ISSN 1392 – 1215 ELECTRONICS AND ELECTRICAL ENGINEERING 2008. No. 7(87)

ELEKTRONIKA IR ELEKTROTECHNIKA

T115 MEDICINE TECHNOLOGY

MEDICINOS TECHNOLOGIJA

Nonlinear Detection of Weak Pseudoperiodic Chaotic Signal Frequencies from Noisy Environment

K. Pukėnas

Department of Sports Biomechanics, Information Science and Engineering, Lithuanian Academy of Physical Education, Sporto str. 6, LT-44221, Kaunas, Lithuania, phone: +370 37 302668; e-mail: k.pukenas@lkka.lt

Introduction

The task of signal detection and separation is a central theme in a wide variety of fields. Many techniques exist to improve the capability of detecting or enhancement a weak target signal corrupted by the additive Gaussian noise or the background noise generated by an unknown nonlinear dynamical mechanism. Due to its simplicity in implementation and efficiency in computation, noise reduction based on phase-space projection has been widely studied in previous literature [1]-[3]. Recently, nonlinear dynamical modeling [1]-[3] has been combined with other techniques such as artificial neural network [4] and time-frequency analysis [5], to spawn off a powerful algorithm for signal detection in real-life interference environments. In work [3] a novel strategy named supervised principal components analysis (SPCA) for the detection of a target signal of interest embedded in an unknown noisy environment has been investigated and a simple detect algorithm based on nonlinear phase-space reconstructor and a principal components analyzer has been developed. By measuring the difference of both eigenvectors of the correlation data matrices from two channels (background noise and the testing data consists of both the background noise and the signal-of-interest) proposed error detector gives a positive results in the detection of weak electromagnetic high-frequency signals hidden in the real-life interference at signal to noise ratio (SNR) up to –15 dB. The main disadvantage of this algorithm – existence of many false frequency peaks in the regions near to main frequency of the signal-of-interest. The nature of these *ghost signals* is explained in Ref. [3], but really is no possibility to preclude they egsistence, as Statistical analysis tools like Principal Component Analysis (PCA), Singular Spectral Analysis (SSA), Independent Component Analysis (ICA) etc. quickly degrade if the signals exhibit a low SNR [6]. Determining the eigen-structure on the basis of one high-noisy data matrix leads usually to poor or unsatisfactory results because such matrices, based usually on an arbitrary choice, may have some degenerate eigenvalues which leads to loss of information. Therefore, from a statistical point of view, in order to provide robustness and accuracy, it is necessary to consider the average eigen-structure by taking into account simultaneously a possibly large set of data matrices [7]. Under the assumption that the signal-of-interest and noise are stationary and that the noise is additive and uncorrelated with the clean signal the average eigen-structure can be easily implemented via linear combination (averaging) of several covariance matrices and applying the standard eigenvalue decomposition (EVD) or singular value decomposition (SVD). An alternative approach to averaged covariance matrix is to apply the time-delayed covariance matrix. The latter approach is widely used in signal processing for Blind Source Separation (BBS) [7], [8].

In this paper the straightforward detect algorithms based on nonlinear phase-space reconstruction, a principal components analysis and frequency analysis are investigated. By performing the standard EVD (or SVD) to the i) averaged covariance matrix or ii) time-delayed covariance matrix of the data of reconstructed phase-space (or – trajectory matrix) detector gives a good performance in the detection of weak pseudoperiodic chaotic signals buried in a large white Gaussian or colored noise background. This class of chaotic time series – pseudoperiodic – has aroused great interest due to their close relation to some important natural and physiological systems [9], [10]. Naturally, the results can be applied also for regular sinusoidal signal. The performances of the algorithms are compared with the SPCA algorithm.

Throughout the paper, the *x* component of the well-known Rossler system, which is chaotic and contains obvious periodic component, for illustration is used.

Description of the averaged covariance matrix based and time-delayed covariance matrix based algorithms

Let $\{z_i\}_{i=1}^{L}$ denote time series with L samples. The testing data consists of both the background noise and the low dimensional pseudoperiodic deterministic signal-of-interest. The phase points can be reconstructed by time delay embedding [11] – i.e. $\{\mathbf{z}_i\}_{i=1}^{L-(d-1)\tau}$:

$$\mathbf{z}_i = \left[z_i, z_{i+\tau}, z_{i+2\tau}, \cdots, z_{i+(d-1)\tau}\right]^T \qquad (1)$$

and a reconstructed phase space matrix $\mathbf{Z}$ with d rows and $M = L - (d-1)\tau$ columns (called a trajectory matrix) is defined by

$$\mathbf{Z} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{L-(d-1)\tau} \\ z_{1+\tau} & z_{2+\tau} & \cdots & z_{L-(d-2)\tau} \\ \vdots & & \ddots & \\ z_{1+(d-1)\tau} & z_{2+(d-1)\tau} & & z_L \end{bmatrix}, \qquad (2)$$

where d – the embedding dimension, τ – time delay and $(\cdot)^T$ denotes the transpose of a real matrix.

The set of data matrices $\mathbf{X}_j$ $(j = 1, \cdots, k)$ with d rows and N columns are obtained by selecting N consecutive columns of $\mathbf{Z}$ at lag $q = floor(M/k)$, i.e.

$$\mathbf{X}_j = \mathbf{Z}(:, q*(j-1)+1 : q*(j-1)+N) \qquad (3)$$

and centered with $\mathbf{X}_j = \mathbf{X}_j - \overline{\mathbf{X}}_j$, where $\overline{\mathbf{X}}_j$ is the column matrix of mean over dimension $1,\cdots,d$. Under the assumption that the signal-of-interest and noise are stationary and that the noise is additive and uncorrelated with the clean signal we calculate k covariance matrices $\mathbf{R}_j = \frac{1}{N-1}\mathbf{X}_j\mathbf{X}_j^T$ and the averaged covariance matrix $\mathbf{R} = \langle \mathbf{R}_j \rangle$. In order that covariance matrices should be to treat as independent and the averaging should be effective, necessary that $q \geq N$. Taking the standard eigenvalue decomposition to the averaged covariance matrix $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\mathbf{T}$ (or singular value decomposition $\mathbf{R} = \mathbf{W}\mathbf{\Sigma}\mathbf{U}^\mathbf{T}$), the projected trajectory matrix is computed via the equation [12], [13]

$$\hat{\mathbf{Z}} = \mathbf{U}_1 \cdot \mathbf{U}_1^T \cdot (\mathbf{Z} - \overline{\mathbf{Z}}) + \overline{\mathbf{Z}} \ , \qquad (4)$$

where the eigenvectors $\mathbf{U}_1$, associated with the m largest eigenvalues diag($\mathbf{\Lambda}$), span the signal subspace and $\overline{\mathbf{Z}}$ is the mean over dimension $1,\cdots,d$. Finally, a enhanced one-dimensional signal is created from the new space, typically by time-aligning and averaging the columns of the trajectory matrix $\hat{\mathbf{Z}}$ (see [12] for more details) and analyzed using the standard frequency estimation.

The delayed covariance matrix $\mathbf{R}_p$ is computed with one matrix $\mathbf{R}_r$ obtained by eliminating the first p columns of $\mathbf{Z}$ and another matrix, $\mathbf{R}_s$, obtained by eliminating the last p columns of $\mathbf{Z}$:

$\mathbf{R}_p = \frac{1}{NN-1}\mathbf{X}_r\mathbf{X}_s^T$, where $NN = M - p$. Time-lag $p \neq 0$ is chosed experimentally, whereas the matrix $\mathbf{R}_p$ is not always positive definite and this can leads to pure results. The further calculation is identical to above described, excepting that only singular value decomposition can be performed in order that avoid complex eigenvectors.

Numerical results

To test the detection performance of both averaged covariance matrix based and time-delayed covariance matrix based algorithms let's consider a high-noisy x component of the Rossler system contaminated with additive white Gaussian noise and with additive colored noise. The Rossler system is given by

$$\begin{cases} \frac{dx}{dt} = -(y+z), \\ \frac{dy}{dt} = x + a \cdot y, \\ \frac{dz}{dt} = b + z(x-c) \end{cases} \qquad (5)$$

with parameters a = 0,398, b = 2 and c = 4.

Additive colored noise are got from the AR process $r_{n+1} = ar_n + b\eta_n$, where η_n is the Gaussian noise term, $a = 0{,}69$ and $b = 0{,}31$. It has been argued that the PCA method can obtain better results for pseudoperiodic signals by over-embedding with time delay $\tau = 1$ [3], [12], [13]. In the detection experiment, the time delay $\tau = 1$ and the data dimension $d = 60$ were chosen. The vector space of the noisy signal is composed of a signal-plus-noise subspace and a complementary noise subspace. For a weak signal hidden beneath the noise floor the subspace is dominated by noise and the power distribution among associated eigenvalues is generally balanced since the noise variance is the same in any direction (for Gaussian noise). Averaging of the covariance matrices allow to suppress the influence of additive noise and several first eigenvectors with the largest eigenvalues can be assigned to the subspace of the true signal. In experiments the relatively short data length – 6 000 and 9000 data points of Rossler system – were used. For algorithm based on averaged covariance matrix $k = 4$ and for algorithm based on time-delayed covariance matrix $p = 10$. Two eigenvectors, associated with the largest eigenvalues were chosen ($m = 2$). The signal frequency extracted from additive white Gaussian noise environment by performing eigenvalue decomposition to the averaged covariance matrix (algorithm conditionally named AMPCA) is shown in Fig. 1 and the signal frequency extracted from additive colored noise environment is shown in Fig. 2. The signal frequency extracted from additive white Gaussian noise environment by performing eigenvalue decomposition to the time-delayed covariance matrix (algorithm conditionally named DMPCA) is shown in Fig. 3. For comparison the signal frequency extracted from additive white Gaussian noise environment by applying SPCA detection algorithm is shown in Fig. 4. It is found that the AMPCA detection algorithm and equally the DMPCA detection algorithm can reliably extract the main signal frequency 1,6 Hz at $SNR \geq -20\ dB$.

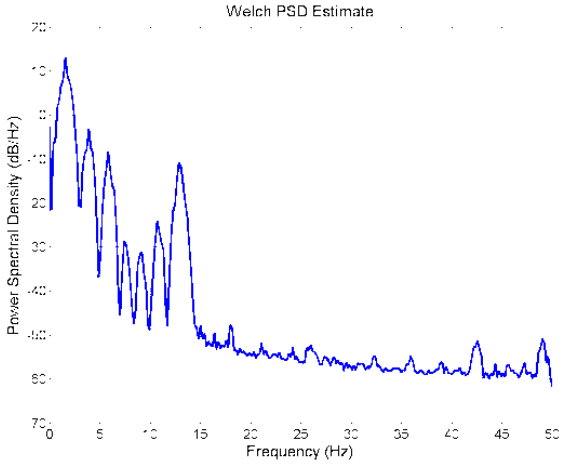

Fig. 1. Frequency spectra of the signal extracted from additive white Gaussian noise environment using AMPCA algorithm at SNR = – 20 dB, where the main frequency (1,6 Hz) of the signal of interest is clearly observable

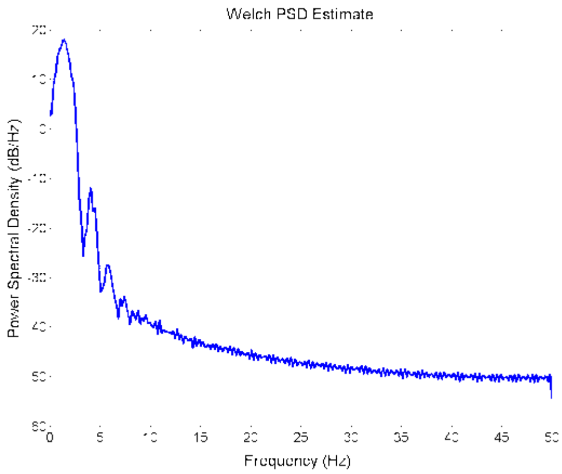

Fig. 2. Frequency spectra of the signal extracted from additive colored noise environment using AMPCA algorithm at SNR = – 20 dB, where the main frequency (1,6 Hz) of the signal of interest is clearly observable

While SPCA detection algorithm besides the main signal frequency also generates many false frequency peaks (*ghost signals*) in the regions over 1,6Hz. The amplitude and frequency of these peaks depend on the series length, etc. In practical application, these false peaks only may be identified by performing the calculations with time series of various lengths – the main frequency peak has the same amplitude and remains in the same place, whereas false peaks are shifted.

Conclusions

In this paper the detection algorithms consisting of nonlinear phase space reconstruction technique, principal components analysis feature selection and frequency analysis are investigated by applying them to high-noisy pseudoperiodic chaotic Rossler signal.

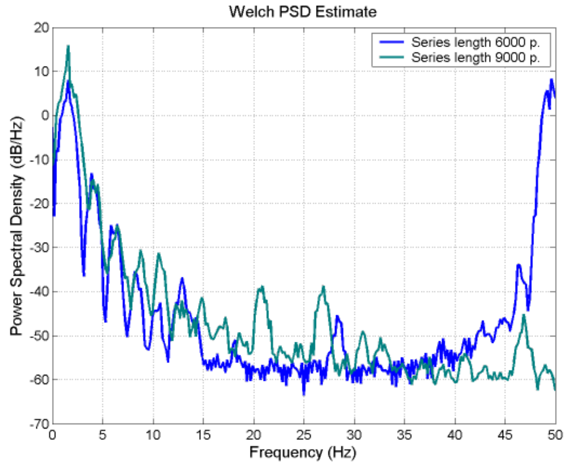

Fig. 3. Frequency spectra of the signal extracted from additive white Gaussian noise environment usind DMPCA algorithm at SNR = – 20 dB, where the main frequency (1,6 Hz) of the signal of interest is clearly observable

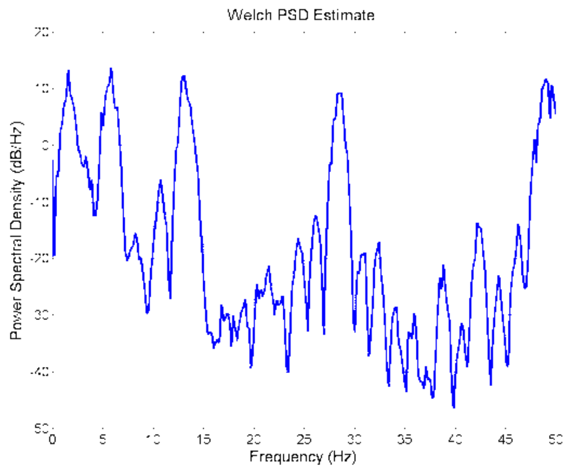

Fig. 4. Frequency spectra of the signal extracted from additive white Gaussian noise environment using SPCA algorithm at SNR= – 20 dB

It is demonstrated, that both algorithms – the detection algorithm based on EVD performing to the averaged covariance matrix and the detection algorithm based on SVD performing to the time-delayed covariance matrix – are able to detect weak pseudoperiodic chaotic (or regular sinusoidal) signals hidden beneath the additive Gaussian or colored noise floor at SNR up to –20 dB. The signal‘s main frequency can be extracted accurately and no false frequency peaks occur in spectrum of enhanced signal by the time series with length of over 6000 – 9000 points. Therefore, it may be concluded, that above-mentioned algorithms are preferable to the SPCA algorithm by detecting weak pseudoperiodic chaotic or sinusoidal signals buried in a additive Gaussian noisy background.

It should be noted that applying of Joint Approximate Diagonalization [7] of a set of time-delayed covariance matrices in this case (high noise level) gives negative results. In this experiment the Joint Approximate Diagonalization algorithm, described in [14] has been used.

References

1. **Ephraim Y., Trees H. L. V.** A Signal Subspace Approach for Speech Enhancement // IEEE Trans. Speech Audio Process. – 1995. – Vol. 3, No. 4. P. 251–266.
2. **Luo X., Zhang J., M. Small.** Optimal phase-space projection for noise reduction // Phys. Rev. E. – 2005. – Vol. 74, No. 4. – P. 046711-1 – 046711-5.
3. **Zhou C. T., Cai T. X., Cai T. F.** Nonlinear real-life signal detection with a supervised principal components analysis // Chaos. – 2007. – Vol. 17, Iss.1. – P. 013108-1 – 013108-5.
4. **Zhou C., Cai T., Lai C. H., Wang X., Lai Y.-Ch.** Model-based detector and extraction of weak signal frequencies from chaotic data // Chaos. – 2008. – Vol. 18, Iss.1. – P. 013104-1 – 013104-12.
5. **Chandre C., Wiggins S., Uzer T.** Time–frequency analysis of chaotic systems // Physica D. – 2003. – Vol. 181, Iss. 3.-4 – P. 171 – 196.
6. **Gruber P., Stadlthanner K., Bohm M., Theis F. J., Lang E. W.** Denoising Using Local Projective Subspace Methods // Neurocomputing. – 2006. – Vol. 69, Iss. 13-15. – P. 1485–1501.
7. **Choi S., Cichocki A., Park H. M., Lee S. Y.** Blind Source Separation and Independent Component Analysis: A Review // Neural Information Processing – Letters and Reviews. – 2005. – Vol. 6, No. 1. – P. 1–57.
8. **Gruber P., Stadlthanner K., Böhm M., Theis F. J., Lang E. W., Tomé A. M., Teixeira A. R., Puntonet C. G., Gorriz Saéz J. M.** Denoising using local projective subspace methods // Neurocomputing – 2006. – Vol. 69, Iss. 13-15. – P. 1485–1501.
9. **Zhang J., Luo X., Small M.** Detecting chaos in pseudoperiodic time series without embedding // Phys. Rev. E. – 2006. – Vol. 73, No. 1. – P. 016216-1 – 016216-5.
10. **Pukenas K.** Detecting deterministic structure from a high noisy pseudoperiodic time series // Electronics and Electrical Engineering. – 2008. – No. 4 (84). – P. 80–85.
11. **Kantz H., Schreiber T.** Nonlinear Time Series Analysis // Cambridge University Press, Cambridge. – 2003. – P. 30–39.
12. **Johnson M. T., Povinelli R. J.** Generalized phase space projection for nonlinear noise reduction // Physica D. – 2005. – Vol. 201, Iss. 3-4. – P. 306–317.
13. **Sun J., Zhao Y., Zhang J., Luo X., Small M.** Reducing colored noise for chaotic time series in the local phase space // Phys. Rev. E. – 2007. – Vol. 76, No. 2. – P. 026211-1 – 026211-6.
14. **Ziehe A., Laskov P., Nolte G., Muller K. R.** A Fast Algorithm for Joint Diagonalization with Non-orthogonal Transformations and its Application to Blind Source Separation // Journal of Machine Learning Research – 2004. – Vol. 5 – P. 777–800.

Received 2008 04 10

K. Pukėnas. Nonlinear Detection of Weak Pseudoperiodic Chaotic Signal Frequencies from Noisy Environment // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 7(87). – P. 81–84.

The extraction of weak pseudoperiodic chaotic signal frequencies from white Gaussian and colored additiwe noise is investigated by applying the nonlinear signal detection algorithms, based on phase-space embedding technique, principal component analysis and power spectral analysis. By analyzing Rossler chaotic time series, it is demonstrated, that the detection algorithm based on standard eigenvalue decomposition performing to the averaged covariance matrix of the reconstructed phase space matrix and the detection algorithm based on eigenvalue decomposition performing to the time-delayed covariance matrix are able to detecting of weak pseudoperiodic chaotic (or regular sinusoidal) signals hidden beneath the additive Gaussian or colored noise floor at SNR up to –20 dB. It is concluded, that these algorithms are preferable to the SPCA algorithm by detecting weak pseudoperiodic chaotic or sinusoidal signals buried in a additive Gaussian noisy background. Ill 4, bibl. 14 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас. Нелинейное детектирование слабых псевдопериодических хаотических сигналов на фоне больших шумов // Электроника и электротехника. – Каунас: Технология, 2008. – № 7(87). – С. 81–84.

Исследуется выделение частот слабых псевдопериодических хаотических сигналов из адитивных гауссовых или цветных шумов при использовании алгоритмов обнаружения сигналов, основанных на реконструкции фазового пространства, анализе главных компонент и спектрального анализа. Путем анализа хаотического сигнала Росслера показывается, что алгоритм, основанный на декомпозиции усредненной ковариационной матрицы данных реконструированного фазового пространства и алгоритм, основанный на декомпозиции задержанной по времени ковариационной матрицы позволяют обнаружить основную частоту сигнала Росслера на фоне белого гауссового шума или цветного шума при отношении сигнал-шум выше – 20 дБ. Делается вывод, что эти алгоритмы более предпочтительны при обнаружении слабых псевдопериодических хаотических сигналов на фоне гауссовых шумов чем алгоритм SPCA. Ил. 4, библ. 14 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Silpnų pseudoperiodinių chaotinių signalų dažnių išskyrimas iš triukšmų naudojant netiesinius metodus // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 7(87). – P. 81–84.

Tiriamas silpnų pseudoperiodinių chaotinių signalų dažnių išskyrimas iš adityvinio baltojo Gauso triukšmo arba spalvotojo triukšmo, signalų detekcijai naudojant algoritmus fazinės erdvės rekonstrukcijos, esminių komponenčių analizės bei spektrinės analizės pagrindu. Atlikus tyrimus su chaotiniu Rosslerio signalu, parodoma, kad algoritmas rekonstruotos fazinės erdvės duomenų suvidurkintos kovariacinės matricos dekompozicijos tikriniais vektoriais pagrindu ir algoritmas suvėlintos kovariacinės matricos dekompozicijos pagrindu įgalina išskirti pagrindinius Rosslerio signalo dažnius iš baltojo Gauso triukšmo ir spalvotojo triukšmo, kai signalo ir triukšmo santykis didesnis kaip –20 dB. Daroma išvada, kad šie algoritmai yra pranašesni už SPCA algoritmą išskiriant silpnus pseudoperiodinius chaotinius signalus iš Gauso triukšmo. Il. 4, bibl. 14 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).