Channel Information Capacity of the Sensomotor System of the Eye

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Introduction

More than half of century scientists around the world investigate eye movements control system which is called oculomotor system. Being part of the vision this system has exclusively important features. There are no other parts of the body which has so well developed sensor and motor subsystems than eye. Visual system gives largest part of the information of the surrounding world. Sensors of the eye scanning around one’s room found object of interest and muscles of the eye ball turns the gaze to the direction until image of the object falls on the fovea which is most sensitive part of the retina. Accuracy and large velocity of eye movements brings to the idea to use visuo-oculomotor system not only traditionally to accept visual information but, measuring position of the gaze, perform control commands. Most common tasks for the oculomotor system are to find coordinates of the object of interest (searching and pointing) and to write texts (typing) [1].

Investigation of the human beings oculomotor system found several types of eye movements which are substantially analyzed [2]. Only a few of them are related with target direction and enough accurate that could be used for control tasks. They are saccadic, smooth pursuit and fixational eye movements. Fixation of the target could be assumed as smooth pursuit eye movements in situation when target velocity is zero [3]. In this case there are only two types: saccadic and smooth pursuit eye movements, which could be used to find direction of the target position. This is the reason why precision and velocity of saccades and smooth pursuit eye movements are so widely analyzed [4,5]. Accuracy of oculomotor system is defined as error between target position and line of sight point on the screen. Looking at the stationary target on the screen line of sight position and target position eliminatig fixational micromovements fit each another. When the target is jumping or moving, line of sight or eye movements are lagging and difference between target and eye appears causing dynamic errors which would become bigger when target velocity increases.

Target jumping from one position to another elicit saccadic eye movements. Error in this situation could be evaluated as difference between new target position and line of sight initial coordinate after jump. Later eye eliminates this error performing one or sometimes two corrective saccades.

When eye is following slowly moving target, smooth pursuit error could be defined as difference between target and line of sight positions at the same time. This error strongly depends on the type of trajectory. When the trajectory is predictable and/or repeatable eye movements control system after couple of cycles defines the shape of trajectory and substantially reduces the size of error. Predictable trajectories have small information capacity and in this research were not analyzed.

In this research we proposed quality of human’s eye movements control system evaluate using not accuracy, as usual, but investigating limit of the amount of information, transmitted thru the oculomotor channel. Instead of accuracy, this characteristic - channel information capacity- has advantage because it defines eye movements control system only by one parameter. Errors, measured during experimental investigation, always depends on the amplitude of the target jump during saccades or type and velocity of trajectories during smooth pursuit and demands to determinate and explicit target motion characteristics.

Using information theory characteristics eye movements control system could be defined as follows. Trajectory of target movement \( T \) on the screen would be defined as source or input information of the oculomotor system, line of sight trajectory on the screen \( A \) as output information and difference between them as lost information \( L \). When the velocity of target would increase or the target trajectory would become more complicated the information rate of the source would increase, eye movement control system is no more capable to follow the target and difference between target and eye position or lost information would be bigger. This let us formulate conclusion that human smooth pursuit oculomotor system has channel information capacity limit and could be used as important characteristic to determinate eye movements control system.

When the target is jumping randomly from one position to another we have discreet source information and lost information would be difference between target and eye position after jump to the new target position. In this behavior information rate transmitted thru oculomotor channel would be difference between input and
lost information rates. When the frequency of target jumps would be increased, saccadic oculomotor system would be no more capable to elicit adequate jumps and lost information at the threshold frequency substantially increases. It means that saccadic oculomotor system also has channel information capacity limit.

Theoretical background

We separately describe calculation of the channel information capacity for smooth pursuit and saccadic eye movements control systems. During smooth pursuit target moves and eye follows by continuous-time analog trajectories and we can apply information theory for analog signals. Saccadic eye movements are elicited separately for every new target position and in this case we can apply information theory for discrete signals.

1. Channel information capacity of the oculomotor system during smooth pursuit.

When the target moves by the continuous-time analog trajectory $T$ in the horizontal axis source information rate determined as the target entropy $H(T)$ depends on target trajectory probability distribution $p(T)$ by the equation:

$$H(T) = - \int_{-\infty}^{+\infty} p(T) \log p(T) dT.$$  

(1)

The largest information rate has trajectory with probability distribution fits Gaussian law

$$p(T) = \frac{1}{\sigma_T \sqrt{2\pi}} \exp(-\frac{T^2}{2\sigma_T^2}),$$

(2)

where $\sigma_T$ - standard deviation of the target $T$ trajectory. Solving 1 and 2 equations we can get

$$H(T) = 0.5\ln(2\pi e \sigma_T^2).$$

(3)

Mutual information between target trajectory $T$ as source information $H(T)$ and line of sight trajectory $A$ as accepted information $I_p(T,L)$ during one moment of time are related by the equation

$$I_p(T,L) = H(T) - H(T|L).$$

(4)

Where $H(T/L)$ conditional probability distribution representing lost information or left uncertainty. If difference between target and line of sight trajectories as continuous-time analog noise signal would be defined by Gaussian probability distribution with standard deviation $\sigma_L$, information rate transmitted threw oculomotor system would be

$$I_p(T,L) = 0.5\ln(1 + \frac{\sigma_T^2}{\sigma_L^2}).$$

(5)

If duration of the smooth pursuit experiment would be $t$ and frequency threshold of the human’s oculomotor system would be $F$, the amount of information transmitted threw it would be

$$I_{pt}(T,L) = Ft\log(1 + \frac{\sigma_T^2}{\sigma_L^2}),$$

(6)

and channel capacity of the one-dimension human’s oculomotor system when the duration of the experiment $t$ would be long enough

$$C_{pl} = \lim_{t \to \infty} \frac{1}{t} \left[ F t \log(1 + \frac{\sigma_T^2}{\sigma_L^2}) \right].$$

(7)

If the target moves in two coordinates $T_x$ and $T_y$, eye movements control system performs two-dimensional smooth pursuits. Target or source information rate $H(T_x,T_y)$ in this case would be defined by two dimensional probability distribution $p(T_x,T_y)$. When the target trajectories in the horizontal and vertical coordinates are not correlated, two-dimensional probability distribution would be completely determined by the sum of the target horizontal and vertical probability distributions: $p(T_x,T_y) = p(T_x) + p(T_y)$. Assuming that two dimensional human smooth pursuit eye movements control system works by two separate horizontal and vertical neural circuits, oculomotor system channel information capacity would be approximately twice bigger.

$$C_{p2} = \lim_{t \to \infty} \frac{1}{t} \left[ F_x t \log(1 + \frac{\sigma_{T_x}^2}{\sigma_{L_x}^2}) + F_y t \log(1 + \frac{\sigma_{T_y}^2}{\sigma_{L_y}^2}) \right],$$

(8)

where $\sigma_{T_x}, \sigma_{T_y}, \sigma_{L_x}, \sigma_{L_y}$ - standard deviations of target and error signals (trajectories) in the horizontal and vertical directions respectively.

Last assumption could be experimentally approved by measuring one and two-dimensional channels information capacity of the human’s smooth pursuit eye movements control system.

2. Channel information capacity of the oculomotor system during saccadic eye movements.

When the target jumps from one position to another, saccadic eye movements control system is in action. In this case we can assume that input or source information for the oculomotor system is discrete. If all target positions have equal probabilities $p(T_n,T_m)$, two dimensional source information rate would be

$$H(T_x,T_y) = - \sum_{x=1}^{n} \sum_{y=1}^{m} p(T_x,T_y) \log p(T_x,T_y).$$

(9)

where $n$ and $m$ number of possible targets in the horizontal and vertical coordinates. If we assume that horizontal and vertical target coordinates are not correlated, source information rate

$$H(T_x,T_y) = H(T_x) + H(T_y) = - \sum_{x=1}^{n} p(T_x) \log p(T_x) -$$

$$- \sum_{y=1}^{m} p(T_y) \log p(T_y).$$

(10)

Accuracy of the saccadic eye movements is limited. Errors after eye jump $E_e$ and $E_{oc}$ could be assumed as lost information, which could be defined by two conditional probabilities of the independent horizontal and vertical eye movement control channels $H(T/L_e)$ and $H(T/L_{oc})$. Transmitted threw oculomotor system information rate during one single saccade could be obtained as difference between source information and lost information rates

$$I_s(T,L) = H(T_x) - H(T_x/L_e) + H(T_y) - H(T_y/L_y).$$

(11)

Duration of the saccade depends on the amplitude of saccade and together with preprogramming time is about 0.25 sec. So after one jump another target could be
presented only after time interval D, which approximately is 0.5-1.5 sec. Channel information capacity during saccadic eye movements, when duration of the experiment is long enough, could be obtained by the equation

\[
C_S = \lim_{t \to \infty} \frac{1}{D} \int_{T}^{L} I_5(T, L) dT = \frac{1}{D} \left[ \log \frac{k}{0.41\sigma_x} + \log \frac{l}{0.41\sigma_y} \right].
\]

(12)

In this equation, \(k, l\) are range of amplitudes and \(\sigma_X, \sigma_Y\) are standard deviations of errors of the saccadic eye movements in the horizontal and vertical directions respectively.

**Method**

Complete data were obtained from five subjects and averaged. Eye movements in all experiments were recorded using LC Technologies, Ltd produced eye movements tracker EyeGaze System. During smooth pursuit experiments subjects tracked stimulus which moves on the computer screen by random trajectory in the 26 degrees range in the horizontal direction and 20 degrees in the vertical. One-dimension tracking was performed only in the horizontal direction and later two-dimensional tracking was conducted keeping horizontal trajectory the same. Smooth pursuit errors were calculated as differences between target position and gaze point on the screen at the same time separately in the horizontal and vertical directions. Channel information capacity \(C\) of the oculomotor system was calculated using equation 8, where frequency limit \(F\) was defined at the experimental situation when ratio between target signal standard deviation \(\sigma_T\) and error signal (noise) standard deviation \(\sigma_e\) is around 10 and calculated using Fourier transformation method. Results obtained from the experimental data and calculated by the equation 8 for all five subjects shown in the Table 1.

**Table 1.** Experimental and calculated data for five subjects during smooth pursuit

<table>
<thead>
<tr>
<th>Subject</th>
<th>F, Hz</th>
<th>(\sigma_{x, \text{deg}})</th>
<th>(\sigma_{y, \text{deg}})</th>
<th>(C_{p, \text{bytes/sec}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB</td>
<td>1.1</td>
<td>1.32</td>
<td>1.87</td>
<td>8.07</td>
</tr>
<tr>
<td>GD</td>
<td>1.0</td>
<td>1.38</td>
<td>2.02</td>
<td>5.35</td>
</tr>
<tr>
<td>RZ</td>
<td>1.2</td>
<td>0.85</td>
<td>1.13</td>
<td>9.62</td>
</tr>
<tr>
<td>AS</td>
<td>1.1</td>
<td>1.03</td>
<td>1.52</td>
<td>7.52</td>
</tr>
<tr>
<td>VL</td>
<td>1.0</td>
<td>1.43</td>
<td>2.13</td>
<td>5.17</td>
</tr>
</tbody>
</table>

For the experimental investigation and calculation of the channel information capacity of the oculomotor system during saccades one-dimension and two-dimension discrete target jumps were used. Targets are placed in 1 degree distance between each another. Range of targets amplitude is: 41 in horizontal direction and 31 in vertical. Position of switched on targets is picked up in random way with inter saccadic interval D. During experimental session inter saccadic interval was tuned as small as possible until responses to all switched on targets are correctly performed. Accuracy of saccades was measured defining standard deviations of errors \(\sigma_x\) and \(\sigma_y\), i.e. differences between target position and eye position separately for horizontal and vertical directions immediately after each jump. Results obtained from experimental data and calculated by the equation 12 for all five subjects placed in the Table 2.

**Table 2.** Experimental and calculated data of saccadic eye movements elicited to the random target jumps

<table>
<thead>
<tr>
<th>Subjects</th>
<th>D, sec</th>
<th>(\sigma_{x, \text{deg}})</th>
<th>(\sigma_{y, \text{deg}})</th>
<th>(C_{p, \text{bytes/sec}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB</td>
<td>0.85</td>
<td>0.65</td>
<td>0.78</td>
<td>6.96</td>
</tr>
<tr>
<td>GD</td>
<td>0.91</td>
<td>0.73</td>
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</tr>
<tr>
<td>RZ</td>
<td>0.76</td>
<td>0.66</td>
<td>0.72</td>
<td>7.92</td>
</tr>
<tr>
<td>AS</td>
<td>0.87</td>
<td>0.78</td>
<td>0.82</td>
<td>6.42</td>
</tr>
<tr>
<td>VL</td>
<td>0.91</td>
<td>0.85</td>
<td>0.93</td>
<td>5.81</td>
</tr>
</tbody>
</table>

**Results and discussion**

Most recent investigations on smooth pursuit and saccadic eye movements based on the oculomotor system reactions to the predictable target jumps or movement trajectories. Some of them used random direction of the movement but target trajectories were sinusoidal or linear. Those situations are artificial. Most real tasks for the visuo-oculomotor system such as searching and pointing or typing deal with random target jumps or unpredictable movement trajectories. Investigation of the visuo-oculomotor system as the information transfer channel could not be based on predictable target trajectories because of zero information rate of the predictable position or trajectory. Therefore, we used random analog time-continuous target trajectory for smooth pursuit and jumps to random position for saccades.

Obtained experimental and calculation results shows that two-dimensional channel information capacity is in the range from 5.2 to 9.6 bytes per second for smooth pursuit and from 5.8 to 7.0 for saccadic eye movements.

It is necessary to point out that results presented in this research obtained with a few assumptions.

1. Target trajectories in the horizontal and vertical directions are independent and its probability distributions fit Gaussian law.
2. Horizontal and vertical eye movements control channels are independent.
3. Smooth pursuit and saccadic eye movements errors also fit Gaussian law.
4. Duration of the experimental sessions is not longer than three minutes therefore fatigue of the subjects during experiments not evaluated.

Influence of these assumptions must be object of the further investigation.

**Conclusions**

Theoretical and experimental investigation of the channel information capacity of the human’s oculomotor system during smooth pursuit and saccadic eye movements shows successful adoption of the information theory to the bioengineering tasks.

Channel information capacity is useful parameter and could be applied defining quantity parameters not only to the visuo-oculomotor system but also analyzing information interaction and visuo-motor control in wide variety man-machine structures.

Further investigation must be concentrated on the comparison of channel information capacities of the visuo-oculomotor and other sensomotor systems such as head, arm and feet visuo-motor control subsystems. Information transfer models must be analyzed separately for visually guided and memory guided visuo-motor behaviour.

Most recent investigations on smooth pursuit and saccadic eye movements based on visuo-oculomotor system reaction to the predictable target jumps or movement trajectories do not fit real working situation. Visuo-oculomotor system of the operator during man and machine interaction and visuo-motor control is involved in searching and pointing or pursuing objects of interest with position or trajectories are random. Study was performed to evaluate capability limit of the visuo-oculomotor system to pursue not predictable target or jump to random stimulus. For this task concept of the information theory was adopted. It was supposed that human visuo-oculomotor system is information transfer channel where target movement trajectory or jump position is input or source information and elicited smooth pursuit or saccadic eye movement errors are lost information. Information rate transferred through channel is defined as difference between input and lost information. Channel information capacity of the visuo-oculomotor system is calculated as a limit of transferred information threw channel during time unit. Experimental investigation of the human visuo-oculomotor system during smooth pursuit and saccadic eye movements was performed using LC Technology, Ltd. produced eye tracker “EyeGaze System”. After testing five subjects and calculations it was found that channel information capacity of the human visuo-oculomotor system during smooth pursuit is around 8 bites per second and around 7 bites per second during saccadic eye movements. During calculations a few theoretical assumptions were accepted such as independence between horizontal and vertical neural channels of the visuo-oculomotor system and application of the Gaussian probability distribution for target and error signals. Bibl. 5 (in English; summaries in English, Russian and Lithuanian).

References

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