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Performance of Nonuniform PAM Constellations for Gaussian Channel

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Introduction

In this paper, the Gaussian source quantization is done by using three different quantization methods[1,2]. The Gaussian source is an optimal information source, with average power as a constraint. We analyzed Pulse Amplitude Modulation (PAM) signal constellation obtained by means of different quantization methods for Gaussian channel. We quantized the Gaussian source applying the method given in [2], which gives the equiprobable appearance of signal constellations points, then, we quantized the same source using Lloyd-Max's iterative quantization method and finally we performed Re-optimal quantization [1]. Re-optimal quantization is utilised because of minimization of average distortion for transmission through binary symetric channel [1], but we utilize Re-optimal quantization because of design of signal constellation for transmission through Gaussian channel in this paper. The first two methods are designed for noise-free transmission whereas the noise influence is considered by the third method. Also this paper presents the analysis method for the determination of the optimal signal constellation points disposition after quantization, when the error probability is minimized under the power constraint[3,4].

The combined method for the signal constellation design is considered in this paper. This method has two steps: the first step is the application nonuniform quantization (quantization methods of the noiseless transmission) and the second one is the nonlinear point transformation. The quantization methods of the noiseless transmission for the signal constellation design are considered in paper [5]. The nonlinear transformation is considered in [6] taking, the equiprobable transmission and great constellation points into account. In this paper, a method for determining, the conditional minimum is used. The minimum of the average error probability caused by the channel noise (in dependence on the information source is discrete or continual) is determined [3,4]. The average transmission power is a constraint in determining the conditional minimum. The analysis for the sixteen points PAM signal constellation is presented.

Nonuniform quantization of Gaussian source

In this section, a Gaussian source quantization, using three different methods of nonlinear quantization, is performed. The analisysis is done on the basis of the signal constellation gain. We quantized the Gaussian source applying the method given in [2], which gives the equiprobable appearance of signal constellation points. After that, the same source is quantized using Lloyd-Max's iterative quantization method and finally we performed Re-optimal quantization.

Using the first method(I) we obtain the decision levels (r_k) and reconstruction levels (m_k) given by:

$$\frac{1}{L}=\frac{1}{\sqrt{2\pi P}}\int_{r_k}^{r_{k+1}}\exp(-x^2/2P)dx\,, \qquad (1)$$

$$m_i=\frac{L}{\sqrt{2\pi P}}\int_{r_k}^{r_{k+1}}x\exp(-x^2/2P)dx\,; \qquad (2)$$

where L-is the number of levels, P-is average power.

The second method (II) is Max's iterative quantization which leads to r_k and m_k given by [1]:

$$r_{k,opt}=\frac{1}{2}(m_{k,opt}+m_{k-1,opt});k=2,3,...,L,$$
$$r_{1,opt}=0, r_{L+1,opt}=\infty\,; \qquad (3)$$

$$m_{k,opt}=\frac{\int_{r_{k,opt}}^{r_{k+1,opt}} rp_r(r)dr}{\int_{r_{k,opt}}^{r_{k+1,opt}} p_r(r)dr};k=1,2,...,L\,; \qquad (4)$$

where is $p_r(r)$-probability density function of the source.

In order to investigate the quantization influence on symbol error probability for the Gaussian channel, Re-optimal quantization(III-method) for model given on Fig. 1 is used.

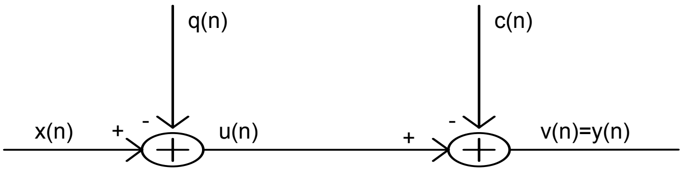

Fig. 1. Transmission of quantized amplitudes through a noisy channel

The reconstruction and decision levels for this model are obtained by the error variance optimization.

$$\sigma_r^2 = \sum_{j=1}^{L}\sum_{k=1}^{L} P_{k,j} \int_{r_k}^{r_{k+1}} (r-m_j)^2 p_r(r)dr \ . \qquad (5)$$

$$P_{k,j} = \int_{r_j}^{r_{j+1}} \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{(r-m_k)^2}{2\sigma_s^2}\right)dr \ ;$$

$P_{k,j}$ -is channel matrix elements, q(n)-is quantization error, c(n)-is noise, σ_s^2-is average noise power.

Decision levels (r_k) and reconstruction levels (m_k) can be calculated from [1]:

$$r_{k,opt} = \frac{\sum_{j=1}^{L} m_{j,opt}^2 (P_{k-1,j} - P_{k,j})}{2\sum_{j=1}^{L} m_{j,opt}(P_{k-1,j} - P_{k,j})} \ ; \qquad (6)$$

$$m_{k,opt} = \frac{\sum_{j=1}^{L} P_{j,k} \int_{r_{j,opt}}^{r_{j+1,opt}} r p_r(r)dr}{\sum_{j=1}^{L} P_{j,k} \int_{r_{j,opt}}^{r_{j+1,opt}} p_r(r)dr} ; k = 1,2,\ldots,L \qquad (7)$$

The average error probability and power of one-dimensional signal constellation can be calculated as

$$P_e = \sum_{j=1}^{N} P_j 1/(\sqrt{2\pi}\sigma_n) \int_{m_j - y_{jl}}^{m_j + y_{jd}} \exp(-(r-m_j)^2/(2\sigma_n^2))dr, P_{av} = \sum_{j=1}^{N} P_j m_j^2,$$

$$y_{jl(d)} = \frac{m_{j(j+1)} - m_{j-1(j)}}{2} + \frac{\sigma_n^2}{m_{j(j+1)} - m_{j-1(j)}} \ln \frac{P_j}{P_{j-1(j+1)}}, P_j = \int_{r_j}^{r_{j+1}} p_r(r)dr,$$

$$P = 2\sum_{j=1}^{L} P_j m_j^2 \ ; \qquad (8)$$

where σ_n^2 is an average noise power, m_j is the j-th representation level, L is a number of levels

$$P_j = \int_{r_j}^{r_{j+1}} p_r(r)dr,$$

.

$p_r(r)$ is a probability density function of the Gaussian source, and r_k are the decision levels. y_{jl} and y_{jr} are defined in the following way (see Fig. 2.).

$$y_{jl} = \frac{m_j - m_{j-1}}{2} + \frac{\sigma_n^2}{m_j - m_{j-1}} \ln \frac{P_j}{P_{j-1}}, \ y_{1l} = -\infty \ .$$

$$y_{jr} = \frac{m_{j+1} - m_j}{2} + \frac{\sigma_n^2}{m_{j+1} - m_j} \ln \frac{P_j}{P_{j+1}}, \ y_{Lr} = +\infty \ ,$$

j=1, 2, ... , L.

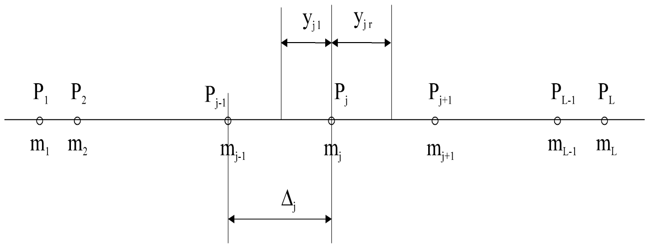

Fig. 2. One-dimensional signal constellation and a decision region example

The analysis for the sixteen points PAM signal constellation is presented. The error probability dependence on signal-to-noise ratio *SNR*, for different quantization methods, is shown in Fig. 3.

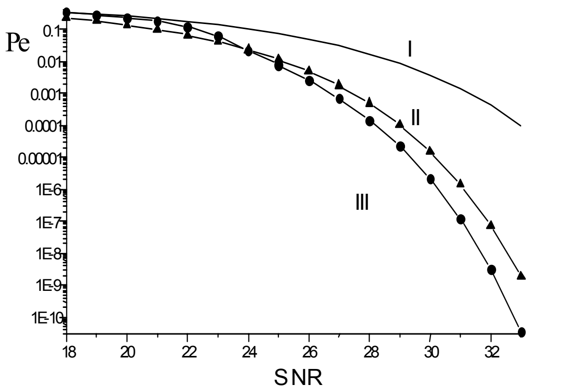

Fig. 3. Error probability for nonuniform constellations is obtained using three different quantization methods

The quantization constellation gain (G) is defined as difference between SNR for nonuniform constellation (SNRn) and SNR for uniform constellation (SNRu), when the error probabilities (P_e) are equal and the bit rates are almost equal [4] :

$$G(Pe) = SNR_n(Pe) - SNR_u(Pe) \ . \qquad (9)$$

The gains for the previous three quantization methods are shown on Fig. 3.

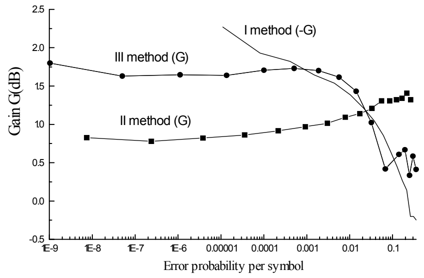

Fig. 4. The gain (G) for three quantization methods

The negative gain (-G) for the first method is evident from Fig. 4. G gain depends very much on quantization method.

Nonlinear transformation of PAM signal constellation points

Better performance of PAM signal constellations obtained by method I and II we can improve using nonlinear transformation of PAM signal constellation. The error probability PAM signal constellation decreasing obtained by using the method in paper [4]. The error probability for the first quantization method, is illustrated in Fig. 5.

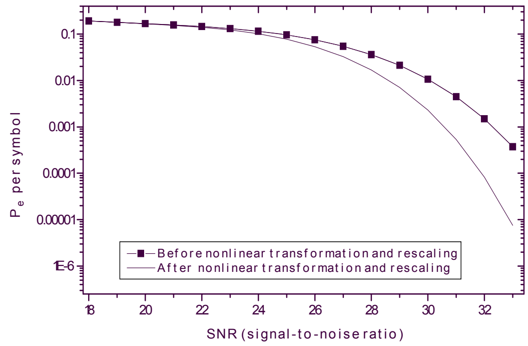

Fig. 5. Error probability per symbol before and after nonlinear transformation and rescaling for the first quantization method

Observing graphic for probability error after nonlinear transformation on signal constellation obtained by method I and before nonlinear transformation (Fig 3) we can conclude then PAM signal constellation obtained by method II is better. Therefore we will calculate nonlinear transformation gain only for PAM constellations obtained by method II now.

The nonlinear transformation gain is defined as a difference between SNR before (SNR_{before}) and SNR after (SNR_{after}) the nonlinear transformation (G) of the signal constellation, when the error probabilities (P_e) are equal

$$G(Pe) = SNR_{before}(Pe) - SNR_{after}(Pe) = \\ = 10\log\frac{P_{av}}{\sigma^2} - 10\log\frac{P_{avopt}}{\sigma_{opt}^2} = 10\log\left(\frac{P_{av}}{P_{avopt}}\cdot\frac{\sigma_{opt}^2}{\sigma^2}\right) \quad (10)$$

is a gain due to the nonlinear transformation. The gain obtained by the nonlinear transformation of the signal constellation points initially generated by the second quantization method of the Gaussian source are shown in Fig. 6. The signal constellation design is done separately for each SNR value, and the constellation having the maximal gain is obtained.

The aim of paper [6] is the analysis of nonlinear transformation named warping transformation of the uniform constellation for equiprobable transmission where the solution is obtained using the continual approximation. An increase in nonlinear transformation gain with a decrease of SNR for equiprobable constellations may be observed as in [6]. On the contrary, with nonequiprobable nonuniform constellations, the gain decreases with SNR decreasing.

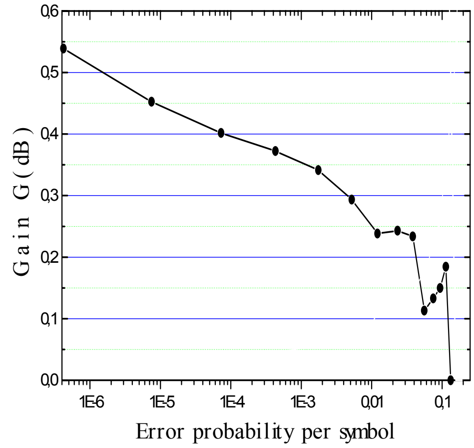

Fig. 6. Gain (G) after nonlinear transformation

Analysis

We will make comparison gains for Re-optimal quantization method and combined method for fixed probability error. From this comparison we will decide which method for design PAM signal constellation is better. The total gain (for combined method) is equal to the sum of the quantization gain and gain caused by the nonlinear transformation. For probability errors which are from practical importance $P_e=10^{-5}$ and $P_e=10^{-6}$ Re-optimal quantization method is better than combined method approximately 0.3 dB. Still one advantage of this method is in total smaller complexity at design PAM signal constellation.

Conclusion

In this paper, the Gaussian source quantization is done by usage of three different quantization methods. The gain depends on probability density function. So, the Gaussian source is used and the quantization method influence on the gain is analyzed . The effective method for PAM system signal constellation construction for transmission over the Gaussian channel is also presented in this paper. The nonlinear transformation for determining the conditional minimum under the average power constraint is used. Re-optimal quantization method for design PAM signal constellation is better and simpler than existing methods.

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Gausiniams kanalams rikiuoti siūloma taikyti skirtingus metodus. Analizuojama impulsų amplitudės moduliacija leidžia optimaliai apskaičiuoti kanalo charakteristikas. Analizuojamu metodu galima apskaičiuoti minimalias paklaidas bei įvertinti įvairias kanalo charakteristikas. Pateikiami kanalų paklaidų teoriniai ir eksperimentiniai rezultatai. Il. 6, bibl. 6 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

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Предлагаются разные методы анализа характеристик гауссовских каналов. При использовании импульсной амплитудно-импульсной модуляции рассчитаны оптимальные характеристики, в том числе рассчитаны минимальные погрешности. Описываются теоретические и экспериментальные результаты анализа гауссовских каналов. Ил. 6, библ. 6 (на английском языке; рефераты на литовском, английском и русском яз.).