ISSN 1392 - 1215 ELEKTRONIKA IR ELEKTROTECHNIKA. 2004. Nr. 5(54)

T 170 ELEKTRONIKA

Synthesis of the Matched Load for the Lumped Lossy Transmission Line

E. Hermanis
Laboratory "Vide"
p.o. Jaunjelgava, LV-5134, Latvia, phone: 271-51-52864, e-mail: evalds.h@apollo.lv

Lossy transmission line

In an infinitely long line there are only forward traveling waves and no reflected waves. This is however also true for a line terminated with its characteristic impedance. A line is called a matched line when the load impedance is equal to the characteristic impedance. In the zero-loss circuit characteristic impedance is active, and the reflected signal can be eliminated by the appropriate resistor.

From the theory of electrical chains the lossy transmission line charecteristic impedence is described by the formula,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}\ , \quad (1)$$

where L - (series) inductance per unit length (H/m); C - (shunt) capacitance per unit length (F/m); R - (series) resistance per unit length (Ω/m, results from conductor loss); G - (shunt) conductance per unit length (S/m, results from dielectric loss); ω - angular frequency.

In this case the synthesis of the matched load becomes the difficult problem. It cannot be reduced to the form of final rational fraction. Consequently, this characteristic impedance has no passive two-terminal network of final order; it is possible to create only some approximations. In higher frequencies, with additional losses, the formula gets even more complicated

$$Z_0 = \sqrt{\frac{r_{dc} + r_{ac}\sqrt{f} + j\omega L_{eff}}{g_{dc} + g_{ac} f + j\omega C_{eff}}}\ . \quad (2)$$

For the majority of practical dielectrics the DC (direct current) conductivity g_{dc} is so low, that it can be disregarded. If the microstrip is etched on a dielectric, when the base layer is semiconductor, then at higher frequencies the AC (alternating current) conductivity g_{ac} becomes important. [1,2,3].Usually the coefficient of g_{ac} in the formula is much smaller than coefficient r_{ac}. (example: r_{ac}=6.9e-5 Ω/(m·Hz$^{1/2}$), g_{ac}= 1.39e-11 S/(m·Hz)) [4]). But the frequency with a coefficient of r_{ac} in the numerator is under the square root, while in the denominator with g_{ac} it is to the first degree. This means that with an increase in frequency the corresponding denominator component increases faster than the numerator component. At 1 GHz both loss components become approximately equal. (see Fig. 4 V [4]). This occurs even with the microstrip on the semiconductor base layer (see Fig.5a, [5]). Inductance and capacity are the prevailing values. This facilitates the matching of a line to load considerably, over a wide frequency range. (The load can be active even up to 1 GHz).

Matching the capacitive input of high-resistance amplifiers is also solved ideally [by 6]. The load is formed from two parallel chains with like time constants, where one of them is the amplifier input capacitance. If the load forms two oscillatory circuits, then together with the matched load it is possible to attain also an improvement in the waveform [7].

Distortionless line

Such lines are used for signal delay and also as the base signal generator to synthesize needed different signal forms. Let us examine a finite member chain consisting of the RLC-ladder network (see Fig. 1, RLC- link). Formula (1), describes the characteristic impedance of a ladder network, where in this case the DC chain resistance R and leakage conductace G are significant. Since any inductance has significant internal resistance, then it is expedient to add leakage conductance for the purpose to build line a without the distortions. This occurs in the frequency band $[0,\ 1/(\pi\sqrt{LC})]$, if $RC=GL$.

In this strip the characteristic impedance can be considered constant, but in the unlimited frequency band formula (1) applies. If we consider equation (2) then we can eliminate one parameter from the formula, for example inductivity L. The formula thus is simplified to:

$$Z_0 = \tfrac{1}{2}\sqrt{r(4R + r(1 + j\omega CR)^2)}\ , \quad (3)$$

where R *is* the in parallel connected resistance to the capacitor and r is the in series inductance connected resistance.

Synthesis of loads

It is well known that in the endless chain only the traveling wave occurs. But each real chain must be finite. In order to eliminate the reflection the chain must be qualitatively coordinated with the load. The difficulty of this task is determined by the fact that formula (4) cannot be represented in the form of a rational fraction, therefore, there is no such two-terminal network, which would have input resistance equal to (4). It is possible only to attempt to build acceptable approximations. The higher the damping in the chain, the less the signal reflected and it to more easily muffled. For the load it is possible to use the following circuits, which qualitatively do not differ from the link chains (Fig. 1 a) or b)).

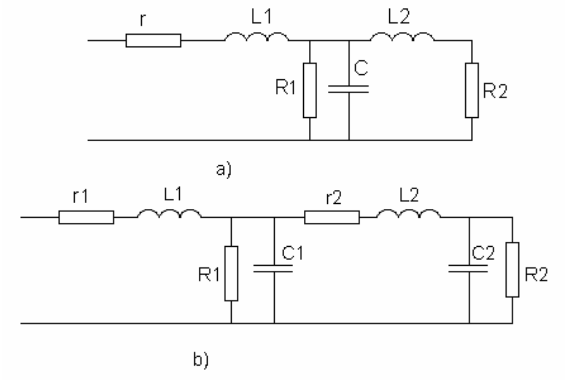


Fig. 1. Load of the RLC circuit

One can also reach a perceptible effect just by using one component of the load chain (see Fig. 1 a), if parameters a_k and b_j of input impedance

$$Z_L = \frac{a_3 p^3 + a_2 p^2 + a_1 p + a_0}{b_2 p^2 + b_1 p + 1} \quad (4)$$

are selected for the purpose of approximation Z_0, where $p=j\omega$, ω - angular frequency, $a_3=(L_1L_2R_1C)/(R_1+R_2)$, $a_2 = (R_1CrL_2+L1R_2CR_1+L_1L_2)/(R_1+R_2)$, $a_1=(R_1rR_2C+L_2R_1 + L_1R_1+L_2r+L_1R_2)/(R_1+R_2)$, $a_0=(rR_2+rR_1+R_1R_2)/(R_1+R_2)$, $b_2 = R_1CL_2/(R_1+R_2)$, $b_1=(R_2R_1C+L_2)/(R_1+R_2)$.

The input impedance of the two-terminal network Fig.1b the numerical calculations is not necessity to be expressed in the compact form: $Z_L=r_2+pL_1+Z_4$, *where* $Z_4=Z_2Z_3/(Z_2+Z_3)$, $Z_2=r_1+pL_2+Z_l$, $Z_3=R_1/(pC_1R_1+1)$, $Z_l=R_2/(pR_2C_2+1)$. Expression for Z_L as in the previous case, can lead to the fraction,

$$Z_L = \frac{a_0 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4}{1 + b_1 p + b_2 p^2 + b_3 p^3}, \quad (5)$$

but then some expressions for *a* and *b* would prove to be too long. So let us further limit ourselves only to the numerical calculations on the computer, then it is possible to manage without these expressions for *a* and *b*.

In order to muffle the signal reflections in a line one has to calculate the load (Fig.1b) parameters r_1, r_2, R_1, R_2, C_1, C_2, L_1, L_2, so as to best satisfy the equality $Z_0 \approx Z_L$ both on module and on phase. For this purpose the following functional was developed

$$\Phi = \sum_{\omega} |(\mathrm{Re}(Z_0) - \mathrm{Re}(Z_L)| \times \sum_{\omega} |\mathrm{Im}(Z_0) - \mathrm{Im}(Z_L)|. \quad (5)$$

This represents the resulting error of the module and phase multiplication for all frequencies in the designated range. It is necessary to find the function $\boldsymbol{Z_L}$, which would minimize the functional.

In order to find the minimum the MATLAB 'fminsearch' operator was used, which makes it possible to find the minimum of the function from the parameters, which does not exceed 10 parameters. Diagram Fig.1 b contains only 8 parameters and does not exceed the limit.

As a practical example a chain was selected, in which according to Fig. 1 L=33μH, C=100pF, R=33kΩ, r=10Ω. As shown equality (3) is satisfied (L/r=3.3μs, CR=3.3μs). For the minimization of function (5) one should select a starting point. The difficulty arises because the function has many minimums, and it is necessary to find the deepest minimum in the 8- dimensional parameter space. MATLAB language algorithm 'fminsearch' will find only the minimum on the slope where the desired start point was located.

The following considerations were used with the selection of starting point. Since the load circuit is similar to the chain circuit, then the last resistor R_2 should be close to the value for the load resistor for low frequencies, i.e. $R_2 = \sqrt{L/C}$. Consequently R_2 = 574.5Ω, and initial R, L and C are such, as in the series chain link. It was impossible to find a deeper minimum within the parameters of the function by varying the starting point.

As a result of minimization of function (5) the following parameters were obtained: L_1=16.7μH; L_2=27.6μH; C_1=96.3pF; C_2=40.9pF; r_1=15.9Ω; r_2=2Ω, R_1=43.67kΩ; R_2=573.7Ω; It is evident that L_1, C_1 and R_2 compared to the start point are modified very little; basic changes came from C_2, R_1, r_1, r_2. Resulting load parameter values are dependent also on the frequency range where said minimization was accomplished.The low frequency limit according to the theory of lines with the lumped parameters, without signal distortion, is equal $\omega_{\max} = \frac{2}{\sqrt{LC}}$. In this case we obtain $f_{\max}$= 5.54 MHz. For the minimization of the function a two-fold wider frequency interval was selected, i.e. 11.08 MHz.

For the similarity of the absolute and angular impedence characteristic graphs lets look at Fig. 2 and Fig. 3. The absolute values of the impedance of an ideal load does have the sharp minimum, with which the phase response does accomplish a smooth change on π/2. (if the chain did not have any losses, then the phase slope would show a sharp rise). It should be obvious that the more numerous load sections you have, the less deviation from the ideal.

Transient processes

The quality of load approximation can be judged from the transient responses of the individual link chain capacitors. For lines without the distortions the transient responses of capacitors are described by formula (6),

$$h_k(t) = \int_0^t \omega_{\max} \exp(-x/\tau) J_{2k-1}(\omega_{\max} x) dx \quad (6)$$

containing Bessel functions of the 1st type, in which τ=CR=L/r. Formula (6) gives us a means to calculate the transition process in the case of an ideal load, and then compare it to the transition process of an approximated load. From the size of the error we can judge the quality of the approximated error. Without the aid of the formula, observing only the process of the approximated load, the error could not be evaluated.

Curves to Fig. 4 and Fig. 5 are obtained by the numerical simulation of transient processes on the capacitor of the 6th component of the line containing 8 sections. With the ideal load according to (6) k=6, but Bessel function index is 11. On other link capacitors, further from the load, the case is similar. The only difference being in signal delay time and its amplitude. Since components 7 and 8 are reconstructed under the load, then the 6th signal component is maximally distorted. Since the 6^{th} component is next to the load the influence of its reflected signal is greater.

For the simulation of processes Simulink language MATLAB was used. An 8 step chain mathematical line model was devised. In this case the differential equation of: $\begin{matrix} dx/dt = Ax(t) + Bv(t) \\ y(t) = Cx(t) + Dv(t) \end{matrix}$ describes the condition and contains matrices with the constant coefficients

$$A = \begin{Vmatrix}
-r_1/L_1\ 0\,0\,0\,0\,0\,0\,0\,0\ -1/L_1\ 0\,0\,0\,0\,0\,0\,0 \\
0\ -r_2/L_2\ 0\,0\,0\,0\,0\,0\,0\,1/L_2\ -1/L_2\ 0\,0\,0\,0\,0\,0 \\
0\,0\ -r_3/L_3\ 0\,0\,0\,0\,0\,0\,0\,1/L_3\ -1/L_3\ 0\,0\,0\,0\,0 \\
0\,0\,0\ -r_4/L_4\ 0\,0\,0\,0\,0\,0\,0\,1/L_4\ -1/L_4\ 0\,0\,0\,0 \\
0\,0\,0\,0\ -r_5/L_5\ 0\,0\,0\,0\,0\,0\,0\,1/L_5\ -1/L_5\ 0\,0\,0 \\
0\,0\,0\,0\,0\ -r_6/L_6\ 0\,0\,0\,0\,0\,0\,0\,1/L_6\ -1/L_6\ 0\,0 \\
0\,0\,0\,0\,0\,0\ -r_7/L_7\ 0\,0\,0\,0\,0\,0\,0\,1/L_7\ -1/L_7\ 0 \\
0\,0\,0\,0\,0\,0\,0\ -r_8/L_8\ 0\,0\,0\,0\,0\,0\,0\,1/L_8\ -1/L_8 \\
0\,0\,0\,0\,0\,0\,0\,0\ -(r_1+R_L)/L_9\ 0\,0\,0\,0\,0\,0\,0\,1/L_9 \\
1/C_1\ -1/C_1\ 0\,0\,0\,0\,0\,0\,0\ -1/C_1/R_1\ 0\,0\,0\,0\,0\,0\,0 \\
0\,1/C_2\ -1/C_2\ 0\,0\,0\,0\,0\,0\,0\ -1/R_2/C_2\ 0\,0\,0\,0\,0\,0 \\
0\,0\,1/C_3\ -1/C_3\ 0\,0\,0\,0\,0\,0\,0\ -1/R_3/C_3\ 0\,0\,0\,0\,0 \\
0\,0\,0\,1/C_4\ -1/C_4\ 0\,0\,0\,0\,0\,0\,0\ -1/R_4/C_4\ 0\,0\,0\,0 \\
0\,0\,0\,0\,1/C_5\ -1/C_5\ 0\,0\,0\,0\,0\,0\,0\ -1/R_5/C_5\ 0\,0\,0 \\
0\,0\,0\,0\,0\,1/C_6\ -1/C_6\ 0\,0\,0\,0\,0\,0\,0\ -1/R_6/C_6\ 0\,0 \\
0\,0\,0\,0\,0\,0\,1/C_7\ -1/C_7\ 0\,0\,0\,0\,0\,0\,0\ -1/R_7/C_7\ 0 \\
0\,0\,0\,0\,0\,0\,0\,1/C_8\ -1/C_8\ 0\,0\,0\,0\,0\,0\,0\ -1/R_8/C_8
\end{Vmatrix}$$

$$B^t = \| 1/L_1\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \|,$$

$$C = \| 0\,0\,0\,0\,0\,0\,0\,0\,R_L\ 0\,0\,0\,0\,0\,0\,0\,0 \|,\ D = \| 0 \|,$$

R_L – the effective resistance of load.

This method makes it possible to simulate the transition process for the different parameter values.

In the case of a resistive load regardless of the index all resistors have a value of: R=33 kΩ, capacitors C=.100pF, and inductors L=33μH, with exception of L1=.L9=.L/2. Series resistors also have similar values: r=10Ω except for r1=.r8=.r/2. in this case RL=574.5Ω.

In the case of a complex load the values of the following parameters changed: L_7=.L/2+16.7; L_8=27.6; L_9 = 10^6; R_L=10^5. They were developed in the minimization process. The value of L_9 and R_L is not critical. Their values were selected in order to disregard the current in resistor R_L, and simultaneously the time constant L_9/R_L would not limit the numerical solution of the differential equation. C_7=9.64e-5; C_8=4.085e-5; r_7=r/2+15.9; r_8=2. All units of induction are expressed in μH, for capacitors in μF, and for resistors in Ω, but the time measurement is the microsecond.

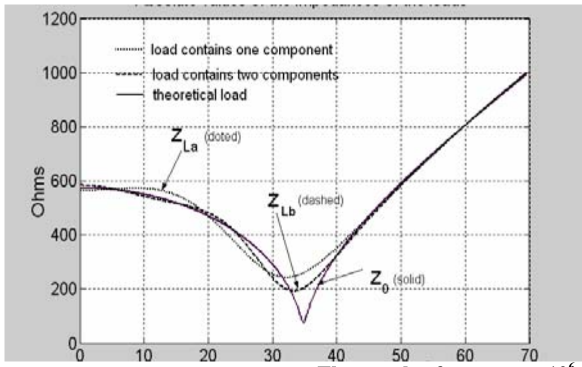


Fig. 2. Absolute values of the impedances of the loads

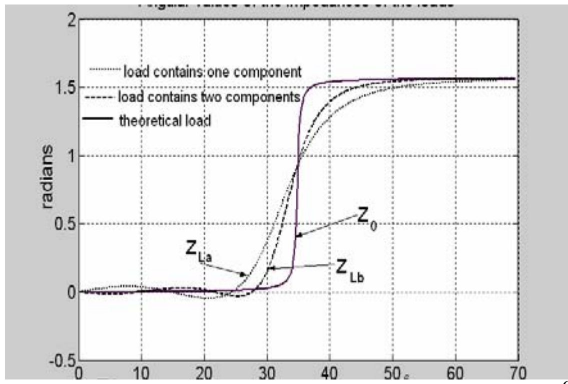
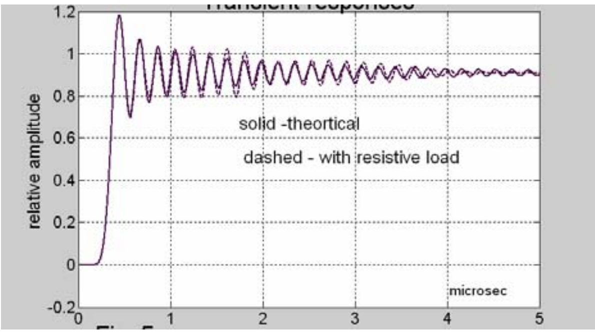


Fig. 3. Angular values of the impedances of the loads

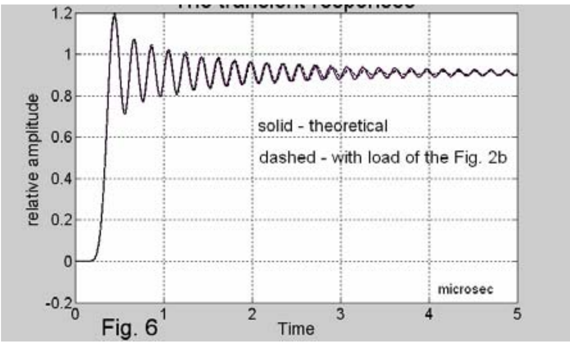
Fig. 4 shows the difference in the transient response in the cases of an ideal and a resistive load. Fig. 5 shows difference in the transient response in the case of an ideal load $R = \sqrt{L/C}$ and reactive load of the type in Fig. 1 b. The maximum error in the case of an approximated load is twice as small in comparisson to a resistive load according to formula $R = \sqrt{L/C}$ { 0.0528/0.0264=2}, but the average quadratic error relation in a 5 microsecond time period is 1.71.

Comparing the distribution of errors it is apparent from Fig. 4 errors appear right after the arrival of the reflected wave. The signal has to travel from one end of the chain to the other and then back to the 6^{th} member. Since the chain has 8 members, the delay corresponds to the time interval the signal travels through 10 members. According to the chain theory the delay time per member is $\sqrt{LC}$ = 57.45 ns, but the delay for all 10 members is 575 ns. Consequently the undistorted part of the process is small. In the case of the approximated load in Fig. 5 the distortions on average are not only smaller, but the less distorted part of the process takes up half of the whole time interval.



Time

Fig. 4. The difference in the transient response



Time

Fig. 5. The approximation in the transient response

Advantages of the Bessel Function Base

In the synthesis of signals and filters based on the ladder network, it is possible to utilize the well developed Bessel Function mathematics. The Bessel functions of the first kind are convenient for the synthesis of pulse signals. They are different from zero in the semi-infinite interval of the argument [0,∞], originating a sharp and oscillatory nature of slow damping. Functions are orthonormalized with weighing $1/\sqrt{x}$ the semi-infinite interval [0, ∞] of the argument $\int_0^\infty \frac{J_{2k-1}(x)J_{2i-1}(x)}{x}dx = \begin{cases} \frac{1}{2(2k-1)}, & if\ i=k \\ 0, & if\ i \neq k \end{cases}$.

This is the basis for the synthesis of the signals of arbitrary form. Since the Bessel functions represent a ladder network, this makes it possible to construct a wide spectrum of analog filters [8].

References

1. **Nickelson L., Tamošiūniene M., Ašmontas S., Tamošiūnas V.** Numerical investigation of strip lines with semoconductor substrates // Electronics and electrical engeneering. - Kaunas: Technologija, 2004, Nr.1(50). – P.16-21.
2. **Tuncer E., Neikirk D.P.** Highly Accurate Quasi-Static Modeling of Microstrip Lines Over Lossy Substrates // IEEE Microwave and Guided Wave Letters 2. - Oct. 1992, P.409-411.
3. **Johnson H.** Slow-wave mode -- *EDN* , 11/8/2001, http://www.sigcon.com/Pubs/5Cedn/5CSlowWaveMode.htm
4. Application Note TDA systems: Practical Characterization of Lossy Transmission Lines Using TDR; http://www.tdasystems.com/library/appnotes/loss0703.pdf
5. **Wang GQi., X., Yu Zhiping, Dutton R.W.** Device Level Modeling of
6. Metal-Insulator-Semiconductor Interconnects, IEEE Transactions On Electron Devices, Vol. 48, No. 8, August 2001.
7. **Johnson H.** Constant-resistance termination, *EDN*, 6/12/2003
8. **Johnson H.** Constant-resistance equalizer, *EDN*, 7/10/2003
9. **Hermanis E.** Comparison of the Physically Realizable Bases for the Synthesis of Signals and Circuits // Electronics and Electrical Engineering. – Kaunas: Technologija, 2003. – No.1(43). – P. 24-27.

Pateikta spaudai 2004 04 12

E. Hermanis. Sutelktųjų parametrų perdavimo linijos su nuostoliais suderintos apkrovos sintezė // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2004. - Nr. 5(54). – P.5-8.

Nagrinėjama perdavimo linijos su nuostoliais suderintos apkrovos sintezė. Tokios linijos turi kompleksinę banginę varžą, kurią dažniniu atžvilgiu neįmanoma pateikti racionalios trupmenos pavidalu. Pateikta jos aproksimacijos pavyzdžių ir atitinkamos apkrovos schemos. Pagrindinis tikslas: sukurti schemą – Beselio funkcijų generatorių kaip pagrindą laikinės srities filtrų sintezei. Il. 6, bibl. 8 (anglų kalba, santraukos lietuvių, anglų ir rusų k.).

E. Hermanis. Synthesis of the Matched Load for the Lumped Lossy Transmission Line // Electronics and Electrical Engineering. – Kaunas: Technologija, 2004. – No. 5(54). – P. 5-8.

Complex characteristic impedance has lossy lines. Surface effect (Skin effect) influences the frequency properties of conductors (linear resistance and inductance); the mobility of charges in the insulator creates the frequency dependent losses of dielectrics and the dependence of linear capacity on the frequency. If the signal receiver, as a load, is not matched to the line, the connecting of this receiver will cause a reflected signal. Reflection losses create noise and reduce signal strength in the receiver. To reduce this signal reflection at the receiver end an additional circuit can be connected to reduce said reflections and improve signal characteristics. In the synthesis of filters and signal generators a linear step chain form can be used, where the impulse response of each link can be represented by a corresponding Bessel function, if the chain is matched with its characteristic impedance. Ill. 6, bibl 8 (in English; summaries in Lithuanian, English and Russian).

Э. Херманис. Синтез согласованной нагрузки линии со сосредоточенными параметрами // Электроника и электротехника. – Каунас: Технология, 2004. – № 5(54). – С. 5-8.

Рассматривается синтез согласованной нагрузки линии со сосредоточенными параметрами. Такие линии имеют комплексное волновое сопротивление, которое в частотном плане невозможно представить в виде рациональной дроби. По этому показаны примеры её аппроксимации и соответствующие нагрузочные схемы. Основная цель: создать схему – генератор функций Бесселя как базис для синтеза фильтров во временной области. Ил. 6, библ. 8 (на английском языке; рефераты на литовском, английском и русском яз.).