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Methodology of Operation Efficiency Prognosis

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Introduction

Sometimes we have situations where decision-making, based on classical optimisation methods, is impossible. It happens, when making of the analytical objective function is complicated, when variables are physically unmeasured, and etc. Such problems associated with objects, action and processes fuzzily described. For example: estimation of reforms and reorganizations expedience. In similar case, methods based on expert estimations are used, that is, human intellect is being used as a measuring instrument.

The easiest way is to process estimations of the single expert. But to appeal to a single expert opinion could be too risky, because influence of subjective extraneous factor is quite distinct. Therefore methods of collection and processing *group* experts' estimations are described.

The well-known and widely used group estimation methods are:

1) based on premise, that competence of all experts is equal;

2) based on robust decisions search.

Both methods have limitations. The fact that different experts have different competence is blinked in the first case. The "extreme" estimations (that necessarily are not incorrect) are usually eliminated in the second case (this is too risky when number of experts is small).

In this paper we analyse a methodology of processing group expert estimations (the methodology is based on search and use of experts' competence rates). The experts competence rates can be get applying *excess* of the experts offered information.

Such methods and algorithms are described in literature [1-4], but they are not universal and applicable just in that case, when importance of goals independent from goals consummation. Therefore, our improved methodology (without noted limitation) of processing group expert estimations is discussed in this paper.

Criterions of Estimation

For estimation of an object, action or process A the following procedures are executed:

The set $S_i | i = \overline{1,n}$ of A goals (that specify objectives of creative object, executable action or realizable process) is formed. The set S has to be full, but minimal, the goals must not overlap.

After interviewing of each expert such information is obtainable:

a) for estimation of each A (there A – object, action or process) goal S_i its subjective ***importance rate*** $g_i \in [0,1]$ is suggested.

b) the ***consummation rate*** h_{s_i} (further h_i) of each goal S_i is prognosticated.

c) the complex rate (criterion) e_A for aggregation of each A goal importance's and consummation's estimations is offered.

Three typical variants of importance g_i and consummation h_i estimations aggregation into one complex rate (criterion) e_A are suggested:

1. When importance of goals is independent from goals consummation;
2. When goals with ***high*** consummation rate h are meaning (the goals with low consummation rate can be "contributed" under the goals with higher consummation rate);
3. When goals with ***low*** consummation rate h are meaning.

Rates $h_i|_{i=\overline{1,n}}$ and e_A have to be quantitative, that is, measurable, calculable or subjectively rateable. They mostly are presented in normal numbers, that is, $e_A, h_i \in [0,1]$.

In the simple cases, when expert direct, that "importance of goals is independent from goals consummation rate h", can be used formula [1-4]:

$$e_A = \sum_{i=1}^{n} g_i h_i // \sum_{i=1}^{n} g_i . \qquad (1)$$

Symbol "//" is sign of division. If fuzzy numbers are dividend, then instead ordinary division its adjective division is executed.

In another case (when aggregation formula (1) unacceptable for expert) the additional test for identification of g_i and h_i estimations aggregation variant into one complex rate e_A is suggested. In this case the test can be formulated as follows:

Let's note value of complex rate e_A, when is given:

	S_1	S_2
Importance rate g_i of each goal	0,6	0,7
Consummation rate h_i of each goal	0,8	0,2

Such expert's answers are usually met:

1) $e_A = 0{,}6$;
2) $e_A = 0{,}48$;
3) $e_A = 0{,}3$;
4) $e_A = 0{,}14$.

All these ("typical") aggregation variants we can express applying parameter g_λ and fuzzy integrals based on it [5].

The point of method is such:

When fuzzy set is given:

$$B = h_1 / g_1 + h_2 / g_2 + \ldots + h_n / g_n, \tag{2}$$

where $0 \le g_i \le 1$,

it is possible to define for it (for set *B*) parameter g_λ, whose rating parameter λ must supply condition

$$-1 < \lambda < \infty; \tag{3}$$

and can be finding as follows:

$$\frac{1}{\lambda}\left[\prod_{i=1}^{n}(1+\lambda g_i)-1\right]=1; \tag{4a}$$

or

$$\frac{1}{\lambda}\left[\prod_{i=1}^{n}\left(1+\lambda g_i h_i\right)-1\right]=1. \tag{4b}$$

We have note, the sets of g_i and h_i estimations have no priority, that is, parameter g_λ can be definite for both g_i and h_i sets or for its product.

If $\lambda = 0$, parameters g_λ is converted into stochastic parameter. But solution $\lambda = 0$ (in equation (4)) is obtainable, when

$$\sum_{i=1}^{n} g_i = 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i = 1. \tag{5}$$

In other cases are available *sub additive mates*

$$\left(\sum_{i=1}^{n} g_i > 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i > 1;\ -1 < \lambda < 0\right);$$

or *super additive mates*

$$\left(\sum_{i=1}^{n} g_i < 1 \quad \text{or} \quad \sum_{i=1}^{n} g_i h_i < 1;\ \lambda > 0\right).$$

The fuzzy set (2) integral is expressed:

$$S = \sup_{\alpha\in[0,1]} \min\{\alpha; g_\alpha\}. \tag{6}$$

Using parameter g_λ, g_α in (6) is as follows:

$$g_\alpha = \frac{1}{\lambda}\left[\prod_{i|h_i\ge\alpha}^{n}\left(1+\lambda g_i\right)-1\right]; \tag{7a}$$

or

$$g_\alpha = \frac{1}{\lambda}\left[\prod_{i|g_i h_i\ge\alpha}^{n}\left(1+\lambda g_i h_i\right)-1\right]. \tag{7b}$$

Formulas (4a), (6), (7a) are applied, when expert gives the first answer of test mentioned above.

When expert gives the second answer, formulas (4b), (6), (7b) are applied.

In all cases the complex rate e_A is expressed:

$$e_A \to S. \tag{8}$$

Here *S* calculated by (6) and (7a) or (7b)

That is fitted for a variant, when goals with high consummation rate h are meaning.

If goals with low consummation rate h are meaning, so it is enough to use some changed formulas (6), (7a) or (7b) as follows:

$$S = 1 - \sup_{\alpha\in[0,1]} \min\{\alpha; g_\alpha\}; \tag{9}$$

$$g_\alpha = \frac{1}{\lambda}\left[\prod_{i|(1-h_i)\ge\alpha}^{n}\left(1+\lambda g_i\right)-1\right]; \tag{10a}$$

or

$$g_\alpha = \frac{1}{\lambda}\left[\prod_{i|(1-g_i h_i)\ge\alpha}^{n}\left(1+\lambda g_i h_i\right)-1\right]. \tag{10b}$$

Formulas (4a), (9), (10a) are applied, when expert gives the third answer of test mentioned above.

When expert gives the fourth answer, formulas (4b), (9), (10b) are applied.

If expert's answers are at variance with 4 "typical" answers, the additional researches to define subjective low of g_i and h_i aggregation are required.

Aggregation of estimations

As we have already mentioned, for estimation of object, action or process *A* goal an expert is invited to suggest its importance rate g_i and consummation rate h_i together with the variant of theirs aggregation into one complex rate (criterion) e_A (or answer to the additional question mentioned above).

If we want to find and use expert's competence rate α_j, we must ask expert to give e_A value by intuition (but not calculated by applying methodology mentioned above), what further is called h_A.

We have note, that e_A and h_A are not the same: the e_A can be calculated by applying methodology mentioned above, while h_A the expert specifies as the overall effect.

Submitting h_A value, the expert by intuition must aggregate (generalize) those data, which he suggests before (h_i , g_i and aggregation variant).

Submitting h_A value the expert specifies a Paretto set of object, action or processes A estimations, while submitting h_i and g_i – an expert specifies a certain item of this set.

If expert can't aggregate information right, his suggested value h_A and calculated value e_A will differ. The degree of their difference can be *the rate of an expert competence*. Naturally, such rate has meaning just in these cases, if estimation of *A* is executed by the more than one expert.

Let's say, that estimation of the object, action or processes *A* is executed by *k* experts.

To put the case that from experts we get all necessary information:

a) estimations g_{ij} , h_{ij} ir h_{jA} , $(j = 1, 2, \ldots, k)$;

b) founded (calculated) complex estimations e_{jA} .

For finding j-th expert's (we have k experts) competence rate α_j ($j = 1, 2, \ldots, k$), we can use methods suggested in literature [1-4].

In that cases, when $h_{jA}(x)$, $h_{ij}(x)$ are the triangle membership functions, j-th expert's competence rate is characterized by:

$$\beta_j = \max_{\alpha \in [0,1]} [\alpha \,\|\, (H_a(h_{jA}(x) \cap H_\alpha(e_{jA}(x)) \neq \varnothing] , \qquad (11)$$

where $H_\alpha = \{x \,\|\, h(x) \geq a\}$;

$H_\alpha(h_{jA}(x))$, $H_\alpha(e_{jA}(x))$ – α-level sets of fuzzy numbers $h_{jA}(x)$ and $e_{jA}(x)$.

The best illustration of β_j is from geometrical point of view – it is a magnitude of both triangles h_{jA} and e_{jA} intersection point (see Fig. 1).

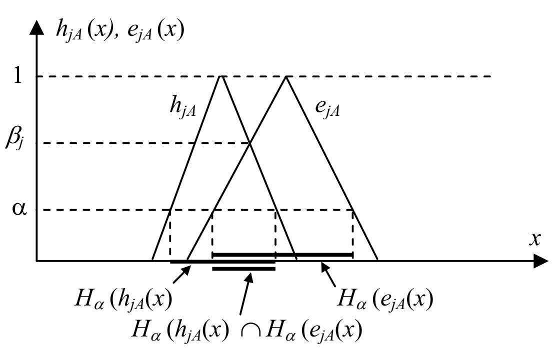

Fig. 1. Illustration of formula (11)

Unfortunately, for the trapezoid membership case [6] formula (11) is unfitted. In this case we would like to propose the complex indicator:

$$\beta_j = \left(\frac{2 \int_X \left(e_{jA}(x) \wedge h_{jA}(x)\right) dx}{\int_X e_{jA}(x)dx + \int_X h_{jA}(x)dx} \right)^{0,5} ; \qquad (12)$$

„ $\wedge$ " means „minimum".

From geometrical point of view, β_j is a magnitude of both trapezoids h_{jA} and e_{jA} intersection point (see Fig. 2). Integrals mean areas limited by both triangles and trapezoids.

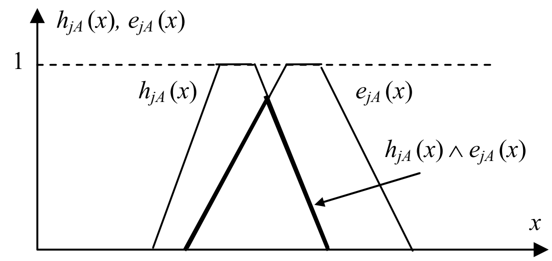

Fig. 2. Illustration of formula (12)

In the case, when $h_{jA}(x)$ and $e_{jA}(x)$ are the equal isosceles triangles membership functions, β_j values calculated by (11) and (12) are the same.

Unfortunately, coefficient β_j underestimates distinctness of expert prognosis: the bigger is the area limited by $h_{jA}(x)$ and $e_{jA}(x)$ or the bigger are values of $\int_X e_{jA}(x)dx$ and $\int_X h_{jA}(x)dx$, the more distinct is j-th expert prognosis (but that make the bigger β_j).

In this case, we would like to propose a weight coefficient, characterizing expert's distinctness of estimations:

$$\gamma_j = \frac{\min_v \left(\int_X e_{vA}(x)dx + \int_X h_{vA}(x)dx \right)}{\int_X e_{jA}(x)dx + \int_X h_{jA}(x)dx} . \qquad (13)$$

The index *v* is the same as *j* ($v = 1, 2, \ldots, k$) – it is the index of the expert.

The complex *expert's weight coefficient* is expressed by:

$$\alpha_j = \beta_j \gamma_j , \quad j = 1, 2, \ldots, k , \qquad (14)$$

and the complex indistinct estimation of an object, process or action *A* is characterized by:

$$H_A(x) = \frac{1}{\sum_{j=1}^{k} \alpha_j} \sum_{j=1}^{k} \alpha_j h_{jA}(x) . \qquad (15)$$

Operating with $h_{jA}(x)$ and $e_{jA}(x)$ defuzzyfied values h_{jA} and e_{jA} the complex distinct group expert estimations of an object, process or action *A* are available that are characterized by:

$$E_A = \frac{1}{\sum_{j=1}^{k} \alpha_j} \sum_{j=1}^{k} \alpha_j e_{jA} \qquad (16)$$

or

$$H_A = \frac{1}{\sum_{j=1}^{k} \alpha_j} \sum_{j=1}^{k} \alpha_j h_{jA} \; . \qquad (17)$$

The question which formula is the best is open (not researched).

Conclusions

1. To get the upper reliability of operation prognosis for data processing it is desirable to use the group expert researches based on methodology of robust decisions search.

The classical method of robust decisions search based on elimination of "extreme" estimations is too risky when number of experts is not big enough.

2. For processing the group expert estimations method offered in this paper based on product averaging of expert estimations and adequate expert competence rate can be used.

3. The expert's competence rate can be founded applying excess of the information offered by experts.

4. For estimation of an expert competence rate, identification of the right low of this expert subjective partial estimations aggregation into one complex rate (criterion) is very important.

5. The aggregation variant of expert's subjective partial estimations into one complex rate can be expressed using fuzzy set integrals.

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The methodology of operation efficiency estimations compilation and processing (according many rates) performed by group expert is discussed in this article. The methodology is based on search and use of experts' competence rates, what can be get applying excess of the experts offered information. Such methods and algorithms are described in literature, but they are not universal and applicable just in that case, when importance of goals independent from goals consummation. Therefore, our improved methodology (without noted limitation) of processing group expert estimations is discussed in this paper. The methodology is based on application of Sugeno criterions and fuzzy integrals. Il. 2, bibl.6 (in English; summaries in English, Russian and Lithuanian).

В. Багдонас, И. Даунорас, А. Дервинене. Методика экспертного прогноза эффективности операций // Электроника и электротехника. – Каунас: Технология, 2005. – № 5(69). – С. 95–98.

В статье анализируется методика экспертной оценки (по множеству критериев) эффективности планируемых операций, включая правила сбора и обработки экспертных данных. Методика предусматривает определение и учет индивидуальных показателей компетенции экспертов, получаемых на основе избыточности экспертных данных. Представленные в научной литературе аналогичные по назначению методы и алгоритмы не обладают достаточной универсальностью, так как не могут быть применены в тех случаях, когда важность целей меняется в зависимости от степени их достижения. Предлагаемая методика свободна от этого недостатка. Методика основана на применении нечетких мер и нечетких интегралов. Ил. 2, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

V. Bagdonas, J. Daunoras, A. Dervinienė. Operacijų efektyvumo prognozės metodika // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 5(69). – P. 95–98.

Straipsnyje nagrinėjama operacijų efektyvumo grupinių ekspertinių įvertinimų (pagal daugelį kriterijų) rinkimo ir apdorojimo metodika, grindžiama ekspertų kompetencijos rodiklių paieška ir panaudojimu. Ekspertų kompetencijos rodikliai gali būti gauti panaudojant jų (ekspertų) teikiamos informacijos perteklių (*excess*). Mokslinėje literatūroje aprašyti šio tipo metodai ir algoritmai nėra pakankamai universalūs, kadangi tinka tik tais atvejais, kai tikslų svarbumas nepriklauso nuo jų pasiekimo laipsnio (*consummation*). Čia siūloma universalesnė (neturinti to trūkumo) grupinių ekspertinių įvertinimų apdorojimo metodika, grindžiama Sugeno matų ir neraiškiųjų integralų panaudojimu. Il. 2, bibl.6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).