ISSN 1392 – 1215 ELECTRONICS AND ELECTRICAL ENGINEERING 2007. No. 6(78)
ELEKTRONIKA IR ELEKTROTECHNIKA

T 190 ELECTRICAL ENGINEERING
ELEKTROS INŽINERIJA

Development of the Phasor Diagram and Equivalent Circuit for the Salient Pole Synchronous Machine

S. Kudarauskas, L. Simanynienė
Department of Electrical Engineering, Klaipėda University,
Bijūnų st. 17, LT-91225 Klaipėda, Lithuania, phone +370 46 398685,e-mail: kudarauskas@klaipeda.omnitel.net, simanyniene@yahoo.com

Introduction

Phasor diagrams and equivalent circuits are widely used for analysis of all AC electrical machines. Different variants of the phasor diagram for the salient pole synchronous machine are composed using the principle of two reactions, which was proposed by A. Blondel in 1895. Variants of the equivalent circuit of such machines are known as well. Despite the fact that the phasor diagrams and equivalent circuits for the considered machines have been used for a long time, development of those by various evaluation of the machine power is presented in this paper.

Main assumptions

The phasor diagrams and equivalent circuits are examined neglecting the armature winding resistance (because evaluation of this resistance does not change principal results). The unsaturated machine is examined. Phasor diagrams are presented for generator action, showing voltage drops across the corresponding reactances (and resistances).

Phasor diagram for the non-salient pole synchronous machine

First of all the phasor diagram for the non-salient pole synchronous generator could be reminded (Fig. 1*a*). Calculation of the output power according to the phasors $\underline{U}$ and $\underline{I}$ yields well-known expression:

$$P = \frac{mEU}{x_s}\sin\theta, \qquad (1)$$

where m is the number of phases, x_s is the synchronous reactance.

The same expression can be got according to phasors $\underline{E}$ and $\underline{I}$, that is according to developed power

$$P = mEI\cos\psi = mE\frac{x_s I\cos\psi}{x_s} = \frac{mEU}{x_s}\sin\theta. \qquad (2)$$

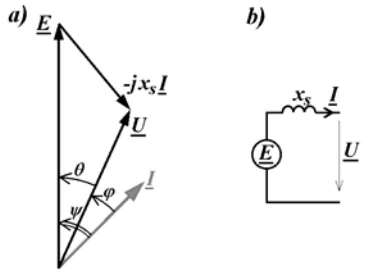

Fig. 1. Phasor diagram (*a*) and equivalent circuit (*b*) for the non-salient pole synchronous generator

The identity of (1) and (2) is apparent, because phasors of the voltage drop $-jx_s\underline{I}$ and current $\underline{I}$ are in quadrature, ant they develop any power. So, the presented phasor diagram in terms of powers is complete.

In Fig. 1*b* the per phase equivalent circuit for the non-salient pole synchronous generator is shown too.

Phasor diagram for the salient pole synchronous machine

Phasor diagram for the salient pole synchronous generator can be constructed from the following equation (that is, finding the terminal voltage $\underline{U}$ by subtracting voltage drops from the excitation electromotive force $\underline{E}$):

$$\underline{U} = \underline{E} - jx_d\underline{I}_d - jx_q\underline{I}_q, \qquad (3)$$

where $\underline{I}_d$ is the direct-axis current, $\underline{I}_q$ is the quadrature-axis current (the corresponding components of the armature current $\underline{I}$), x_d is the direct-axis synchronous reactance, x_q is the quadrature-axis synchronous reactance. It is to notice that these reactances are treated as fixed parameters.

The conventional phasor diagram according to (3) is shown in Fig. 2.

Calculation of the output power from the phasors $\underline{U}$ and $\underline{I}$ results well-known expression:

$$P = \frac{mEU}{x_d}\sin\theta + \frac{mU^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\sin 2\theta. \qquad (4)$$

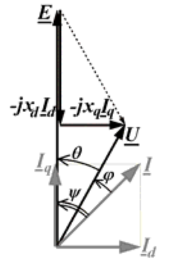

Fig 2. Phasor diagram for the salient pole synchronous generator

Still another expression for power results when the developed power is calculated according to the phasors $\underline{E}$ and $\underline{I}_q$ (because the phasors $\underline{E}$ and $\underline{I}_d$ are perpendicular):

$$P = mEI_q = \frac{mEU}{x_q}\sin\theta. \qquad (5)$$

This wrong result is got despite the fact that phasors of the voltage drops across the corresponding synchronous reactances and phasors of the current components are always in quadrature, and these pairs of phasors develop any power. That signifies that the conventional phasor diagram, as is shown in Fig. 2, is not complete. Possible complement of such diagram is presented in the next section.

Exact expression of the machine power should be derived from the conventional phasor diagram (Fig. 2), if the sum of voltage drops across the synchronous reactances x_q and x_d was evaluated as the voltage drop across some equivalent non-linear reactance x_{sn}. Such phasor diagram is presented in Fig. 3.

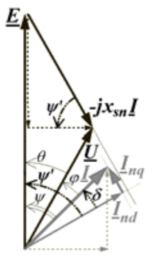

Fig. 3. Developed phasor diagram for the salient pole synchronous generator

In Fig. 3 the armature current $\underline{I}$ is divided into components $\underline{I}_{nd}$ and $\underline{I}_{nq}$, where the first component is in quadrature with the voltage drop across the reactance x_{sn}, and the last component is in cophasal state with the mentioned voltage drop. Thus, this current component $\underline{I}_{nq}$ develops some power, and it is necessary to evaluate that as additional power in the expression (5). This additional power ΔP (per phase) can be derived as follows:

$$\Delta P = -x_{sn}I \cdot I_{nq} = -x_{sn}I^2\sin\delta = -x_{sn}I^2\sin(\psi' - \psi) =$$
$$= -x_{sn}I^2(\sin\psi' \cdot \cos\psi - \sin\psi \cdot \cos\psi') =$$
$$= -I_d x_d I_q + I_d x_q I_q = I_d I_q (x_q - x_d) =$$
$$= \frac{E - U\cos\theta}{x_d} \cdot \frac{U\sin\theta}{x_q}(x_q - x_d) =$$
$$= EU\left(\frac{1}{x_d} - \frac{1}{x_q}\right)\sin\theta - \frac{U^2}{2}\left(\frac{1}{x_d} - \frac{1}{x_q}\right)\sin 2\theta. \qquad (6)$$

It is easy to ascertain that the sum of (5) and (6) leads to exact expression of the salient pole synchronous machine power (4):

$$\frac{mEU}{x_q}\sin\theta +$$
$$+ mEU\left(\frac{1}{x_d} - \frac{1}{x_q}\right)\sin\theta - \frac{mU^2}{2}\left(\frac{1}{x_d} - \frac{1}{x_q}\right)\sin 2\theta =$$
$$= \frac{mEU}{x_d}\sin\theta + \frac{mU^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\sin 2\theta. \qquad (7)$$

As we can see, the presented in Fig. 3 development of the phasor diagram enables to calculate rightly the machine power in different way.

Equivalent circuit for the salient pole synchronous machine

Construction of equivalent circuit for the salient pole synchronous machine based on the voltage equation (3) are problematical because it is necessary to sum up the voltage drops caused by different components of the armature current. Therefore mostly only abstract view of an equivalent circuit is presented, as it is shown in the sample of Fig. 4 [1].

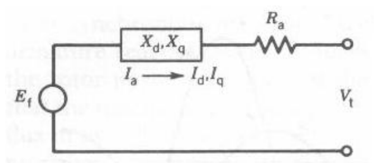

Fig. 4. Equivalent circuit for the salient pole synchronous machine [1]

As we can see in Fig. 4, the synchronous reactance is depicted in generalised mode (as some set of the direct-axis and quadrature-axis synchronous reactances). Here the components of armature current are presented schematically too.

There are other variants of the considered equivalent circuit. For example, we can find an equivalent circuit, which is composed according to principle of the transformer equivalent circuit (e.g. Fig. 5 [2]). Unfortunately, such equivalent circuit is not convenient to analyse salient pole synchronous machine.

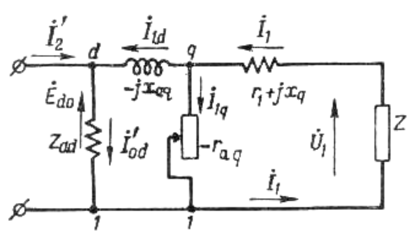

Fig. 5. Equivalent circuit for the salient pole synchronous machine [2]

Naturally, the generalised equivalent circuit can be presented with non-linear synchronous reactance x_{sn}, as it is shown in Fig. 6. However, such equivalent circuit also is not convenient to analyse the considered machine.

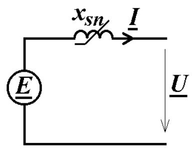

Fig. 6. Generalised equivalent circuit for the salient pole synchronous machine

If we would like to specify the equivalent circuit, first of all the armature current $\underline{I}$ must bifurcate into two branches with different components of the current $\underline{I}_d$ and $\underline{I}_q$. This procedure is presented in Fig. 7.

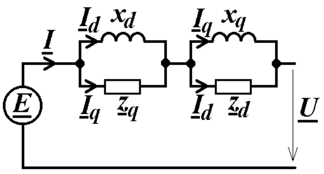

Fig. 7. Specified equivalent circuit for the salient pole synchronous machine

The current $\underline{I}$ is divided in the branch points into their components shunting the direct-axis and quadrature-axis reactances by additional elements $\underline{z}_q$ and $\underline{z}_d$ (indefinite thus far). Parameters of these elements are determined by condition that voltage of the parallel elements caused by different components of the current are the same. Thus, this condition gives:

$$jx_d \underline{I}_d = \underline{z}_q \underline{I}_q \,, \tag{8}$$

$$jx_q \underline{I}_q = \underline{z}_d \underline{I}_d \,. \tag{9}$$

Upon solving equations (8) and (9) for $\underline{z}_q$ and $\underline{z}_d$, there results:

$$\underline{z}_q = r_q = x_d \tan\psi \,, \tag{10}$$

$$\underline{z}_d = r_d = -x_q \cot\psi \,. \tag{11}$$

It is naturally that the addition elements are the resistances because of the phasors $\underline{I}_d$ and $\underline{I}_q$ are being in quadrature. On the other hand, these additional resistances are depending on the phase angle ψ, that is, are non-linear. It is very important that the resistance r_d is negative (that is possible in equivalent circuits).

The equivalent circuit in Fig. 7 can be rearranged into ultimate form, which are shown in Fig. 8.

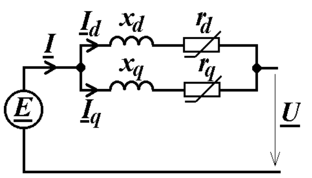

Fig. 8. Developed equivalent circuit for the salient pole synchronous machine

It is to accentuate that the non-linear reactance x_{sn} (Fig. 6) in fact has included some resistances r_q and r_d (positive and negative).

The developed electrical power of the considered machine can also be expressed from the equivalent circuit depicted in Fig. 8. In this case, it is necessary to subtract from the power according to phasors $\underline{E}$ and $\underline{I}_q$ (5) the power ΔP_r in resistances of the equivalent circuit r_d and r_q. Thus, this power ΔP_l (per phase) is:

$$\Delta P_r = r_d I_d^2 + r_q I_q^2 = -x_q \cot\psi \cdot I_d^2 + x_d \tan\psi \cdot I_q^2 =$$

$$= -x_q \frac{I_q}{I_d} I_d^2 + x_d \frac{I_d}{I_q} I_q^2 = I_d I_q (x_d - x_q) = -\Delta P \,. \tag{12}$$

As we can see, this power is equal to the negative additional power from the intermediate expression (6) (in fact, this is an additional power developed in the salient pole synchronous machine due to variable permeance [3]). Consequently, the calculated according to the equivalent circuit Fig. 8 electrical power will be the same true power presented in expression (4). This situation corroborates expedience of the proposed equivalent circuit for the salient pole synchronous machine.

The phasor diagram can also be constructed from the equivalent circuit. Such phasor diagram is shown in Fig. 9. In fact, here the completed conventional (see Fig. 2) phasor diagram is got. Naturally, the true electrical power of the considered machine can be calculated in any way from this phasor diagram. So, the proposed equivalent circuit enables to complement properly the conventional phasor diagram for the salient pole synchronous machine.

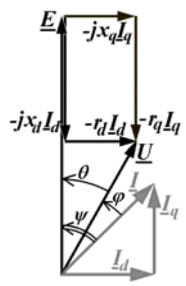

Fig. 9. Complemented phasor diagram for the salient pole synchronous machine

Conclusions

Traditional phasor diagram for the salient pole synchronous machine constructed on a basis of two reactions is not complete, and it enables to calculate machine power only in separate way (from the $\underline{U}$ and $\underline{I}$ phasors).

It is possible to develop an equivalent circuit for the salient pole synchronous machine, in which the direct-axis and quadrature-axis synchronous reactances and additional resistances are presented, as well as the corresponding components of armature current are shown.

The developed phasor diagrams and equivalent circuits for the salient pole synchronous machine enable to calculate power of the machine in various ways.

References

1. **Sen P. C.** Principles of Electric Machines and Power Electronics. – John Wiley & Sons Pb, 1997. – 640 p.
2. **Петров Г. Н.** Электрические машины, ч.2 (Electrical Machines – in Russian). – Госэнергоиздат, 1963. – 704 с.
3. **Kudarauskas S.** Introduction to Oscillating Electrical Machines. – Klaipeda University Pb, 2004. – 183 p.

Submitted for publication 2007 03 01

S. Kudarauskas, L. Simanynienė. Development of the Phasor Diagram and Equivalent Circuit for the Salient Pole Synchronous Machine // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 6(78). – P. 13–16.

Phasor diagrams and equivalent circuits are widely used for analysis of AC electrical machines for a long time. Different variants of the phasor diagram for the salient pole synchronous machine are composed using the principle of two reactions. Variants of the equivalent circuit of such machines are known too. Traditional phasor diagram enables to calculate machine power only in separate way (from the phasors of external voltage and armature current). The developed phasor diagrams and equivalent circuit, in which the direct-axis and quadrature-axis synchronous reactances and additional resistances with the components of armature current are used, are presented in the paper. They enable to calculate power of the considered machine in various ways and more successful to analyse them. Ill. 9, bibl. 3 (in English, summaries in English, Russian and Lithuanian).

С. Кудараускас, Л. Симанинене. Развитие векторной диаграммы и схемы замещения синхронной явнополюсной машины // Электроника и электротехника. – Каунас: Технология, 2007. – № 6(78). – С. 13–16.

Векторные диаграммы и схемы замещения уже долгие годы используются для анализа электрических машин переменного тока. Различные варианты векторных диаграмм синхронных явнополюсных машин строются на основе метода двух реакций. Однако традиционные векторные диаграммы позволяют выразить мощность машины единственным способом – с использованием векторов внешнего напряжения и тока якоря. В статье представлены развитые векторные диаграммы, а также схема замещения с использованием продольного и поперечного синхронных реактивных сопротивлений и добавочных активных сопротивлений с выделением составляющих тока в отдельных ветвях. Это позволяет получить выражение мощности машины различными способами, а также более удобно анализировать свойства машины. Ил. 9, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

S. Kudarauskas, L. Simanynienė. Ryškiapolės sinchroninės mašinos vektorinės diagramos ir ekvivalentinės schemos plėtojimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 6(78). – P. 13–16.

Kintamosios srovės mašinoms analizuoti nuo seno naudojamos vektorinės diagramos ir ekvivalentinės schemos. Ryškiapolės sinchroninės mašinos vektorinių diagramų įvairūs variantai sudaromi taikant dviejų reakcijų metodą, tačiau tradicinės vektorinės diagramos tinka mašinos galiai skaičiuoti tik vienu būdu – pagal išorinę įtampą ir srovę. Straipsnyje pristatomos išplėtotos vektorinės diagramos, taip pat ekvivalentinė schema, kurioje panaudotos skersinė ir išilginė induktyviosios varžos bei papildomos aktyviosios varžos ir atskirose grandinės šakose yra išskirtos inkaro srovės skersinė ir išilginė dedamosios. Tai įgalina teisingai apskaičiuoti mašinos galią įvairiais būdais, sėkmingiau analizuoti jos savybes. Il. 9, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).