ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2009. No. 2(90)
ELEKTRONIKA IR ELEKTROTECHNIKA

ELECTRICAL ENGINEERING
T190
ELEKTROS INŽINERIJA

Determination of Heat Conductivity Coefficient of a Cable Bundle by Inverse Problem Solution Method

R. Čiegis, M. Meilūnas, G. Jankevičiūtė
Mathematical modeling department, Vilnius Gediminas Technical University,
Sauletekio ave. 11, LT-10223, Vilnius, Lithuania; e-mail: {rc,mm, gerda.jankeviciute}@fm.vgtu.lt
A. Ilgevičius
Automation department, Vilnius Gediminas Technical University,
Naugarduko str. 41, LT-03227, Vilnius, Lithuania; e-mail: audrius.ilgevicius@el.vgtu.lt

Introduction

In modern mobile systems like hybrid vehicles (e.g. Toyota Prius) or air planes [e.g. Airbus A380], electrical and electronic equipment plays the major role. With increasing number of electrical drives (over 50 in a mid-class passenger car) and other equipment using electrical power, the amount of wires and the wire sizes rise too. Due to the space limitation and weight reduction in mobile systems, wire conductor sizes must be kept as small as possible while ensuring the voltage drop and mechanical restrictions. At present, the wires for the cable harness for mobile applications are still selected according to design rules and specifications, which were elaborated for stationary use and not necessary suited for mobile applications. Thus, the main aim of this problem is to determine the optimal conductor cross sections for continuous electrical loads in mobile systems.

The modeling of these processes is a complex mathematical problem. To get a quantitative description of the thermo-electrical characteristics in the electrical cables, one requires a mathematical model for it. It must involve the different physical phenomena occurring in the electrical cables, i.e. heat conduction, convection and radiation effects, description of heat sources due to ohmic heating (direct current). At the next stage the model is discretized and numerical algorithms must be developed.

Using the discrete model extensive simulation experiments are done [1], and the results of the simulations have to be verified by the experiments. The final goal is optimization of the commercial products with the main goal to minimize the subsequent weight and costs of electro cables used in car industry.

Fundamentals of the heat transfer in cables are given in [5]. For further readings we refer to [3, 4, 6].

Numerical algorithms for solution of parabolic and elliptic problems with discontinuous coefficients have been widely investigated in many papers. Conservative finite-difference schemes for approximation of such problems were derived by Tichonov and Samarskii [7, 8]. Recently new finite difference schemes were proposed, which approximate with the second order of accuracy not only the solution, but also the normal flux through the interface.

There are specialized commercial software tools, such as CableCad, Ansys, which can be used for solution of heat conduction problems. For example, Ansys software is a coupled physics tool combining structural, thermal, computational fluid dynamics (CFD), acoustic and electromagnetic simulation capabilities in a single engineering software solution [6].

Our project deals with the numerical algorithms developed in [1] to simulate heat transfer in the cable bundles in order to determine temperature distributions of the wires. Some coefficient identification methods for the heat transfer differential equations were carried out too.

The aim of this paper is to identify thoroughly the mixed heat conduction coefficients k_0 of the cable bundles using the inverse problem solution method.

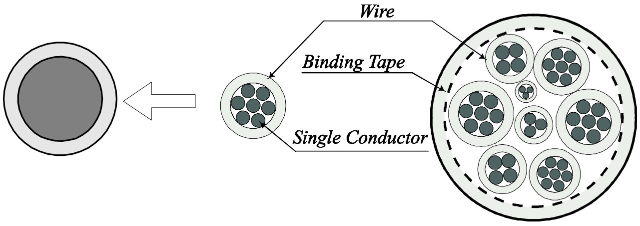
Determination of the heat conductivity coefficients of an electrical cable bundle is one of the main problems when simulating the heat transfer of such a problem. Although, the properties of wire materials and air (air gaps between the wires) are well known, one requires determining the so-called “mixed” conductivity coefficient of the wire isolation material (e.g. polyvinyl chloride (PVC)) and air, if the complicated heat convection and radiation processes in air are replaced by the heat conduction in PVC with the smaller conduction value.

A heuristic formula for the determination of mixed heat conductivity coefficient was presented in [1]. More general procedure using non-linear least square technique for determination of the optimal “mixed” heat conductivity coefficient can be applied too.

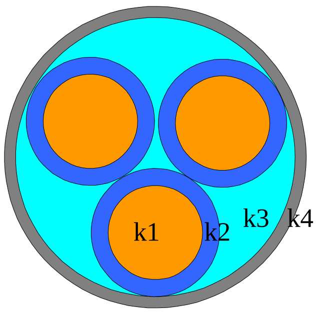
In this paper identification of the mathematical model [1] from numerical computational experiments and the real-world measurements is presented.

Two dimensional mathematical model

The multi-wire cable consists of M various single isolated electric wires, see Fig. 1, a. In Fig. 1, b material properties of a multi-wire cable are indicated.



a)



b)

Fig. 1. The multi-wire cable: a) cross sectional cable structure, b) material heat conductivity properties

The main mechanism of heat transfer can be structured as following:

1. Conduction in a solid (copper conductors, PVC isolation), which is defined by the coefficients *k1* and *k4*;
2. Conduction in the air and PVC mixture, which is defined by the coefficients *k2* and *k3;*
3. Convection and radiation from the outer side of the bundle isolation to the environment.

Since the heat transfer mechanism in air between the wires of a bundle is complicated and non-relevant, the model can be simplified. This non-relevance can be explained by the fact that in a close vicinity to a wire, the main heat transfer mechanism is heat conduction, while in a distance, the dominating mechanism is the motion of molecules. Since all the wires in a bundle are tightly pressed together, only heat conduction is relevant. This simplification helps to increase the efficiency of the numerical algorithm.

The mathematical model consists of the parabolic differential equation:

$$\begin{cases} c(X,T)\rho(X)\dfrac{\partial T}{\partial t} = \sum\limits_{i=1}^{2} \dfrac{\partial}{\partial x_i}\left(K(X)\dfrac{\partial T}{\partial x_i}\right) + f(X,T), \\ (X,t) \in D \times (0, t_F). \end{cases} \quad (1)$$

With the initial condition

$$T(X,0) = T_a, \; X \in \overline{D} = D \cup \partial D \quad (2)$$

and the nonlinear boundary conditions of the third kind:

$$-k(x)\frac{\partial T}{\partial n} = \alpha_k(d,T)\left(T(x,t) - T_a\right) + \\ + \varepsilon\sigma\left(\left(T^4(x,t)\right)^4 - (T_a)^4\right), x \in \partial D. \quad (3)$$

Coefficients of the governing equation (1) are described below [3]:

a) the specific heat capacity $c(X,T)$:

$$c = \begin{cases} 381 + 0.17T, \;\; 0 \le T \le 200^0 C, \textit{ for copper}; \\ 920 - 1.3T + 0.074T^2, \; 0 \le T \le 100^0 C, \textit{ for PVC}; \end{cases} \quad (4)$$

b) the density $\rho(X)$:

$$\rho = \begin{cases} 8960, \textit{ for copper}; \\ 1350, \textit{ for PVC}; \end{cases} \quad (5)$$

c) the heat conductivity coefficient $k(X)$:

$$k = \begin{cases} 401, \textit{ for copper}; \\ 0.17, \textit{ for PVC}; \end{cases} \quad (6)$$

d) the density of the energy source $f(X,T)$ is defined as:

$$f = \left(\frac{I}{A}\right)^2 \rho_0(1 + \alpha_\rho(T - 20)), \quad (7)$$

where I – a direct electrical current, A – an area of the conductor cross-section, ρ_0 – the specific resistivity of the conductor, T_a – the ambient temperature.

Identification procedure of the model

Mixed heat conductivity of air and PVC k_0 can be computed by the following simple relationship between the heat conductivity of air k_A, of PVC k_I and the filling factor F of the bundle [2]:

$$k_0 = k_A(1 - F) + k_I F\,, \quad (8)$$

where the filling factor F of the bundle is computed from the relationship between the total area of the wires

$d = \sum\limits_{i=1}^{n} d_i$ and the inside area of a bundle D,

$$F = \sum_{i=1}^{n} d_i / (D - S)\,, \quad (9)$$

here S – the area of the outer isolation of the bundle.

We use the experimental data from a benchmark suit for the computation of temperatures of electrical cables. The experimental setup and measurement procedure was carried out by Volkswagen AG. There were measured two different cable bundles with 32 wires.

Table 1. "Standard" cable bundle, Volkswagen data

Wire type	Cross section mm^2	Number of wires	Load (Summer) Amps	Load (Winter) Amps	Ambient temperature Degrees Celsius
FLRY-A	0,35	1	4	4	
FLRY-A	0,5	10	0,4 - 7	0,4 - 7	
FLRY-A	0,75	2	4,5	4,5	
FLRY-A	1,0	2	-	-	
FLRY-A	1,5	6	12	9,5-12	70
FLRY-B	2,5	4	-	12-18	
FLRY-B	4	3	7,3-23	14,9-23	
FLRY-B	6	3	-	12-37	
FLRY-B	10	1	35	53	

Table 2. "Reduced" cable bundle, Volkswagen data

Wire type	Cross section mm^2	Number of wires	Load (Summer) Amps	Load (Winter) Amps	Ambient temperature Degrees Celsius
FLRY-A	0,35	11	0,34 - 7	0,34 - 7	
FLRY-A	0,5	1	4,5	4,5	
FLRY-A	0,75	2	-	-	
FLRY-A	1,0	6	12	12	
FLRY-B	2,5	5	-	12-18	70
FLRY-B	4	3	7,3-23	14,9-23	
FLRY-B	6	3	-	12-37	
FLRY-B	10	1	35	53	

In the Table 1 are given data for the "Standart" bundle, which contains wires with the greater cross section compared to the second cable bundle named "Reduced", whose data are given in the Table 2. Both bundles were measured at different load regimes: "Summer" and "Winter". This means that during the summer time the electrical load profile (direct current) in a car is lower compared to the winter time. Ambient temperature at which the measurements were carried out was 70 degrees due to the fact that this temperature is the averaged value of different compartments in a car.

For each experiment we have the following information:

$$(I_m, T_m),\ m = 1,...,M\ , \qquad (10)$$

where T_m – the temperature of the m-th wire after the stationary distribution of the temperature was achieved (from 30 to 60 min.), I_m – the density of the current, M – the total number of wires.

It is important to note that the positions of wires were not fixed during the experimental measurements in the cable bundle and these positions depend on the coordinate z.

In order to simulate this property of cables for a given set of wires L 10 distributions of wires are generated in a random order and for each distribution a stationary solutions defined by the discrete scheme [1] are found:

$$T(l) = \{T_{ij}(l),\ \ X \in D_h(l)\},\quad l = 1,...,L\ , \qquad (11)$$

where L = 10. Then, the averaged values of temperatures of different wires are computed

$$T_m = \frac{1}{L}\sum_{l=1}^{L} T_m(l),\ m = 1,....,M\ . \qquad (12)$$

Fitting of the simulation results to the experimental data

In Table 3 the simulation results fitted to the experimental data are summarized. Here, 4 different distributions of wires are generated. The first 3 distributions are generated in a random order while the last one in a way that the wires with biggest current density are brought to the centre of the bundle. In this way, the 4th computational experiment represents the "worst-case" scenario. Data $T_{e,min}$, $T_{e,max}$, denote the minimal and maximal temperatures of experimental data, T_{min}, T_{max}, are the minimal and maximal averaged temperatures obtained in computations, $T_{min}(l)$, $T_{max}(l)$ denote the minimal and maximal temperatures for the distribution of wires when the maximal temperature was computed in simulations. As mentioned above, two different load-regimes are used: Summer and Winter (see Tables 1 and 2).

Fitting of the simulation results to the experimental data is done by identifying the mixed heat conductivity coefficient k_0. At first, in the identification procedure we use the heuristical formula (8) to compute the parameter k_0 and later we adjust this parameter in a way that the simulation results can repeat the experimental data. The first three computations with randomly distributed wires show quite sufficiently good agreement with the experimental data. The last computational test, in which the "worst-case" scenario is simulated, proves that even in this situation the maximal temperature (T_{max}= 93,6) is very close to the measured one ($T_{e,max}$= 90,2).

Table 3. Simulation results

No.	Test	Regime	k_0	$T_{e,min}$	$T_{e,max}$	T_{min}	T_{max}	$T_{min}(l)$	$T_{max}(l)$
1.	"Standard"	Summer	0,14	80,0	90,2	84,3	89,7	82,5	90,7
2.	"Standard"	Winter	0,13	102,6	117,8	103,2	111,3	101,7	117,5
3.	"Reduced"	Summer	0,13	85,6	96,3	88,2	96,2	86,1	98,9
4.	"Standard"	Summer	0,13	80,0	90,2	-	-	83,4	93,6

Conclusions

The proposed evaluation procedure of the parameter k_0, is applied to generalize the mathematical model of the heat transfer in a cable bundle (up to 40 single wires) for passenger vehicles. From the fitted results (Table 3) one can see a very good agreement with the measurements. Obtained k_0 value of 0,13-0,14 shows that the filling factor of a bundle F is 0,7. For larger numbers of wires (over 40), one can avoid the consideration of the factor F (since $F \approx 1$) and use the coefficient k of PVC material. On the other hand, if the size of wires within the bundle has large deviations from each other, the quality of filling the bundle by the wires is decreasing. In such situations, the factor F

and the mixed heat conductivity if air and PVC must be used.

The term “a very good agreement” can be explained by the fact that experiments were carried out under laboratory conditions corresponding to the “worst-case” scenario. In reality, heat transfer processes of the cable bundles placed in a car, are affected by too many factors, which can barely be simulated by any computer software. The car manufacturer would never relay on the simulation software alone for this kind of calculations, however the real-world tests will be preferred. Such software can be used during the pre-development phase of a passenger vehicle to gain a general picture of electrical power supply system.

Therefore, the precision of the simulation results giving less than 5% error with respect to the experimental data is more than sufficient for the industrial applications.

Acknowledgements

The authors have been supported by the Agency for International Science and Technology Development Programmes in Lithuania within EUREKA Project E!3691 OPTCABLES.

References

1. **Čiegis R., Ilgevičius A., Liess H.-D., Meilūnas M. and Suboč O.** Numerical simulation of the heat conduction in electrical cables // Mathematical Modelling and Analysis. – 2007. – No. 12(4). – P. 425–439.
2. **Liess H.-D and Ilgevičius A.** Analytical versus numerical calculations of physical problems. The benefits of its combination // Mathematical Modelling and Analysis. – 2003. – No. 8(4). – P. 291–302.
3. **Ilgevicius A.** Analytical and numerical analysis and simulation of heat transfer in electrical conductors and fuses / Dissertation. – Universität der Bundeswehr München. – 2004.
4. **Ilgevičius A. and Liess H.-D.** Thermal analysis of electrical wires by finite volume method // Electronics and Electrical Engineering. – Kaunas: Technologija, 2003. – No. 4(46). – P. 87–92.
5. **Incropera F., DeWitt P. and David P.** Introduction to Heat Transfer. – New York: John Willey and Sons, 1985.
6. **Tarvydas P. and Noreika A.** Usability Evaluation of Finite Element Method Equation Solvers // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 7(74). – P. 13–16.
7. **Samarskii A. A.** The theory of difference schemes. – Marcel Dekker, Inc., New York-Basel. – 2001.
8. **Tichonov A. N. and Samarskii A. A.** Homogeneous finite difference schemes. – Zhurn. Vychisl. Mat. Mat. Fiziki. – 1961. – No. 1(1). – P. 5–63.

Received 2008 11 10

R. Čiegis, A. Ilgevičius, M. Meiliūnas, G. Jankevičiūtė. Determination of Heat Conductivity Coefficient of a Cable Bundle by Inverse Problem Solution Method // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 2(90). – P. 77–80.

The aim is to identify thoroughly the mixed heat conduction coefficients k_0 of the cable bundles using the inverse problem solution method. Determination of the heat conductivity coefficients of an electrical cable bundle is one of the main problems when simulating the heat transfer of such a problem. The so-called “mixed” conductivity coefficient of the wire isolation material (e.g. polyvinyl chloride (PVC)) and air is replaced by the heat conduction in PVC with the smaller conduction value. The proposed evaluation procedure of the parameter k_0, is applied to generalize the mathematical model [1] of the heat transfer in a cable bundle (up to 40 single wires) for passenger vehicles. From the fitted results a very good agreement with the measurements can be seen. Ill. 1, bibl. 8 (in English; summaries in English, Russian and Lithuanian).

Р. Чиегис, А. Илгевичюс, М. Мейлюнас, Г. Янкевичюте. Нахождение коэффициента теплопроводности волокон кабеля, используя метод обратного решения задачи // Электроника и электротехника. – Каунас: Технология, 2009. – № 2(90). – С. 77–80.

Целью является нахождение коэффициента теплопроводности k_0 волокон кабеля, используя метод обратного решения задачи. Установление коэффициента теплопроводности k_0 волокон кабеля представляет собой основную задачу при постановке вопроса теплопередачи. Независимо от того, что свойства материала кабеля и воздуха (воздушных промежутков между проводами) хорошо изучены, необходимо определить коэффициент теплопроводности из смеси вещества изоляции проводов поливинилхлорида (ПВЦ) и воздуха. Для этого используется уменьшенная теплопроводность в веществе ПВЦ. Предлагаемая методика установления коэффициента k_0 применяется для обобщения математической модели автомобильного кабельного волокна (до 40 проводов). Итог подтверждает достаточно высокую точность результатов расчетов и замеров. Il. 1, библ. 10 (на английском языке; рефераты на английском, русском и литовском яз.).

R. Čiegis, A. Ilgevičius, M. Meiliūnas, G. Jankevičiūtė. Laidų pluošto šilumos laidumo koeficiento nustatymas atvirkštinio uždavinio sprendimo metodu // Elektronika ir Elektrotechnika. – Kaunas: Technologija, 2009. – No. 2(90). – P. 77–80.

Darbo tikslas – rasti laidų pluošto šilumos laidumo koeficientą k_0 naudojant atvirkštinio uždavinio sprendimo metodą. Laidų pluošto šilumos laidumo koeficientų nustatymas yra vienas pagrindinių uždavinių modeliuojant šilumos pernešimo uždavinį. Nors laidų medžiagų ir oro (oro tarpai tarp laidų) savybės yra gerai žinomos, tačiau būtina rasti laidų izoliacijos medžiagos polivinilchlorido (PVC) ir oro mišinio šilumos laidumo koeficientą. Šiuo atveju naudojama mažesnio šilumos laidumo PVC medžiaga. Pasiūlyta koeficiento k_0 nustatymo metodika yra pritaikyta laidų pluošto (iki 40 laidų), skirto automobiliams, matematiniam modeliui apibendrinti. Iš gautų rezultatų matyti, kad skaičiavimų ir matavimų rezultatai sutampa. Il. 1, bibl. 10 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).