Peculiarities of Digital Infinite Impulse Response Filters Design

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Introduction

Signal filtering is essential in control, telecommunication, data gathering, measurement and other systems. Pure analog systems are replaced with mixed signal systems, because of the prices and power consumption of microcontrollers and digital signal processors are dropping. It tends growing signal processing sizes in the systems digital part.

Digital signal processing is critical in the systems, which require physical space and power consumption minimization. The example of such system is the system of physiological parameters monitoring in a real time [1]. With an eye to systems physical space minimization, the main signal processing part is handed in a digital way.

Infinite impulse response digital filters are more efficient in comparison with finite impulse filters and application of these filters is limited with non linear phase and in a rare case – non stability. Infinite impulse response digital filters are chosen in many applications there the linear phase is not necessary, because of calculation efficiency.

Demand of software for easy and flexible digital filters design is growing with growing digital filter application sphere.

Transfer function realization

There are several ways for digital infinite impulse response transfer function realization. Each of realization has its own noises level and different sensitivity for coefficients quantization. It is very important when digital filters are used in fixed point digital signal processors. These processors are chosen in many applications because of low power consumption and low prices.

First realization of digital infinite impulse response filters is:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{N} a_i z^{-i}}{\sum_{i=0}^{N} b_i z^{-i}}; \]

where \( b_0 = 1 \).

Output is picked up, if first direct realization is chosen:

\[ y(n) = \sum_{i=0}^{N} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i). \]

Second direct realization is derived by introducing intermediate expression \( G(z) \):

\[ H(z) = \frac{Y(z)}{G(z)} \cdot \frac{G(z)}{X(z)}; \]

where

\[ Y(z) = \sum_{i=0}^{N} a_i z^{-i} \]

and

\[ G(z) = \frac{1}{1 + \sum_{i=1}^{N} b_i z^{-i}}. \]

Output is picked up:

\[ y(n) = \sum_{i=0}^{N} a_i g(n-i) \]

and

\[ g(n) = x(n) - \sum_{i=0}^{N} b_i g(n-i). \]

Direct filter transfer functions realizations are very sensitive for coefficients quantization, particularly for higher order filters. So, second order filter cascade realization was chosen:

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{G_1(z)}{X(z)} \cdot \frac{G_2(z)}{G_1(z)} \cdot \cdots \cdot \frac{G_k(z)}{G_{k-1}(z)} = \]

\[ = H_1 \cdot H_2 \cdots H_k = \prod_{i=1}^{k} H_i(z). \]

\( H_i \) – second order sections:

\[ H_i(z) = \frac{a_{0i}}{1 + b_{2i} z^{-2} + b_{1i} z^{-1} + a_{0i} z^{2}}. \]

n/2 quadratic sections for a cascade realization are necessary for even order filter. If filter is odd order, there are need of (n-1)/2 quadratic sections and one first order section. n is filter order.
The analog filter transfer function

Digital infinite impulse response filters are designed according to duplicate analog filter transfer functions, by doing shift from Laplace to z transformation. So, transfer function has to be made of normalized (cutoff frequency \( \omega_c = 1 \)) analog lowpass filter. Cascade filter realization is chosen. So, it has to be made second order transfer function sections for even order filter. One first order section has to be made if filter order is odd. Ad hoc stable poles and zeroes are calculated.

Stable Butterworth filter poles could be found:
\[
s_k = -\sin\left(\frac{(2K + 1)\pi}{2n}\right) + j\cos\left(\frac{(2K + 1)\pi}{2n}\right);
\]
where \( n \) – filter order, \( K = 0,1,2,..,n/2-1 \).

Butterworth transfer function is made if poles are known:
\[
H(s) = (-1)^i \prod_{k=0}^{n/2-1} \frac{s_k}{s - s_k}.
\]

Poles are symmetrical across real axis. For making second order section poles \( s_i \) and \( s_{n-1-i} \) are multiplied. Here \( i = 0,1,..,n/2-1 \). So the imaginary part is eliminated. Section has to be made if filter order is odd. Ad hoc stable Butterworth poles could be found:
\[
s_k = -\sin\left(\frac{(2K + 1)\pi}{2n}\right) \cdot \sinh\left(\frac{\sin^{-1}\left(\frac{1}{n}\right)}{\sinh(1/\varepsilon)}\right) + j\cos\left(\frac{(2K + 1)\pi}{2n}\right) \cdot \cosh\left(\frac{1}{n}\sin^{-1}\left(\frac{1}{\varepsilon}\right)\right);
\]
\[
\varepsilon = \sqrt{10^{0.1Ap} - 1};
\]
where \( n \) – filter order, \( Ap \) – passband ripple.

Chebyshev filter transfer function could be made if filter order is odd:
\[
H(s) = \prod_{k=0}^{n/2-1} \frac{s_k}{s - s_k};
\]
if filter order is even:
\[
H(s) = \frac{1}{\sqrt{1 + \varepsilon^2}} \prod_{k=0}^{n/2-1} \frac{s_k}{s - s_k}.
\]

Next step in digital filter design is transfer function z transformation.

Digital filter design according analog filter transfer functions

It is necessary at first get transfer function of analog filter with needed characteristics and then transform it to z plane for infinite impulse response filter design. Ad hoc step invariant, impulse invariant and bilinear transformations could be used. It is hard to implement impulse invariant or step invariant transformations and they could be used just for a lowpass and bandpass filter design. Therefore, bilinear transformation was chosen, which could be used in lowpass, highpass, bandpass and bandstop filter design.

To get digital filter transfer function, \( H(z) \), Laplase operator must be replaced with:
\[
s \rightarrow \frac{\omega_c}{s}.
\]

Lowpass filter transformation to bandpass is made:
\[
s \rightarrow \frac{s^2 + \omega_c^2}{2s};
\]
where \( \omega_c \) – geometrical mean of upper \( \omega_u \) and lower \( \omega_l \) corner frequencies:
\[
\omega_c = \sqrt{\omega_u \omega_l}.
\]
\( B \) – bandwidth of the filter is
\[
B = \omega_u - \omega_l.
\]

For lowpass filter transformation, Laplase operator is replaced:
\[
s \rightarrow \frac{Bs}{s^2 + \omega_c^2}.
\]

For replacing lowpass filter to highpass, with cutoff frequency \( \omega_c \), Laplase operator must be replaced with:
\[
s \rightarrow \frac{\omega_c}{s}.
\]

Lowpass filter transformation to bandpass is made:
\[
s \rightarrow \frac{s^2 + \omega_c^2}{2s};
\]

Analog filter frequency transformation

Frequency transformation is made for filter cutoff frequency normalization and for replacing lowpass filter with highpass, bandpass or bandstop [4].

Laplase operator \( s \) is replaced for lowpass filter normalization this way:
\[
s \rightarrow \frac{s}{\omega_a};
\]
where \( \omega_a \) – passband cutoff corner frequency.

For lowpass filter transformation, Laplase operator is replaced:
\[
s \rightarrow \frac{Bs}{s^2 + \omega_c^2}.
\]

Next step in digital filter design is transfer function z transformation.
\[ a_{ib} = \frac{2\omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2}; \tag{27} \]

\[ a_{i2} = \frac{\omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2}. \tag{28} \]

Denumerator coefficients:

\[ b_{i1} = \frac{2\omega_a^2 - 8f_d^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2}; \tag{29} \]

\[ b_{i2} = \frac{4f_d^2 - 2kf_d\omega_a + \omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2}; \tag{30} \]

where \( \omega_a \) – according 24 recalculated cutoff frequency; \( f_d \) – discretization frequency.

Quantity \( k \) could be found:

\[ k = 2 \cdot \left| s_{\omega a} \right| \tag{31} \]

\( S_{\omega a} \) – real part of pole \( i \).

The same operations were made to get Butterworth highpass, and Chebyshev, Cauer lowpass and highpass digital filter coefficients expressions.

To derive bandpass filter calculation formulas operator \( s \) were replaced:

\[ s \rightarrow B \left( \frac{1 - 2Ae^{-1} + z^{-2}}{1 - z^{-2}} \right). \tag{32} \]

where

\[ A = \cos \left( \frac{2\pi (f_u + f_l)}{2f_d} \right); \tag{33} \]

\[ B = \cos \left( \frac{2\pi (f_u - f_l)}{2f_d} \right). \tag{34} \]

Quantity \( B \):

\[ B = \tan \left( \frac{2\pi (f_u - f_l)}{2f_d} \right). \tag{35} \]

Readjustments were made to get digital transfer function denumerator coefficient by \( z \) in zero degree value become equal one after \( z \) transformation. Expression was obtained after this readjustment of all other coefficient calculation. Fourth order digital bandpass and bandstop filter sections are obtained from quadratic analog filter sections.

Obtained coefficients expressions were used in software for digital filter design development.

**Digital filter design software**

Software for digital filter design was developed in C++ programming language. System for object – oriented software development – C++ Builder was used for these purposes. User interface of this program is presented in figure 1.

*Fig. 1. Interface of software for digital filter design*

User has to set desirable filter characteristics, such as: maximal passband attenuation or ripple, minimal stopband attenuation, and frequencies of passband edge stopband edge and discretization frequency for digital filter design. Filter type: Butterworth, Chebyshev or Cauer has to be chosen by scrollable list component ComboBox. Filter frequency characteristic: lowpass, highpass, bandpass or bandstop has to be chosen by alternative choice component – RadioGroup. After filter parameters is chosen, filter design is made by pushing one button. Filter coefficients are shown in the C++ Builder text display component Memo. The output of these coefficients is made in a text file as well. Output of stable poles is made into another text component Memo. Output of normal poles, zeroes and digital filter coefficients is made in dynamic arrays.

**Verification of software**

Verification was made by doing mathematical modeling. Ad hoc routine of MathLab 6.5 - Simulink was used. The scheme of modeling blokes used in verification is presented in figure 2. Here number of Discrete filter blocks depends on filter order.

*Fig. 2. Block scheme of verification*

Output of Chirp Signal block is unity amplitude, linearly growing in time frequency sine wave. Designed filter coefficients are brought into Discrete Filter block. Output signal is graphically showed in a Scope block.

Output of modeling results example is shown in figure 3 for designed Chebyshev lowpass filter with such characteristics:

- Passband ripple – 2dB;
- Stopband minimal attenuation – 60 dB;
- Passband cutoff edge frequency – 2 KHz;
- Stopband edge frequency – 8 KHz;
- Discretization frequency – 30 KHz.
Conclusions

1. Cascade filter realization is chosen to minimize sensitivity of coefficient quantization. It minimizes limitations for filter using in fixed form digital signal processors.
2. Bilinear z transform was chosen because of possibility to apply it in miscellaneous frequency characteristic filter design.
3. Results of filter mathematical modeling showed adequacy of set and obtained filter characteristics.
4. Algorithms for finding of normalized poles, zeroes and transfer functions coefficients could be used for active analog filter design.

Literature


Presented 2005 11 07