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# The Investigation of Stress Distribution in Pulsed Magnets

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## Introduction

It is well established that high magnetic fields are a powerful tool for scientific research. Apart from the promise of new discoveries in previously uncharted territory, higher fields give better resolution and enhanced sensitivity to many experiments; therefore the quest for ever higher fields is an important facet of contemporary solid state research [1].

On one side, an increase in magnetic field, for a given coil, is accompanied by a quadratic increase in thermal and mechanical loads [2]. On the other side, the possibility to increase the maximum available magnetic field is limited by thermal and mechanical strength of applicable materials in general. The coil construction should be able to contain the magnetic forces without undue deformation and stresses in wires and reinforcements should not exceed the ultimate tensile strength limit [3].

After the analysis of thermodynamic processes in multilayer inductors [4] the structural-stress analysis should follow and that is a logical step, while during the thermodynamic analysis obtained maximum current value in inductor windings is used to calculate the distribution of magnetic field density, which is then used to evaluate the maximum magnetic (Lorence) forces and stresses that act on inductor's windings and enforcement layers. The calculated axial, radial, hoop and, finally, Von Misses stresses give us an idea of coil's reliability for multi-time-use. Stresses that exceed material's yield strength at any point may lead to the plastic material deformation which might have following consequences: if stresses would exceed materials' tensile strength, inductor would be damaged, its windings short-circuited, reinforcement would be damaged and no longer function as it should, inductor would become unusable; if stresses would exceed materials' yield strength, inductor's geometry would change which would lead to the change of inductor's inductivity, resistance and therefore to unpredicted processes in it during the next pulse. If a long-life inductor is constructed, all the above-mentioned consequences must be avoided.

Although precise finite element technique software based models [5,6] are often used for stress distribution calculations in pulsed magnets, they are difficult to use when many configurations are tested because of their inflexibility.

The approximate stress distribution calculation is a part of an universal and flexible simulation package, which would help to predict pulsed inductor's life expectancy, and multi-criteria (geometry, heating, magnetic field density, stresses) evaluation is the only way to do this because each of these criteria has an influence/is dependent on each other.

## Inductor for simulation

For the simulation the coil, which parameters are shown in Table 1 was used. Earlier thermodynamic simulations [4] showed that inductor was not overheated. The targeted magnetic field was 50±5 T and the coild generated a maximum axial magnetic field of 49T when a 5mF capacitor bank was charged to 3kV and discharged through the coil switching on the power thyristor. The maximum current of 52 kA was withdrawn (see Fig. 2). Inductor‘s cross-section and layers are shown in Fig. 1.

**Table 1.** Pulsed coil. parameters

| **Parameter** | **Value** |
|---|---|
| $a_1$,mm | 6 |
| $a_2$,mm | 18,8 |
| $2b$,mm | 48 |
| $\lambda$ | 0,6913 |
| $n_1$ | 10 |
| $n_2$ | 4 |
| Material | CuNb |
| Composition | 82%Cu |
| $\sigma$,% IACS | 63(293K) |
| $T_{\text{Experiment}}$, K | 77 |

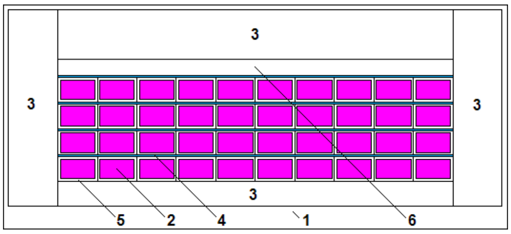

**Fig. 1.** Inductor windings and surounding layers. 1 – liquid $N_2$, 2 – CuNb, 3 – steel shell, 4 – Zylon+epoxy reinforcement, 5 – S2 glass fiber, 6 – outer Zylon+epoxy reinforcement

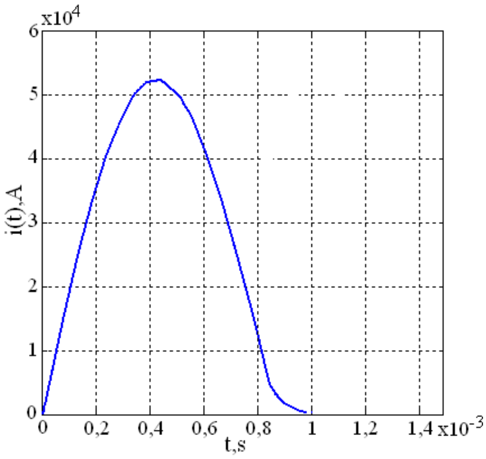

**Fig. 2.** Current transient process in pulsed inductor given in Table 1

## Magnetic field distribution

Magnetic field density distribution at a maximum current is needed while interacting with the current it gives maximum Lorence forces and stresses subsequently.
The axial and radial magnetic fields created by a current loop might be calculated at any given point [7]:

$$B_z(r,z) = (\mu_0 / 2\pi) I_{\max} \frac{1}{\sqrt{(a+r)^2 + z^2}} \times \left( \frac{a^2 - r^2 - z^2}{(a-z)^2 + z^2} E(m) + K(m) \right), \tag{1}$$

$$B_r(r,z) = (\mu_0 / 2\pi) I_{\max} \frac{1}{\sqrt{(a+r)^2 + z^2}} \times \left( \frac{a^2 + r^2 + z^2}{(a-z)^2 + z^2} E(m) - K(m) \right), \tag{2}$$

where: $r, z$ – radius and height of any point where B is calculated (from the inductor centre point (0,0)); $a$ – radius of a loop that conducts current; $E(m), K(m)$ – elliptic first and second order integrals, when $m = 4ar/((a+r)^2 + z^2)$.

When Inductor has $n_1$ windings in a layer and $n_2$ layers, the distribution of axial and radial magnetic fields would be calculated:

$$B_z(r,z) = \sum_{i=1}^{n2 \cdot cz} \sum_{j=1}^{n1 \cdot cr} B_{z,ji}(r,z), \tag{3}$$

$$B_r(r,z) = \sum_{i=1}^{n2 \cdot cz} \sum_{j=1}^{n1 \cdot cr} B_{r,ji}(r,z), \tag{4}$$

where $cz, cr$ – multiplication factors.

The radial and axial magnetic field density distribution in inductor‘s bore and windings are shown in Fig. 3 and 4.

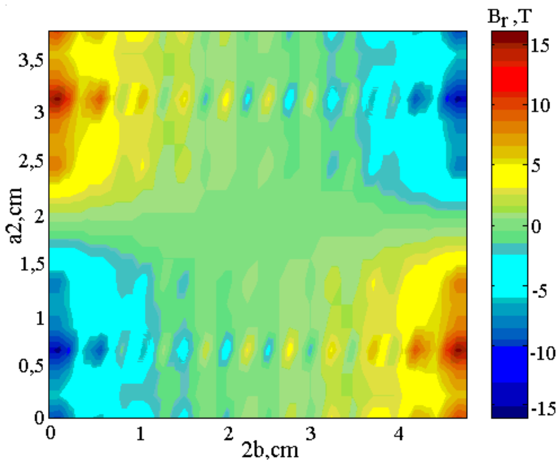

**Fig. 3.** Radial magnetic field distribution

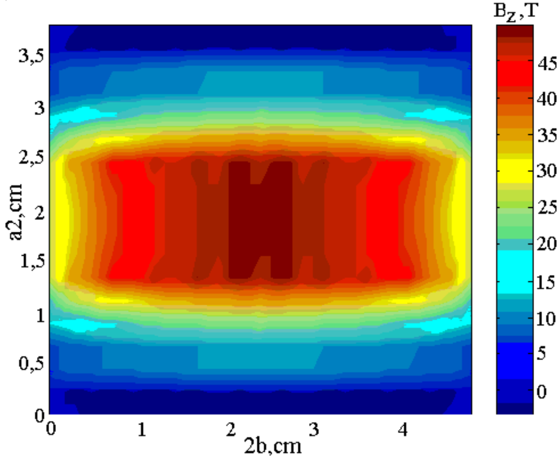

**Fig. 4.** Axial magnetic field distribution

## Stress distribution

As radial stresses are caused by axial magnetic field, they act as inductor‘s expanding forses towards outer reinforcement cylinder and could be computed:

$$\sigma_r(r,z) = j \int_{a2}^{r} B_z(r,z) dr. \tag{5}$$

Axial stresses are caused by radial magnetic field and act as inductor‘s compressive force along inductor‘s bore axis

$$\sigma_z(r,z) = j \int_{0}^{b} B_r(r,z) dz. \tag{6}$$

Hoop stress for each given $z$ is calculated according to formula given in [7]:

$$\sigma_h(r,z) = \frac{(B_z^2 + B_r^2)}{2\mu_0} 2\frac{r}{a}\frac{\alpha - r/a}{(\alpha - 1)^2}, \quad (7)$$

where $a$ – the radius of the first inner winding; $r$ – the radius of a calculated point.

The Von Misses criteria to evaluate the sum stresses then is:

$$\sigma_{VM} = \sqrt{\frac{(\sigma_h - \sigma_r)^2 + (\sigma_h - \sigma_z)^2 + (\sigma_r - \sigma_z)^2}{2}}. \quad (8)$$

Results are shown in Fig. 5-8.

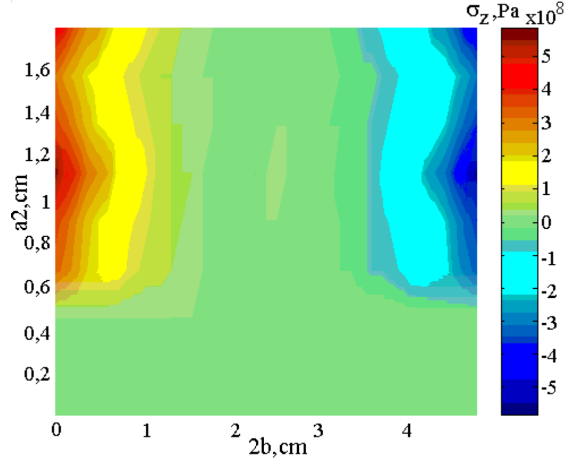

**Fig. 5.** Axial stress distribution

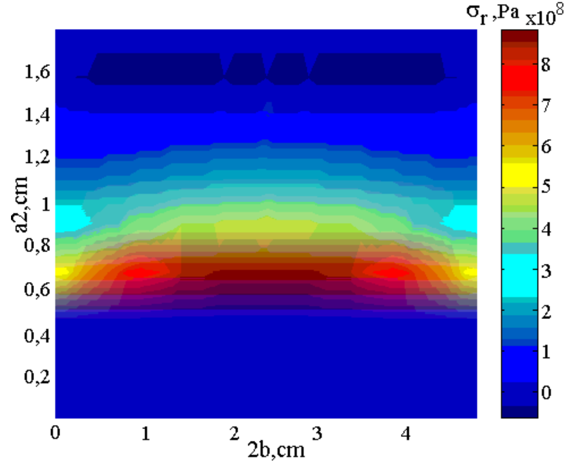

**Fig. 6.** Radial stress distribution

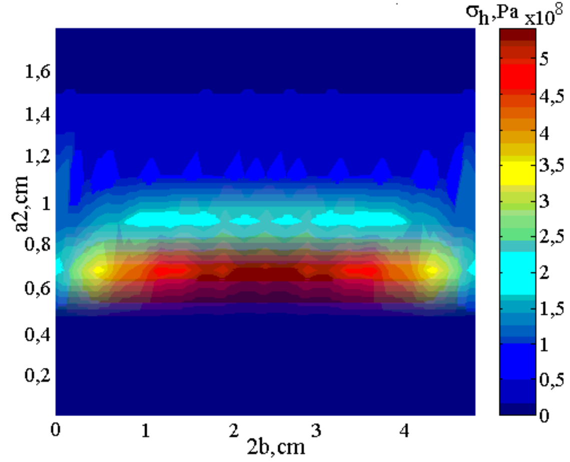

**Fig. 7.** Hoop stress distribution

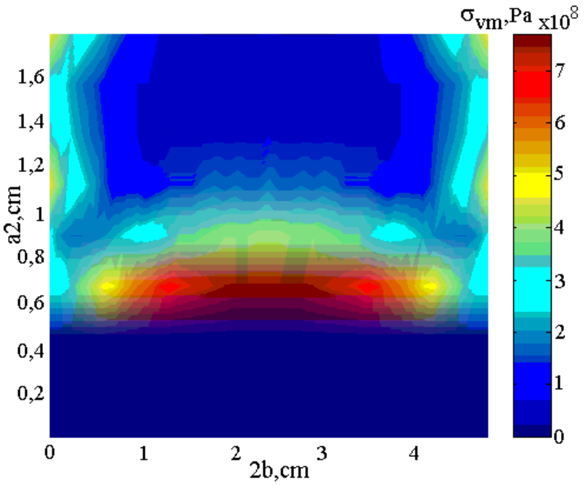

**Fig. 8** Von Misses stress distribution

## Conclusions

The analysis of stress distribution in a pulsed 4 layer, 10 windings/layer coil has been carried out. It was proved that maximum sum stresses in inductor didn't exceed the Yield strength (see Table 2) of any material which was used to construct inductor and was responsible for inductors operation. The maximum Von Misses stress in the inner layers (first and second) didn't exceed 750 MPa (Fig. 8) and it means that all the construction parts, including CuNb wire, overcome only elastic deformations. For other parts, like Zylon© reinforcement layers, S2 glass fiber© or Kapton© isolation, stresses are 3-4 times weaker than their Yield strength. Steel shell in the bore of the inductor and on the sides of it is also deformed only elastically because magnetic forces are directly acting only on to first windings towards outer inductor layers and the middle plane. Finally, taking into account all what was said above and evaluating all the results, we can claim that the inductor analyzed here would not be destroyed by magnetic forces and could be used for multi-time operation which was the main target of this analysis.

**Table 2.** Material properties for structural analysis

| Material | Yield strength, $R_{0.2}$, MPa | Tensile strength, $R_m$, MPa |
|---|---|---|
| Cu-Nb wire | 850 | 1100 |
| Zylon | 3200 | 4500 |
| Epoxy | 42 | 70 |
| S2 GF | 2600 | 3660 |
| Steel shell | 200 | 520 |

## Acknowledgements

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**S. Bartkevičius, J. Novickij. The Investigation of Stress Distribution in Pulsed Magnets // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 7(87). – P. 7–10.**

The analysis of stress distribution in pulsed coils applicable for pulsed magnetic field generation has been done. Process modelling was carried out using *Matlab® Simulink®* software and *Matlab®* programming tools that in combination allow to create easy to modify, flexible and universal model for the magnetic field and stress calculation in pulsed coils. The target of the simulation was to calculate the amplitude of mechanical stresses in inductor cross-section. The stresses that exceed the yield strength of materials used for inductor construction, deform the inductor plastically and change its electrical parameters or, if tensile strengths are exceeded, may lead to a total destruction. 4 layer and 10 windings per layer pulsed inductor with interlayer reinforcement and outer steel shell was analyzed here and it was proved that, while generating a 49T magnetic field pulse, the stresses in inductors cross-section were not larger than ones allowed which tells us that inductor could be multi-time used. The created model might be used to analyze any inductor during the design phase no matter what the geometrical configuration or material composition is. Such methodology enables us to solve inductor geometry and material application problems and design inductors that can be used to generate many magnetic field pulses that are above 50T. Ill. 8, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

**С. Барткевичюс, Ю. Новицкий. Исследование механических нанряжений в импульсных магнитах // Электроника и электротехника. – Каунас: Технология, 2008. – № 7(87). – С. 7–10.**

Представлен анализ механических напряжений в импульсных индукторах при генерации сильных магнитных полей. Моделирование магнитомеханических процессов произведена с помощью *Matlab®* и *Simulink®* программных пакетов, что позволило создать достаточно гибкую и универсальную модель для оценки магнитного поля и механических напряжений в импульсных индукторах. Целью расчётов было нахождение критических значений механических напряжений, превышающих пределы текучести конструкционных материалов и приводящих, в конечном итоге, к необратимым деформациям и разрушению импульсного магнита. Анализируя 4-х слойную (по 10 витков в каждом слое) конструкцию индуктора, было установлено, что при генерации импульса с плотностью магнитного потока 49 Т механические напряжения не превышают предельных значений используемых материалов. Представленная модель импульсного индуктора позволяет оперативно менять исходные данные и находить оптимальную геометрию индуктора. Полученные результаты расчётов использованы для дальнейших исследований по разработке неразрушающегося прототипа индуктора импульсного магнитного поля порядка 50 Т. Ил. 8, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

**S. Bartkevičius, J. Novickij. Impulsinių magnetų mechaninių įtempių tyrimas // Elektrotechnika ir elektronika. – Kaunas: Technologija, 2008. –Nr. 7(87). – P. 7–10.**

Pateikta mechaninių įtempių, susidarančių impulsiniam magnetiniam laukui generuoti naudojamuose induktoriuose, skaitmeninė analizė. Imitacijai pritaikytas programų paketas *Matlab® Simulink®* bei *Matlab®* programavimo kalba, įgalinanti sudaryti lankstų ir universalų, lengvai tobulinamą bei keičiamą sistemos elektromagnetinį ir įtempių skaičiavimo modelį. Imitavimo tikslas buvo nustatyti mechaninių įtempių pasirinkto induktoriaus skerspjūvyje dydžius. Kai jie viršija medžiagų elastingumo ribas, induktorius sugenda arba negrįžtamai pasikeičia jo geometrija ir elektriniai parametrai. Analizuotas 4 sluoksnių induktorius, kurio kiekviename sluoksnyje yra dešimt vijų, jog, sugeneravus 49T magnetinio lauko impulsą, magnetinių jėgų lemiami įtempiai induktoriuje neviršijo leistinų ir tai reiškia, jog toks induktorius galėtų būti naudojamas daug kartų. Sudarytas modelis gali būti taikomas bet kuriam projektuojamam induktoriui nepriklausomai nuo jo geometrinės konfigūracijos ir medžiaginės sudėties. Tokia metodika įgalina spręsti induktorių konfigūracijos, medžiagų gerinimo ir paprastinimo uždavinius, siekiant jį panaudoti daugeliui impulsų, kurių amplitudė >50T, generuoti. Il. 8, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).