Model for Simulation of Dynamic Characteristics of the System Frequency Converter – AC Induction Motor

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Introduction

An advanced electric drive is a mechatronic system, which contains both mechanical and electronic units. The frequency converter acts as an electronic unit in drives based on the AC induction motors. The software Matlab is typically used for the simulation of electric drives. However, in case when the drive includes such a complex electronic unit as a frequency converter, the electronic circuit simulation programs offer an advantage over the program Matlab. Unfortunately, as a rule the electronic circuit simulation programs do not include a suitable model of the AC induction motor. Therefore, the development of AC induction motor models adapted for the simulation using the electronic circuit simulation software becomes important. The model of the induction motor and frequency converter developed for electronic circuit simulation program PSpice is considered in the work. The results of simulation of performance characteristics of the mechatronic system frequency converter – AC induction motor using the program PSpice are presented as well.

Model of the Mechatronic System Frequency Converter – AC Induction Motor

An induction motor is a complex electromechanical system with both electromagnetic and mechanical processes to be considered. The mathematical model of the induction motor includes differential equations, ascribing both electromagnetical and mechanical processes [1, 2, 3].

\[
\begin{align*}
\dot{u}_S &= R_S \dot{i}_S + d\psi_S/dt + \omega_S j\alpha_k, \\
\psi_S &= L_S \dot{i}_S + M_{SR} i_R, \\
\dot{u}_R &= R_R \dot{i}_R + d\psi_R/dt + \psi_S j(\alpha_k - \omega), \\
\psi_R &= L_R \dot{i}_R + M_{SR} \dot{i}_S,
\end{align*}
\]

where \( \dot{u}_S, \dot{u}_R \) – the supply phasor voltages of stator and rotor windings; \( \dot{i}_S, \dot{i}_R \) – phasor currents of stator and rotor windings; \( \psi_S \) and \( \psi_R \) – are phasor flux linkages of stator and rotor windings; \( R_S, R_R \) – resistances of stator and rotor windings; \( L_S, L_R \) – inductances of stator and rotor windings; \( M_{SR} \) – the mutual inductance between stator and rotor windings; \( \omega \) – the angular speed of the rotor; \( \alpha \) – the angular speed of reference frame.

Windings of the stator and rotor are coupled by the mutual inductance. To develop a model of the induction motor for the program PSpice, the inductances of stator and rotor windings should be expressed in terms of self-induction, which is related to the flux, linking the winding and mutual inductances as:

\[
L_S = L_{SR} + M_{SR}, \quad L_R = L_{RS} + M_{SR},
\]

where \( L_{SR}, L_{RS} \) – the self inductance of stator and rotor windings.

Substituting (2) into (1) and expressing phasors by their projections give a set of motor equations for the \( x \) axis:

\[
\begin{align*}
\dot{u}_{s}\delta &= R_{s}\delta i_{s} + L_{s}\frac{di_{s}}{dt} + M_{SR}\left(\frac{di_{s}}{dt} + \frac{di_{r}}{dt}\right) - \omega_S \alpha_k, \\
\psi_{s}\delta &= L_S \frac{di_{s}}{dt} + M_{SR} i_R, \\
\dot{u}_{r}\delta &= R_{r}\delta i_{r} + L_{r}\frac{di_{r}}{dt} + M_{SR}\left(\frac{di_{r}}{dt} + \frac{di_{s}}{dt}\right) - \psi_{s}\delta (\alpha_k - \omega), \\
\psi_{r}\delta &= L_R \frac{di_{r}}{dt} + M_{SR} \frac{di_{s}}{dt}.
\end{align*}
\]

In the same way is derived system of equations for variables, aligned along \( y \) axis:

\[
\begin{align*}
\dot{u}_{s}y &= R_{s}i_{s} + L_{s}\frac{di_{s}}{dt} + M_{SR}\left(\frac{di_{s}}{dt} + \frac{di_{r}}{dt}\right) + \psi_{s}y \alpha_k, \\
\psi_{s}y &= L_S i_S + M_{SR} i_R, \\
\dot{u}_{r}y &= R_{r}i_{r} + L_{r} \frac{di_{r}}{dt} + M_{SR}\left(\frac{di_{r}}{dt} + \frac{di_{s}}{dt}\right) + \psi_{r}y (\alpha_k - \omega), \\
\psi_{r}y &= L_R i_R + M_{SR} i_S.
\end{align*}
\]
The expressions of equation sets (3) and (4) can be presented as closed loops with appropriate electric elements using the program PSpice. Each equation corresponds to one loop.

The loops presenting the x axis are modeled according to (3), and those presenting the y axis according to (4). One loop corresponds to the stator, the other – to rotor circuits. The last terms in the equations, which include the magnetic flux, can be presented using voltage sources connected to the corresponding loops. The sources take into account the influence of loops presented on a different axis to each other, e.g. the influence of loops on the x axis to the loops on the y axis.

Magnetic fluxes \( \psi_{Sx} \), \( \psi_{Rx} \), \( \psi_{Sy} \), and \( \psi_{Ry} \) are modeled by two current controlled voltage sources connected in series, the gain of which is equal to the corresponding inductance.

Mechanical part is described by equation:

\[
\omega = \frac{1}{J_R} \int T_{em} dt = \frac{pM_{SR}}{J_R(L_R L_S + M_{SR})} \left[ \psi_S \times \dot{\psi}_R \right] dt ,
\]

where \( J_R \) – the inertia of the motor and drive; \( T_{em} \) – the developed electromagnetic torque.

The PSpice is a program specialized for the simulation of electronic circuits [4], therefore, all mechanical variables, such as speed and torque must be replaced by electrical variables. The current in the developed model replaces the torque, the capacity – inertia and the voltage – speed.

The equivalent circuit of the induction motor model is shown in Fig.1 [7]. Each nod has its number, which is presented in oval label. The labels of nodes are required for creating of motor model. The further explanation uses node label.

![Fig. 1. Equivalent circuit of the induction motor model](image)

The part of the circuit with nodes 1 – 7 corresponds to the set of equations (3) and the part of the circuit with nodes 8 – 14 to the set of equations (4). Sources \( u_{Rx} \) and \( u_{Ry} \) are used to establish currents of the rotor.

As it was mentioned, magnetic flux linkages are calculated using the circuit with two current controlled voltage sources connected in series and choosing the gains, equal to the corresponding inductances. This connection produces voltages at nodes 16, 18, 21 and 23 (Fig.1) equal to the corresponding fluxes. The resistance Ra between nodes 16, 18, 21, 23 and ground has any influence on the operation of the circuit and is used only to create closed loops.

The torque is calculated according to magnetic fluxes using (5), multiplying it by the adjustable current source. The current of the voltage controlled current source is equal to one component of flux, voltage controlled resistance – to the other. In this way the voltage drop across the adjustable resistance is equal to the product of both components. The controlled current source and
resistance connected between node 25 and ground (Fig.1) generate the product $\psi_S \psi_{RC}$. An analogous circuit connected between 26 and ground gives the product $\psi_S \psi_{RC}$. The torque is equal to the current of the source, connected between node 27 and ground. This source is controlled by the voltage difference between 25 and 26 nodes.

The rotor inertia is modeled using the circuit that contains capacitance connected to node 27 and the current source, which corresponds to the torque. The voltage of node 27 by its numerical value is equal to the angular speed of the rotor.

Voltages of nodes 19 and 24 are equal to the product of magnetic flux and difference in the speed of reference frame and the rotor angular speed. These products are formed in the same way as for calculation of the torque.

The algorithm for creating the frequency converter output voltage is selected according to the requirements of the drive performance parameters, for example, allowed pulsation of the speed, simplicity of the algorithm, contents of harmonics [5]. Let us analyze the frequency converter operating according to the well-known six-step algorithm. Following this algorithm the period of the voltage is divided into six intervals and the voltage is constant inside of each interval.

The developed model of the motor contains just two phases, therefore the three-phase supply source should be changed by the two-phase one.

The phasor of supply voltage is calculated as:

$$\bar{u}_S = \frac{2}{3} \left[ u_A + u_B \exp\left( j \frac{2\pi}{3} \right) + u_C \exp\left( -j \frac{2\pi}{3} \right) \right], \quad (6)$$

where $u_A$, $u_B$, $u_C$ – instantaneous phase voltages.

Supply voltages of the two-phase equivalent electrical machine are calculated using expressions:

$$u_{Sx} = \text{Re}(\bar{u}_S) \quad \text{and} \quad \bar{u}_{Sy} = \text{Im}(\bar{u}_S). \quad (7)$$

The values of three-phase motor phase voltages and corresponding two-phase motor voltages for each of 6 intervals are given in Table 1.

<table>
<thead>
<tr>
<th>Time</th>
<th>$u_A$</th>
<th>$u_B$</th>
<th>$u_C$</th>
<th>$u_{Sx}$</th>
<th>$u_{Sy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – T/6</td>
<td>$U_{max}$</td>
<td>$U_{max}$</td>
<td>0</td>
<td>$U_{max}/3$</td>
<td>$U_{max}/3$</td>
</tr>
<tr>
<td>T/6 – T/3</td>
<td>0</td>
<td>$U_{max}$</td>
<td>0</td>
<td>$-U_{max}/3$</td>
<td>$U_{max}/3$</td>
</tr>
<tr>
<td>T/3 – T/2</td>
<td>0</td>
<td>$U_{max}$</td>
<td>$U_{max}$</td>
<td>$-2U_{max}/3$</td>
<td>0</td>
</tr>
<tr>
<td>T/2 – 2T/3</td>
<td>0</td>
<td>0</td>
<td>$U_{max}$</td>
<td>$-U_{max}/3$</td>
<td>$-U_{max}/3$</td>
</tr>
<tr>
<td>2T/3 – 5T/6</td>
<td>$U_{max}$</td>
<td>0</td>
<td>$U_{max}$</td>
<td>$U_{max}/3$</td>
<td>$-U_{max}/3$</td>
</tr>
<tr>
<td>5T/6 – T</td>
<td>$U_{max}$</td>
<td>0</td>
<td>0</td>
<td>$2U_{max}/3$</td>
<td>0</td>
</tr>
</tbody>
</table>

The pulse voltage generator of the PSpice component library is used to create the values written in the last two rows of Table 1, used for the case of two-phase voltage. As the standard PSpice generator gives a constant level signal during the period, the three generators connected in series are used to create the voltage $u_{Sx}$ and two generators – to create $u_{Sy}$.

**Results of Simulation**

By employing the developed models on the basis of the program PSpice, the simulation of the mechatronic system frequency converter – AC induction motor type 4A100L2 was provided. The results are presented in Fig. 2–4.

![Fig. 2. Current in the stator winding of phase A at starting](image)

The frequency converter produces the pulse output voltage created by the output switches. The change in the state of switches causes the transient. It is seen in Fig. 2 that during the transient the instantaneous values of the current in the motor windings and, consequently, in the switches increase. The increased value of the instantaneous stator current causes a larger torque. This fact induces the ripples (pulsations) of the developed torque (Fig. 3).

![Fig. 3. Starting transients of the motor torque](image)

![Fig. 4. Starting transients of the motor speed](image)
The transient process of the speed is shown in Fig. 4. The torque pulsations are damped by the mechanical system of the motor and do not influence the motor speed. The dashed line shows transients of the motor speed supplying it with the sine voltage. The duration of the starting process of the motor supplied with the frequency converter is longer due to the reverse component of magnetic field, which develops the braking torque.

Conclusions

1. The implementation of the developed model of the motor allows us to present the mechatronic system frequency converter – AC induction motor, which includes the electrical and mechanical parts as a unified model that can be realized using the program PSpice.

2. The instantaneous values of the phase current of the motor supplied with the frequency converter increase significantly during the switching of the motor windings. This fact should be taken into account during the development of the frequency converter power stage.

3. The torque developed by the motor supplied with the frequency converter oscillates and this fact should be taken into account during the design of the mechanical part of the mechatronic system.

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References


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