Large Amplitude Regime of Two Electron Stream Magnetron Frequency Multiplier

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Introduction

Nonlinear theory of two-cascade two electron stream magnetron frequency multiplier with stepped interaction space in an output cascade was represented in a paper [1]. This theory includes integro-differential equations of motion of both electron streams in an input and output cascades as well as equations of excitation of slow-wave systems (SWS) in the cascades by prebunched electron cascades. Equations of motion of the first stream in the first and second cascades are:

\[
\frac{dX_1}{d\xi} = -A_1(\xi) \frac{ch Y_1}{sh Y_{in1}} \sin X_1; \\
\frac{dY_1}{d\xi} = A_1(\xi) \frac{sh Y_1}{sh Y_{in1}} \cos X_1
\]  
(1)

(equations of motion of the first stream in the first cascade);

\[
\frac{dA_1(\xi)}{d\xi} = \frac{1}{2\pi} \frac{X^c}{X^c} \frac{sh Y_1}{sh Y_{in1}} \cos X_1 dX_0
\]  
(2)

(equation of excitation of the first SWS);

\[
\frac{dX_2}{d\xi} = -A_{n2}(\xi) \frac{ch n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \sin nX_2 - S_2 I_2^{12}; \\
\frac{dY_2}{d\xi} = A_{n2}(\xi) \frac{sh n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \cos nX_2 + S_2 I_2^{12}
\]  
(3)

(equations of motion of the first stream in the second cascade);

\[
\frac{dX_2}{d\xi} = -A_{e2}(\xi) \frac{ch n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \sin nX_2 - S_2 I_2^{12}; \\
\frac{dY_2}{d\xi} = A_{e2}(\xi) \frac{sh n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \cos nX_2 + S_2 I_2^{12}
\]  
(4)

(equations of motion of the second stream in the second cascade);

\[
\frac{dA_{e2}(\xi)}{d\xi} = R_2 \left[ \frac{1}{2\pi} \int X_2^{21} \frac{sh n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \cos nX_2 dX_0 + \frac{I_{02}}{2\pi d_0} \int X_2^{21} \frac{sh n(Y_2 + n\beta_e a)}{sh (nY_{in2} + n\beta_e a)} \cos nX_2 dX_0 \right].
\]  
(5)

(equation of excitation of the second SWS by the second electron stream).

All designations we shall use there coincide with ones in the paper [1]. The two main parameters of the frequency multiplier – gain and electronic efficiency – may be written as follows:

\[
K = 10 \log \left( \frac{P_{out}}{P_{in}} \right)_{m0} = 10 \log \frac{R_1 A_{n2}^2(\xi) R_2 / R_2 - A_1^2(0)}{P_0 \left( 1 + \frac{I_{02}^2}{I_{01}^2} d_1 + a \right)}
\]  
(6)

\[
\eta_e = \frac{(P_{out})_{m0} - (P_{in})_{m0}}{P_0} = \frac{A_{n2}^2(\xi) R_1 / R_2 - A_1^2(0)}{2\beta_e d_1 \left( 1 + \frac{I_{02}^2}{I_{01}^2} d_1 + a \right)}
\]  
(7)

Here \((P_{out})_{m0}\) and \((P_{in})_{m0}\) - output and input signal powers; \(P_0\) – power of feedstock; \(A_{n2}(\xi)\) and \(A_1(0)\) – amplitudes of output and input signals on frequencies of \(n\)-th and fundamental harmonics; \(R_2\) and \(R_1\) – interactions impedances of the second and first SWSs at a level of injection of the first stream into interaction space; \(\beta_e = \frac{\omega}{v_e}\) – phase constant of electron stream; \(v_e\) – velocity of the stream; \(d_1\) – distance between negative electrode and SWS in the input cascade; \(a\) – distance between surfaces of negative electrodes in the first and second cascades; \(I_{02}\) and \(I_{01}\) – permanent components of electron currents in cascades.

There we are going to investigate the most important dependencies such as output signal amplitude \(A_{n2}(\xi),\)
Electronic efficiency $\eta_e$ and gain $K$ as well as increase in a gain (gain increase) $\Delta K$, due to step in a negative electrode on standardized interaction space length $\xi$ and standardized height of the input cascade $\beta d_1$ at different values of the ratios $I_{02}/I_{01}$ and $R_2/R_1$, space charge density of the second stream $S_{2s}$, levels of injection of both streams $\beta_0 n_1$ and $\beta_0 n_2$ and standardized height of the output cascade $\beta d_2$.

Moreover, the bigger this ratio, the higher electronic efficiency.

It is interesting to remark that according to equations presented in a paper [1] distributions of the HFFs strengths along $d_1$ in both devices are different. In a multiplier without step longitudinal component of the field at a level of second electron stream is about twice bigger than in a stepped one. (gain of a stepped multiplier is about 3 dB less than in a smooth one). This fact might be explained by quicker exponent abatement of the longitudinal field in $d_1$ direction in the stepped device.

Now consider influence of the ratios $I_{02}/I_{01}$ on the main parameters of multiplier. Fig. 2 shows distribution of HFF amplitude $A_{x_2}(\xi)$ along interaction space for three values of $I_{02}/I_{01}$. Here current $I_{01}$ is kept up constant and the ratio grows for the sake of increase in $I_{02}$. Calculations carried out in accordance with (6) and (7) show that at fixed length of interaction space $\xi = 10$ and $I_{02}/I_{01} = 10$, gain $K=22.8$ dB and electronic efficiency $\eta_e = 74\%$. At $I_{02}/I_{01} = 5$, $K=19.5$ dB, $\eta_e = 66\%$ and at $I_{02}/I_{01} = 1$, $K=11.5$ dB and $\eta_e = 37\%$.

Fig. 2 shows that increase in $I_{02}$ gives rise to electronic efficiency as well as to gain. This may be explained by the bigger contribution of more powerful second electron stream into energetic balance of the device. On the other hand, shortening of longitudinal dimension of the multiplier makes worse bunching of the second stream.

Let a length of the first cascade be small enough, so that certain part of electrons do not reach the surface of the first SWS. Moreover, let space charge of the second stream be finite. Fig. 3 represents distribution of HFF amplitude against standardized length of the device at different values of space charge parameter $S_e$. Another parameters used for calculations are: $\beta_0 n_1 = \beta_0 n_2 = 0.5$; $\beta d_1 = 1.5$; $\beta d_2 = 3.5$. If the space charge of the second stream is vanishingly small ($S_e = 0$), electronic efficiency of the device at saturation conditions does not depend on amount of electrons landing on the first SWS. But at a decrease of this amount length of multiplier can be done less. This

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig_1.png}
\caption{Distribution of dimensionless amplitude of HFF on the 4-th harmonic along interaction space at different standardized height of the first cascade. $\beta_0 n_1=\beta_0 n_2=0.5; \beta d_1=3; \beta d_2=5,5$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig_2.png}
\caption{Distribution of HFF amplitude along the interaction space of multiplier at different values of $I_{02}/I_{01}$}
\end{figure}
possible shortening becomes especially evident at large values of $I_{o2}/I_{o1}$.

Composition of the curves at $S_2=0$ and $S_2=0.5$ shows that growth of a space charge parameter causes rise of HFF amplitude in an initial part of the first cascade. However, further increase of the amplitude stops at about $\xi \geq 3$ and at $S_2=1$ becomes equal zero at $\xi = 4.5$. When $S_2 \geq 1$ some of assumptions of the nonlinear theory [1] (adiabatic equations of motion, model with infinitely thin electron streams) become partly incorrect.

Analysis of HFF amplitude distribution taking into account existence of a space charge should include diocotron effect in second stream leading to a growth of HFF in an initial part of the second SWS (regime of relatively small deflections of second stream electrons) as well as negative influence of the second stream space charge on electrons of the first stream which displays itself at large declinations accomplished by electrons of the second stream. This negative influence is caused by Coulomb’s forces of the second stream electrons, situated under electron clusters of the first stream, and results in elongation these clusters in a direction of wave propagation. Therefore, certain part of electrons of the first stream can find themselves in an accelerating phase of HFF and takes energy from the field of a traveling wave.

**Gain. Electronic efficiency. Electrical length**

In order to evaluate merits of the stepped device with respect to the identical multiplier with smooth interaction space introduce parameter of an increase in a gain (or gain increase) as a difference between gain of stepped $K_s$ and non-stepped $K_{non}$ devices in a saturation regime, when all electrons land on a surface of the second SWS:

$$\Delta K = K_s - K_{non}.$$  

(8)

Fig. 4 represents a dependence between gain increase $\Delta K$ at saturation conditions and standardized height of the input cascade $\beta d_1$, at different ratio of interaction impedances of the second and first SWSs at a level of injection of the first stream. The interval of $R_2/R_1$ was chosen taking into account real values of this ratio which depends mainly on working frequency (in our case on number of working harmonic) and ratio of areas taken by faces of SWS segments and that occupied by HFF. One can see that step on a negative electrode in an output cascade gives similar effect as it takes place when step is made on SWS of input cascade [2]. Comparison with identical dependencies for magnetron amplifiers [3] confirms our inferences.

In Fig. 5 is shown a family of three curves corresponding to the same values of the ratio $R_2/R_1$ as in Fig. 4 representing dependence of electronic efficiency of a stepped multiplier. In a saturation regime all electrons reach the SWS of output cascade and maximum potential energy obtained by HFF from electrons is known exactly, therefore limit electronic efficiency at adiabatic approach may be appreciated as follows:

$$\eta_{e,lim} = \frac{d_1 - y_{at} + (I_{o2}/I_{o1})(d_2 - y_{at})}{d_1 + (I_{o2}/I_{o1})d_2}. $$  

(9)

Calculations carried out in accordance with (9) at $I_{o2}/I_{o1}=\text{const}$ show the increase in $\eta_e$ due to saturation of about (4-8)% with respect to optimal pre-saturation state. The opposite nature of the curves in Fig. 4 and Fig. 5 confirms the fact about existence of incompatible contradictions between gain and electronic efficiency in crossed-field microwave devices [2].

The last Fig. 6 shows how standardized length of the stepped frequency multiplier (normalized electrical length) depends on standardized height of the input cascade at the same interval of the ratio of interaction impedances. It is quite natural that at a higher interaction space in the first cascade bunching extent of the first stream is less and for achievement perfect grouping and effective interaction in the output cascade it is necessary longer second SWS.

![Fig. 3. Distribution of HFF amplitude at different values of space charge parameter of the second electron stream.](image)

![Fig. 4. Gain increase against height of interaction space in the input cascade](image)

![Fig. 5. Electronic efficiency versus height of interaction space of the first cascade](image)
Dependence between $\xi$ and $R_2/R_1$ shows that the smaller interaction impedance in the output cascade with respect to that in the input one, the worse processes of grouping and interaction between electron stream and HFF on a frequency of working harmonic. This deterioration is just reflected in Fig. 6 as a growing length of the device at a decrease of the ratio $R_2/R_1$.

Thus, analysis of the two-cascade two-electron stream magnetron frequency multiplier with a step on a negative electrode of the output cascade carried out on a base of nonlinear theory [1] shows that the device under investigation may have sufficiently higher gain and smaller electrical length at a big enough electronic efficiency in comparison with identical device with smooth interaction space. Distinguishing feature of represented nonlinear theory is possibility to describe a performance of the device in extreme saturation regime.

References


Fig. 6. Standardized length of interaction space against height of input cascade

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