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Blind Separation of Noisy Pseudoperiodic Chaotic Signals

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Introduction

The aim of "Blind Source Separation" (BSS) is to recover mutually independent unknown source signals only from observations obtained through an unknown linear mixture system. Given observation matrix $\mathbf{X}=[\boldsymbol{x}(1),\cdots,\boldsymbol{x}(N)]\in\mathbb{C}^{M\times N}$ the general linear instantaneous mixing signal model is

$$\mathbf{X}=\mathbf{AS}+\mathbf{V}, \qquad (1)$$

where N is the number of available samples, M denoting the number of observations (output dimension), K denoting the number of sources (input dimension), $\mathbf{S}=[\boldsymbol{s}(1),\cdots,\boldsymbol{s}(N)]\in\mathbb{C}^{K\times N}$ contains the corresponding latent (hidden) components which represent unknown source signals, $\mathbf{A}\in\mathbb{C}^{M\times K}$ represents the unknown mixing matrix describing the input-output relation and $\mathbf{V}\in\mathbb{C}^{M\times N}$ is a matrix of additive noises which are mutually uncorrelated and are also uncorrelated with the sources. The goal is therefore to estimate both unknowns ($\mathbf{A}$ and $\mathbf{S}$) from the measurements $\mathbf{X}$ and in principle all there is to do is to invert the mixing process

$$\hat{\mathbf{S}}=\mathbf{WX}, \qquad (2)$$

where $\mathbf{W}\approx\mathbf{A}^{-1}$ is called the separating matrix. The general BSS problem requires $\mathbf{A}$ to be an $M\times K$ matrix of full rank, with $M\geq K$ (i.e. there are at least as many mixtures as independent sources). In most algorithmic derivations, an equal number of sources and sensors is assumed. As resolutions of the problem, many methods have been proposed (see [1] for instance). The approximate joint diagonalization of a set of real m-square symmetrical correlation matrices (second-order statistics) is an essential tool in blind source separation algorithms [2], [3]. Given a matrix set $\mathcal{M}=\{\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_K\}$, where $\mathbf{M}_k\in\mathcal{C}^{N\times N}$, $1\leq k\leq K$, the approximate joint diagonalization problem seeks a nonsingular diagonalizing matrix $\mathbf{W}\in\mathcal{C}^{N\times N}$ and K associated diagonal matrix $\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\cdots,\boldsymbol{\Lambda}_K\in\mathcal{C}^{N\times N}$ (which are usually not of interest in the context of BBS) such that the following common structures are best fitted:

$$\mathbf{M}_k=\mathbf{W}\boldsymbol{\Lambda}_k\mathbf{W}^{\mathbf{T}},\ 1\leq k\leq K. \qquad (3)$$

The "goodness of fit" is evaluated by some criterion (cost or objective function). It is proved, that matrix $\mathbf{W}$ is closely related to $\mathbf{A}^{-1}$ – inverse of mixture matrix $\mathbf{A}$. The existing algorithms for approximate joint diagonalization are generally divided into two categories: orthogonal and nonorthogonal diagonalizations. In BSS, using orthogonal diagonalization, observations are prewhitened so that they are uncorrelated and have unity variance [1]. However, due to the limitations of orthogonal joint diagonalization, the nonorthogonal joint approximate diagonalization (JAD) has received increasing attention in recent years [4], [5].

In this paper the efficacy of the BSS algorithm based on nonlinear phase-space reconstruction and nonorthogonal joint approximate diagonalization of several time-delayed covariance matrices [6] is investigated in the case that the observation noise exists. The algorithm is applicable for mixed pseudoperiodic chaotic signals and another sources, that have temporal structures and non-vanishing temporal correlation.

Description of the algorithm

Briefly algorithm can be characterized as follows. Given a group of M sensor signals with N samples $\mathbf{X}=[\boldsymbol{x}(1),\cdots,\boldsymbol{x}(N)]\in\mathbb{C}^{M\times N}$ a reconstructed phase space matrix [7] $\mathbf{X}^{(k)}$, $k=1,\cdots,M$ with d rows and $L=N-(d-1)\tau$ columns (called a trajectory matrix) for the mixture received by k^{th} sensor is defined by

$$\mathbf{X}^{(k)}=\begin{bmatrix} x_1^{(k)} & x_2^{(k)} & \cdots & x_{N-(d-1)\tau}^{(k)} \\ x_{1+\tau}^{(k)} & x_{2+\tau}^{(k)} & \cdots & x_{N-(d-2)\tau}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1+(d-1)\tau}^{(k)} & x_{2+(d-1)\tau}^{(k)} & \cdots & x_N^{(k)} \end{bmatrix}, \qquad (4)$$

where d – the embedding dimension and τ – time delay. A high-dimensional system, i. e. overembedding at $\tau = 1$ is preferable. For M sensors, we obtain M embedding matrices generally with the same values for τ and d. Using the i^{th}, $i = 1,\cdots,d$ rows of the embedding matrices $\mathbf{X}^{(k)}$ we can form a data matrix for all sensors for every embedding dimension, i. e.

$$\mathbf{X}_i = \begin{bmatrix} x^{(1)}_{1+(i-1)\tau} & x^{(1)}_{2+(i-1)\tau} & \cdots & x^{(1)}_{N-(d-i)\tau} \\ x^{(2)}_{1+(i-1)\tau} & x^{(2)}_{2+(i-1)\tau} & \cdots & x^{(2)}_{N-(d-i)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x^{(K)}_{1+(i-1)\tau} & x^{(K)}_{2+(i-1)\tau} & \cdots & x^{(K)}_{N-(d-i)\tau} \end{bmatrix}, \quad (5)$$

where $i = 1,\cdots,d$.

The time-delayed covariance matrixes $\mathbf{R}_j \in C^{M\times M}$ has the form

$$\mathbf{R}_j = \frac{1}{L-1}\mathbf{X}_1\mathbf{X}^T_{j+1}, \quad (6)$$

where $j = 1,\cdots,d-1$.

Given a set of time-delayed covariance matrixes $\mathcal{R} = \{\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_j\}$, where $\mathbf{R}_j \in C^{M\times M}$, the goal of a joint diagonalization algorithm is to find a diagonalizing matrix $\mathbf{U} \in C^{M\times M}$, called separating matrix in BSS, so that the matrices $\mathbf{U}\mathbf{R}_j\mathbf{U}^T$, $(j = 1,\cdots,d-1)$ are as diagonal as possible. In this work the numerical algorithm FFDIAG (Fast Frobenius Diagonalization) [5], [8] as iterative scheme to approximate the solution of the following optimization problem

$$\min_{\mathrm{V}\in\mathrm{R}^{\mathrm{M}\times\mathrm{M}}} \sum_{j=1}^{d-1}\sum_{k\neq l}\left(\left(\mathbf{U}\mathbf{R}_j\mathbf{U}^T\right)_{kl}\right)^2 \quad (7)$$

are used. The matrix of source signals is estimated as

$$\hat{\mathbf{S}} = \mathbf{U}\mathbf{X} \quad (8)$$

in which each row represent a separate signal.

Numerical results

The proposed algorithm was applied to artificially mixed synthetic signals. In the first experiment two mixed x components of the Rossler system, defined by

$$\begin{cases} \dfrac{dx}{dt} = -(y+z), \\ \dfrac{dy}{dt} = x + a\cdot y, \\ \dfrac{dz}{dt} = b + z(x-c). \end{cases} \quad (9)$$

with parameters a = 0,398; b = 2; c = 4 and a = 0,2; b = 0,2; c = 4,6 respectively were considered. The embedding dimension of the reconstructed phase space $d = 60$ and time delay $\tau = 1$ for mixed signals were defined and 2000 samples were used in this experiment. In the second experiment two mixed signals of the Mackey-Glass differential-delay equation, defined by

$$\frac{dx}{dt} = \frac{ax(t-\tau d)}{1 + x(t-\tau d)^c} - bx(t), \quad (9)$$

were used. The two sequences are generated with the same parameters (a = 0,2; b = 0,1; c = 10 and $\tau d = 30$) but with different integrating conditions. The equation is solved numerically by using the algorithm described in [9]. The embedding dimension of the reconstructed phase space $d = 50$ and time delay $\tau = 1$ for mixed signals were defined. The lenght of sequences – 1600 points. As mentioned above it is assumed that the number of sensors M is equal to the number of signals K. That is, two sensors are used and the coupling (mixing) matrix in both cases is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0{,}5 \\ 1{,}1 & 1 \end{bmatrix}. \quad (10)$$

First, the additive noise $\mathbf{n}(t)$ is modeled as a stationary, temporally white, zero-mean random process independent of the source signals, i. e. the covariance matrix of the noise satisfying:

$$E\left[\mathbf{n}(t+\tau)\mathbf{n}(t)^{\mathrm{T}}\right] = \sigma^2\delta(\tau)\mathbf{I}, \quad (11)$$

where E is the expectation operator, σ^2 denotes the variance of the noise, $\delta(\tau)$ – the Kronecker delta, and $\mathbf{I}$ denotes the identity matrix. In this case adopting delayed correlation matrices resolves the influence of the noise, whereas the autocovariance of noise equals to zero for time lag $\tau \neq 0$. In a simulation environment (the true matrix $\mathbf{A}$ is known) the performance of blind separation can be characterized by one single performance index defined by [10]

$$J(\mathbf{P}) = \frac{\left\|\mathbf{P} - \mathrm{diag}(\mathbf{P})\right\|^2}{\left\|\mathbf{P}\right\|^2}, \quad (12)$$

where the permutation matrix $\mathbf{P} = \mathbf{W}\mathbf{A}$, $\mathbf{P} \in C^{M\times M}$ and $\|\cdot\|$ denotes the Frobenius norm of a matrix. Note that $J(\mathbf{P})$ is non negative and if $\mathbf{W} = \mathbf{A}^{-1}$ $J(\mathbf{P}) = 0$ holds.

Fig. 1 shows that the nonorthogonal joint approximate diagonalization algorithm is robust to the white Gaussian noise – it provides a perfect separation for signal to noise ratio (SNR) up to 5 dB. Since the separated signals remain noisy, they must be enhanced at postprocessing stage. It should be noted that nonlinear noise reduction, as a preprocessing (before blind separation), can adversely affect the total performance of the signals separation, since the errors, committed in this preprocessing stage, lead to the greater errors in the joint diagonalization stage. As the criterion that evaluates the total performance of the separation and denoising of the signals the relative mean square error ε between the normalized original signal matrix $\mathbf{S}$ and estimated signal matrix $\mathbf{Z}$ is used

$$\varepsilon = \frac{\left\| \mathbf{S} - \mathbf{Z} \right\|^2}{\left\| \mathbf{Z} \right\|^2}. \qquad (13)$$

Separated noisy signals are denoising by applying nonlinear noise reduction based on the local phase space singular value decomposition method [11], [12], when the covariance matrix $\mathbf{R}_n$ is defined as $\mathbf{R}_n = \frac{1}{N-1}\mathbf{S}_n\mathbf{S}_n^T$, where $\mathbf{S}_n$ – the centered neighborhood $\mathbf{N}_n$ matrix for every the reference point $\mathbf{s}_n$ of reconstructed phase space and N – the number of neighbors in $\mathbf{N}_n$. The embedding dimension $d = 60$ (i. e. overembedding), time delay $\tau = 1$ and the first 60 nearest neighbors for each reference phase space point were used. For large amount of noise it becomes a nontrivial problem to identify the correct neighbors, whereas all neighborhoods merge. As a result the ε is considerably higher at SNR = 5 dB (Fig. 1). Therefore, for large amounts of noise the nonlinear noise reduction based on the global phase space singular value decomposition method [11], [12] is preferable.

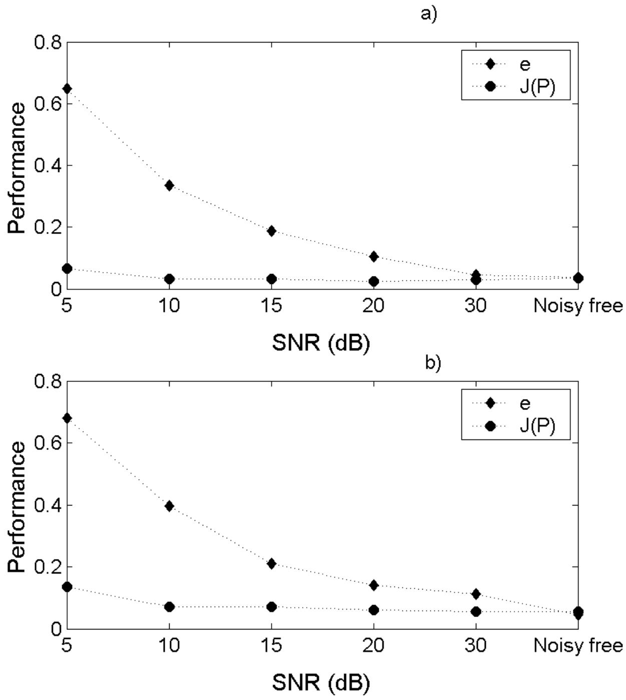

Fig. 1. The performance index $J(\mathbf{P})$ and the relative mean square error ε versus white Gaussian noise level for separation the a) Rossler signals and b) Mackey-Glass signals

Further, the case for chaotic data with colored noise generated from a three-order autoregressive process [AR(3)] is studied $w_n = 0{,}8 \cdot w_{n-1} - 0{,}5 \cdot w_{n-2} + 0{,}6 \cdot w_{n-3} + \varepsilon_n$, where $\varepsilon_n \sim N(0{,}1)$ follows the normal distribution. Fig. 2 shows that blind source separation performance remains approximately at the same level, but nonlinear noise reduction error is lower.

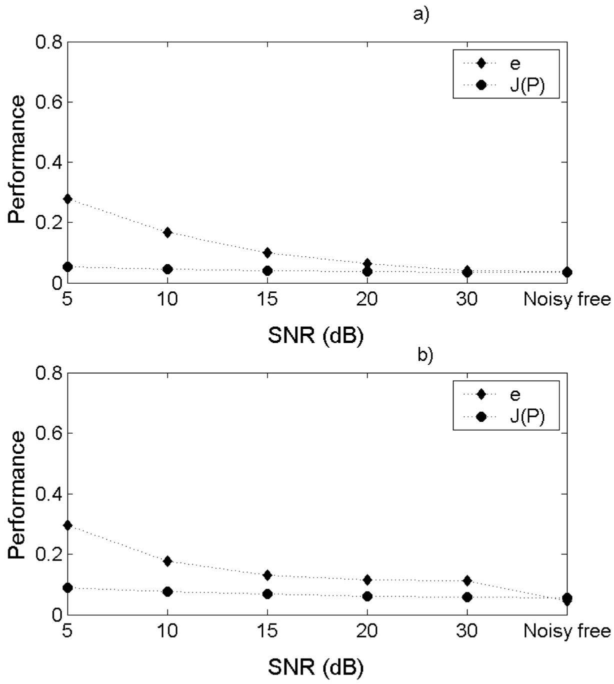

Fig. 2. The performance index $J(\mathbf{P})$ and the relative mean square error ε versus colored noise level for separation the a) Rossler signals and b) Mackey-Glass signals

Conclusions

In this paper the BSS algorithm based on nonlinear phase-space reconstruction, nonorthogonal joint approximate diagonalization of several time-delayed covariance matrices and nonlinear noise reduction are investigated by applying them to noisy mixed pseudoperiodic chaotic Rossler signals and Mackey-Glass signals. The time-delayed covariance matrices are estimated corresponding to the data matrix of first embedding dimension and data matrix of the every another embedding dimension. A high-dimensional system, i. e. overembedding, at the nonorthogonal joint approximate diagonalization stage and at the postprocessing – nonlinear noise reduction stage is used.

Simulation results show that algorithm is able to separate mixed pseudoperiodic chaotic or similar to pseudoperiodic signals, which have temporal structures and each source has non-vanishing temporal correlation, in the presence of a white Gaussian noise or stationary colored noise up to SNR=(5–10) dB.

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The blind source separation (BSS) algorithm based on nonlinear phase-space reconstruction, nonorthogonal joint approximate diagonalization (JAD) of several time-delayed covariance matrices and nonlinear noise reduction is investigated by applying it to noisy mixed pseudoperiodic chaotic Rossler signals and Mackey-Glass signals. The time-delayed covariance matrices are estimated corresponding to the data matrix of first embedding dimension and data matrix of the every another embedding dimension. Simulation results show that algorithm gives a good performance in the separation and denoising of mixed noisy signals in the presence of a white Gaussian noise or stationary colored noise up to SNR=(5–10) dB and can be applied to separation signals, that have non-zero autocorrelation function for a non-zero time lag, i. e. when analysis based on the second-order statistics (SOS) is applicable. Ill 2, bibl. 12 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас. «Слепое разделение» смеси псевдопериодических хаотических сигналов при наличии шумов // Электроника и электротехника. – Каунас: Технология, 2009. – № 3(91). – С. 31–34.

Исследуется алгоритм «слепого разделения источников» (*Blind Source Separation* – BSS), основанный на реконструкции фазового пространства, совместной приблизительной неортогональной диагонализации нескольких ковариационных матриц сигналов реконструированного фазового пространства, определенных с использованием матрицы данных первой меры и матрицы данных каждой другой меры фазового пространства, а также нелинейной фильтрации. Путем анализа смеси хаотических сигналов Росслера и сигналов Маккей-Гласс показывается, что алгоритм обеспечивает хорошее разделение и фильтрацию сигналов при наличии белого или стационарного цветного шума при отношении сигнал-шум выше (5–10) дБ и может применяться для разделения псевдопериодических хаотических или им подобных сигналов, когда каждый источник обладает ненулевой автокорреляционной функцией при ненулевом сдвиге, т. е. когда применимы статистики второго порядка. Ил. 2, библ. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Triukšmingų pseudoperiodinių chaotinių signalų atskyrimas „akluoju metodu“ // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 3(91). – P. 31–34.

Tiriamas “aklo šaltinių atskyrimo” algoritmas fazinės erdvės rekonstrukcijos, bendros apytikslės kelių rekonstruotos fazinės erdvės signalų kovariacijos matricų neortogonalios diagonalizacijos ir netiesinės filtracijos pagrindu. Kovariacijos matricos sudaromos pirmo rekonstruotos fazinės erdvės matmens duomenų matricos ir visų kitų rekonstruotos fazinės erdvės matmenų duomenų matricų pagrindu. Atlikti tyrimai su sumaišytais triukšmingais chaotiniais Rosslerio signalais ir Mackey-Glass signalais parodo, kad algoritmas įgalima gerai atskirti ir filtruoti signalus esant baltajam Gauso arba stacionariajam spalvotam triukšmui iki santykio signalas-triukšmas (5–10) dB ir gali būti naudojamas pseudoperiodiniams chaotiniams bei į juos panašiems signalams atskirti, kada kiekvienas atskiriamas šaltinis turi nenulinę autokoreliacinę funkciją prie nenulinio postūmio, t. y. kada galima taikyti antros eilės statistikas. Il. 2, bibl. 13 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).