[ k_R = c_0 / \nu_{ph} = \theta / kL, \]
\[ f = k_c / 2\pi, \]

where \( \nu_{ph} \) is the phase velocity of the traveling-wave, \( c_0 \) is the light velocity and \( L \) is the step of the conductors of the multiconductor line.

The input impedance \( Z_{in} \) of the system is dependent on the coordinate \( x \). At \( x = 0 \), according to (1) and (2)

\[ Z_{in}(0) = \frac{U_{in}(0)}{I_{in}(0)} = \frac{A_2 + A_4}{j \cdot Y(\theta) \cdot A_1 + Y(\theta + \pi) \cdot A_3}. \]

During calculations, values of characteristic admittances \( Y(\theta) \) and \( Y(\theta + \pi) \) can be found using formula [12, 13]

\[ Y_i(\theta) = Y_i(0) + \left[ Y_i(\pi) - Y_i(0) \right] \sin^2(\theta / 2). \]

Values of characteristic admittances \( Y_i(0), Y_i(\pi) \) and corresponding characteristic impedances \( Z_i(0) = 1 / Y_i(0) \), \( Z_i(\pi) = 1 / Y_i(\pi) \) can be determined using numerical (finite difference or finite element) methods [14].

**Results of simulation**

Results of simulation of meander system (retardation factor and input impedance versus frequency) are presented in Fig. 3, Fig. 4 and Fig. 5.

At constant characteristic impedances of the meander conductors, frequency characteristics of the symmetrical meander system are continuous (curves 1 in Fig. 3(a, b)). Relatively great dispersion of retardation and small variation of input impedance are characteristic features of a homogeneous meander system. These results correspond to [7].

In traveling-wave deflection systems meander electrodes containing wide central parts of meander conductors and narrowed peripheral parts are used (in order to have high sensitivity of the cathode-ray tube and high impedance of its signal path). Characteristics of the model of such meander system are presented by curves 2 in Fig. 3(a, b). According to Fig. 3(b), input impedance of the system changes rapidly and, according to analysis, the stop-band appears when phase angle \( \theta \) approaches to \( \pi \) and frequency approaches to

\[ f_{c,n} = \frac{1}{2t_4}, \]

where \( t_4 = x_1 k_0 / L c_0 \) is the delay time corresponding to the step of meander conductors.

Increase of variation of characteristic impedances \( Z(0) \) or \( Z(\pi) \) is followed by increase of the width of the stop-band at \( \theta = \pi \) (curves 3 in Fig. 3(a, b)).

Characteristics presented in Fig. 4(a, b) illustrate influence of asymmetrical inhomogeneities on properties of meander systems.

Curves 2 are obtained at different peripheral parts of meander system, curves 3—at shifted central part of mean-
der electrode (different lengths of peripheral parts). Comparing characteristics 2 and 3 with characteristic of meander system with transverse symmetry (curves 1), we see that asymmetry of the system causes radical changes of its frequency properties. At asymmetry, input impedance rapidly change when phase angle $\theta$ approaches to $\pi/2$. As a result, discontinuity of frequency characteristics and the stop band appear at frequency

$$f_{c,\pi/2} = \frac{1}{4d_0}.$$

Taking into account $[10]$, we can easily explain the effects caused by asymmetry taking into account that at transverse asymmetry the period of inhomogeneities along the meander conductor becomes two times greater with respect to the period of inhomogeneities at absence of transverse asymmetry.

Characteristics of meander systems at transverse asymmetry are similar to that of meander systems with different widths of neighbor meander conductors $[9]$. At different widths of neighbor conductors the period of inhomogeneities along the meander conductor is also two times greater than the length of the meander strip. As a result the discontinuities of characteristics and the stop-band exist at $\theta = \pi/2$.

Characteristics presented in Fig. 5(a, b) are obtained using the model consisting of two sections of multiconductor lines. They illustrate the influence of asymmetry of characteristic impedances $Z_i(0)$, $Z_j(\pi)$ on properties of meander system.

At constant characteristic impedances along meander conductors, frequency characteristics of the meander system are continuous (curves 1 in Fig. 5(a, b)). At variation of $Z_i(0)$ or $Z_j(\pi)$ along the meander conductor, the stop-band appears when phase angle $\theta$ approaches to $\pi/2$ (curves 2–4 in Fig. 5(a, b)). Increase of variation of characteristic impedances $Z_i(0)$ or $Z_j(\pi)$ is followed by increase of the width of the stop-band (curves 2–3 in Fig. 5(a, b)).

### Conclusions

Due to periodical inhomogeneities and multiple reflections from them, meander systems obtain properties of the stop-band filters.

In order to increase sensitivity and have high impedance of the signal path, meander electrodes containing wide central parts of meander conductors and narrowed peripheral parts are used. Input impedance of such system changes rapidly and the stop-band appears when phase angle $\theta$ approaches to $\pi$.

At transverse asymmetry, the period of inhomogeneities along the meander conductor becomes two times greater with respect to the period of inhomogeneities at absence of transverse asymmetry. Wherefore input imped-

![Fig. 4. (a) Retardation factor and (b) input impedance versus frequency at $x_3 = l_1 + l_2 + l_3 = 20$ mm, $L = 2$ mm and other data given in the Table 2](image)

**Table 2. Initial data for simulation**

<table>
<thead>
<tr>
<th>Curve</th>
<th>$l_1 + l_2$ (mm)</th>
<th>$Z_i(0)/Z_i(\pi)$</th>
<th>$Z_j(0)/Z_j(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5+10+5</td>
<td>70/50</td>
<td>50/30</td>
</tr>
<tr>
<td>2</td>
<td>5+10+5</td>
<td>70/50</td>
<td>50/30</td>
</tr>
<tr>
<td>3</td>
<td>5+5+10</td>
<td>70/50</td>
<td>50/30</td>
</tr>
</tbody>
</table>

![Fig. 5. (a) Retardation factor and (b) input impedance versus frequency at $x_3 = l_1 + l_2 = 20$ mm, $L = 2$ mm and other data given in the Table 3](image)

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<td>70/40</td>
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<td>60/30</td>
<td>60/40</td>
</tr>
<tr>
<td>4</td>
<td>10+10</td>
<td>60/30</td>
<td>70/40</td>
</tr>
</tbody>
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ance rapidly changes and the stop-band appears when phase angle $\theta$ approaches to $\pi/2$.

Increase of variation of characteristic impedances $Z(0)$ or $Z(\pi)$ along the meander conductor is followed by increase of the width of the stop-band at $\theta = \pi$ or $\theta = \pi/2$ depending on the type of inhomogeneities.

References


Influence of transverse asymmetry of meander systems on their frequency characteristics and rejection properties is considered. The model of the systems based on the multiconductor line method is used. Examples of calculated characteristics are presented. Properties of non-homogeneous meander structures, asymmetrical with respect to the longitudinal plane perpendicular to the central part of the system, are revealed. In the meander system containing electrodes with wide central parts of meander conductors and narrowed peripheral parts, the stop-band appears at $\theta = \pi$ where $\theta$ is the phase angle between voltages or current on neighbor conductors. At transverse asymmetry, the period of inhomogeneities along the meander conductor becomes two times greater with respect to the period of inhomogeneities at absence of transverse asymmetry, wherefore input impedance rapidly changes and the stop-band appears when phase angle $\theta$ approaches to $\pi/2$. Increase of variation of characteristic impedances $Z(0)$ or $Z(\pi)$ is followed by increase of the width of the stop-band. Ill. 5, bibl. 14, tabl. 3 (in English; abstracts in English and Lithuanian).


Nagrinėjama skersnio asimetriškumo įtaka nevienaličiose meandrinių lėtinimo sistemose užtvariniams savybėms. Sistema modeliuojama vienėčių daugiailių linijos atkarpą. Pateliaiomi ir nagrinėjamos apskaičiuotos lėtinimo koefficiento ir įejimo varžos dažninės charakteristikos. Parodyta, kad dėl nevienaličiškumo meandrinius lėtinimus, kurios laidininkas sudarytas iš plačių centrinės sričių ir siauresnių kraštinio sričių, įgyja užtvarinių filtrų savybių, kai, didėjant dažnui, faza kampas $\theta$ tarp greičių meandro laidininkų įtampų ar srovio artėja prie $\pi$. Atsiradus meandro laidininko skersiniame asimetriškumui (paslinkta centrinė laidininko dalis arba skiriasi kraštinio sričių būdingieji impedansai), dvigubai išauga netolygumų periodas ir užtvarinė juosta atsiranda ties $\theta = \pi/2$. Labiau keičiantis būdingiesiems impedansams $Z(0)$ arba $Z(\pi)$, užtvarinė juosta platieja. Il. 5, bibl. 14, lent. 3 (angulį kalba; santraukos angulį ir lietuvių k.).